Tenth Tbilisi Symposium on Language, Logic and Computation

Gudauri, Georgia

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Centre for Language, Logic and Speech

and

Razmadze Mathematical Institute at the Tbilisi State University

Institute for Logic, Language and Computation

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Georgia and the Tbilisi Symposia

The Symposium series is organized by the Institute for Logic, Language and Computation (ILLC) of the University of Amsterdam in conjunction with the Centre for Language, Logic and Speech and Razmadze Mathematical Institute at the Tbilisi State University. There have been nine installments of this series of biennial Symposia. The preceding meetings took place in the Georgian mountain resort Gudauri (1995), at the capital of Georgia Tbilisi (1997), in the Black see coastal resort Chakvi (1999), in the spa resort Likani situated in the Borjomi Canyon (2001), Tbilisi (2003), the Black sea resort Batumi (2005), Tbilisi (2007), the ski resort Bakuriani (2009), and in the second largest city of Georgia and the capital of the western region of Imereti Kutaisi (2011).

Georgia

Georgia is an ancient country situated between the Black Sea, the Kaspian Sea and the Caucasus Mountains. The is the country of the Golden Fleece, the myth of the Argonauts, Jason and Medea, and Promethee, who was chained to the Caucasus Mountains. Georgia is famous for its ancient language and writing, for its rich and diverse social, religious and artistic culture, for its exquisite cuisine and high quality wines, and for its cordial hospitality. Nowadays Georgia is also known for hosting the Tbilisi Symposia on Language, Logic and Computation.

Gudauri

Gudauri is the mountain resort developing in recent years on the southern slopes of the Greater Caucasus mountain range in Georgia. The resort is near Jvari Pass (2379 m above sea level) and in 43 km from the Kazbegi summit (5033 m), which is the second highest in Georgia. Gudauri lies on a small plateau, at the height of 1900-2200 meters above sea level, in some 120 km from Tbilisi. In winter it is the most popular skiing destination in Georgia. Outside the skiing season, Gudauri is often used as a convenient stopping point on the way to Kazbegi mountain by mountaineers and tourists. It offers great scenery, trekking and other outdoor activity opportunities.

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Programme

General programme grid

	Monday	Tuesday		Wednesday	Thursday	Friday
09.00-10.00	*Tutorial* Computation 1 Samsom Abramsky	*Tutorial* Logic 1 Rosalie Iemhoff		*Tutorial* Computation 2 Samson Abramsky	*Tutorial* Computation 3 Samson Abramsky	*Tutorial* Logic 2 Rosalie Iemhoff
Break						
10.15–11.15	*Tutorial* Language 1 Daniel Altshuler	*Tutorial* Language 2 Daniel Altshuler		*Tutorial* Logic 3 Rosalie Iemhoff	*Session* La-3 / Lo & Co-3 4 talks	*Tutorial* Language 3 Daniel Altshuler
Break						
11.30–12.30	*Invited lecture* Logic Alexandra Silva	*Workshop on aspect*	*Workshop on algebraic proof theory*	*Invited lecture* Language Galit Sassoon	*Invited lecture* Logic Agata Ciabattoni	*Invited lecture* Computation Balder ten Cate
Lunch						
14.00–15.30	*Session* La-1 / Lo & Co-1 6 talks			*Excursion*	*Session* La-4 / Lo & Co-4 6 talks	*Session* Lo & Co-6 2 talks
Break						
16.00–17.30	*Session* La-2 / Lo & Co-2 6 talks				*Session* La-5 / Lo & Co-5 6 talks	*Departure*
Break						
18.00–19.00	*Invited Lecture* Language Sergei Tatesov				*Invited Lecture* Computation Thomas Colcombet	

Workshop schedules

Programme workshop on aspect

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12.00–12.30	Ana Arregui & Maria Biezma *Cancelling Aspect-Based Counterfactuality Implicatures*
Lunch	
Session 2: Aspectual Composition (Chair: Daniel Hole)	
14.00–14.30	Berit Gehrke & Louise McNally *Event individuation by objects: Evidence from frequency adjectives*
14.30–15.00	Wiebke Petersen & Ralf Naumann *Towards a frame-based analysis of event structures and aspectual classes*
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Session 3: Aspectual Composition (Chair: Daniel Hole)	
15.30–16.30	Roumi Pancheva (invited speaker)
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Break	
17.00–18.00	Hans Kamp (invited speaker) *From 'today' to 'since this morning': how perfects combine with different types of temporal adverbials*
18.00–18.15	Questions
18.15–19.00	General discussion

Programme workshop on algebraic proof theory

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Lunch	
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15.00–15.30	Rostislav Horcik *Density Elimination and the Corresponding Algebraic Construction*
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16.00–17.00	Alessio Guglielmi (Invited) *Geometric Ideas in the Design of Efficient and Natural Proof Systems*
17.00–17.30	Michel Parigot *Atomic lambda-calculus: a typed lambda-calculus with explicit sharing extracted from deep inference*
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17.45–18.15	Norbert Preining *One monadic predicate symbol*
18.15–18.45	Bartosz Wieckowski *Graded Natural Deduction Rules*

Detailed schedule contributed talks

Monday	Session Logic & Computation	Session Language
14.00–15.30	Paula Henk	Kata Balogh
	Philippe Balbiani and Levan Uridia	Ralf Naumann and Wiebke Petersen
	Katsuhiko Sano and Minghui Ma	Matthijs Westera
16.00–17.30	Virginie Fiutek	Liana Lortkipanidze, Marina Beridze and David Nadaraia
	Vladimír Svoboda	Albert Ortmann
	Dominik Klein and Eric Pacuit	Paul Meurer
Thursday	Session Logic & Computation	Session Language
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	Dick de Jongh and Zhiguang Zhao	Camilo Thorne and Jakub Szymanik
14.00–15.30	Emil Jerábek	Gary Mar, Yuliya Manyakina and Amanda Caffary
	Jeroen Goudsmit	Kerstin Schwabe
	Philippe Balbiani and Cigdem Gencer	Isidora Stojanovic
16.00–17.30	Ayse Bölük, I.layda Ates, and Cigdem Gencer	Martin Aher
	Johannes Werner and Thomas Studer	Jeroen Groenendijk and Floris Roelofsen
	Marta Bilkova	Martin Aher and Jeroen Groenendijk
Friday	Session Logic & Computation	Session Language
14.00–15.30	Mikheil Rukhaia, Cvetan Dunchev, Alexander Leitsch and Daniel Weller	Maria Spychalska
	Stefan Hetzl, Tomer Libal, Martin Riener and Mikheil Rukhaia	Maarten Janssen
	Vit Puncochar	Natia Dundua, Lela Samushia and Armin Hoenen
16.00–17.30		Ondrej Majer
		Sicco Pier Van Gosliga and Frans Groen

Workshops

Workshop on aspect

Invited lectures

Cross-linguistic variation in the perfect from the perspective of the Perfect Time Span theory
Roumyana Pancheva

Variation among the perfects in different languages presents a challenge for the original Result State and Extended Now / Perfect Time Span theories of the perfect. But rather than abandon the idea of a uniform semantics for the perfect, the theories have sought to make modifications that allow for other grammatical factors, themselves subject to cross-linguistic variation, to interact with the perfect.

In this talk I will focus on the cross-linguistic variation with respect to the availability of different perfect interpretations (universal and different kinds of existential readings) and with respect to the interaction of the perfect with present and past positional adverbials like today and yesterday. I will suggest that the Perfect Time Span theory is naturally suited to account for the variation in both domains. In treating the perfect as a relative tense, the Perfect Time Span theory can integrate naturally with an independent theory of viewpoint aspect, and derive variability in perfect interpretations through constraints on the viewpoint aspects that can combine with the perfect and temporally locate the underlying eventuality relative to the Perfect Time Span. A further modification of the theory that allows for cross-linguistic variation in the location of the Perfect Time Span relative to the time interval determined by tense can naturally account for the variati on with respect to the interaction of the perfect and temporal adverbials like today and yesterday.

Ultimately, this argument for the Perfect Time Span theory may turn out to be primarily conceptual, if the Result State theory can also be suitably modified to capture the cross-linguistic facts, but hopefully it will highlight the question of how a theory of the perfect should integrate in a larger theory of temporality.

From 'today' to 'since this morning': how perfects combine with different types of temporal adverbials
Hans Kamp

Simple result state accounts of the perfect predict that the sentence 'I have submitted my Abstract today' can be true when the submission took place before today. (And some such accounts even predict that the sentence can be true **only** when the submission took place before today.) I take it that this predication is empirically false. So the result state approach must either be abandoned or it must be modified in such a way that this wrong prediction is no longer made. I have, for some considerable period of time, been among those who have advocated a simple result state account. In this talk I will outline a modification of that account in which the unacceptable predictions are no longer made. The main feature of this modified form of the result state theory is that the English perfect, and also perfects in other languages are treated as 'feature shifting operators', where the different 'features' that are shifted have to do with the temporal localisation of eventualities by tenses and temporal adverbs, respectively.

In the second part of the talk I will be looking specifically at the interaction of perfects with interval-denoting adverbials such as 'since'-phrases and clauses and phrases of the form 'in the course of/during the last three hours'. In this way I hope to set the stage for a comparison between the account I will outline and the perfect time span accounts of, among others, Iatridou et al. and Rothstein.

Much of the talk will be based on an unfinished monograph 'Perfects as Feature Shifting Operators' (Kamp, Reyle & Rossdeutscher).

Contributed talks

Cancelling aspect-based counterfactuality implicatures

Ana Arregui and Maria Biezma

Introduction

This paper investigates the cancellation of counterfactuality in backtracking Anderson-style perfect *would*-conditionals. We observe that, contrary to the plain cases, it is not possible to cancel aspect-based counterfactuality implicatures in Anderson backtrackers. We explain this in terms of differences in the way conditional strengthening inferences are derived, arguing that while cancellation is possible in cases of discourse-driven strengthening, general language principles of *discourse rationality* and *pragmatic economy* make it impossible in case of semantically-driven strengthening. The proposal puts together an antipresupposition view of counterfactuality in perfect *would*-conditionals based on aspectual composition, two types of conditional inferences and an epistemic analysis of backtrackers.

Preliminaries

Counterfactuality

The (plain) Anderson-example in (1) shows that *would*-conditionals with perfect antecedents do not presuppose the falsity of the antecedent (i.e. counterfactuality) [1].

(1) **A**: When would have Jones shown these symptoms?
B: If Jones *had taken* arsenic, he would have shown just exactly those symptoms which he does in fact show. So, he probably took arsenic/(#So, he took arsenic).

We show in (2) that this contrasts with 'backtracking' Anderson-examples (i.e. the consequent temporally precedes the antecedent), where trying to cancel counterfactuality results in infelicity:

(2) **A**: #If Jim had asked Jack for help, there would have to have been no quarrel yesterday. And there was no quarrel yesterday. So it is likely/possible that Jim asked Jack for help.

Antipresuppositions Given (1), it has been proposed that counterfactuality be treated as an implicature triggered by aspectual composition (e.g. [12, 13, 14]). We follow [14] in treating counterfactuality as an 'antipresupposition' derived from the need to 'Maximize Presuppositions' (see a.o. [16, 7, 18]).

Backtracking : [15] noted that backtrackers were mostly false, but could be helped with strong contextual support or 'special' syntax (i.e. a double layer of auxiliaries as in (2)). Proposals in the literature have made sense of backtrackers in terms of an 'extra' layer of (epistemic) modality (e.g. [8]) or best-explanation ([5]). We follow [3, 2] in claiming that in backtrackers like (2) the consequent carries an extra layer of modality and is understood as the *expected* conclusion (considering the way people usually behave): *in the most similar worlds in which Jim asked Jack for help, the 'normal' / expected thing would have been for there not to have been a quarrel* (the actual facts, of course, could lead to a departure from normality). The additional auxiliary is interpreted as a (universal) epistemic modal, triggering an interpretation based on epistemic expectations/laws: in smooth backtrackers, the consequent is understood as the *normal* precursor for the antecedent.

Conditional strengthening (CS) Building on [9] we argue that CS is a discoursive implicature: CS arises when a conditional *if* α, β leads the addressee to infer that β is brought about only in situations. In particular, CS arises when participants understand that the utterance of the conditional constitutes an exhaustive answer to the (implicit) question under discussion (QuD) ([6, 17]) regarding the consequent (*when* β?). [Note that the presumed speaker's authority as well as the QuD (tracking speaker's intentions) affect the availability of CS (e.g. A: *When would you give me 5\$?* B: *If you mow the lawn, I'll give you 5\$* = by CS 'Only if you mow the lawn I'll give you 5\$' [9, 10]).]

Counterfactuality in plain and backtrackigng Anderson examples

Case 1: plain Anderson-style examples (1) provides various pieces of information: (i) the conditional structure tells the addressee that all relevant α-situations are β-situations ($\alpha \to \beta$); (ii) the aspectual make-up gives rise to theimplicature that the antecedent is not true (antipresupposition); (iii) the discourse situation leads to the implicature of conditional strengthening (cs) that *only* α *situations are* β *situations* ($\neg\exists\gamma, \gamma \neq \alpha, / \to \alpha : \gamma \to \beta$) ([11, 9] a.o). **Why does CS matter?**: As a corollary from (i)-(iii), the addressee also obtains (**iv**) the implicature ($\rightsquigarrow$) that the consequent is not true ($\rightsquigarrow \neg\beta$) (unavailable without CS). But the B-speaker also asserts β, hence cancelling $\rightsquigarrow \neg\beta$. Since $\rightsquigarrow \neg\beta$ is cancelled, the conclusion is that either α is true (cancels counterfactuality) *or* that CS is false. So, as far as we know, α might be true. (5) details cancellation options when if α, β is followed by β:

$$\left(\begin{pmatrix} (i) & \alpha \to \beta \\ (ii) & \rightsquigarrow \neg\alpha \\ (iii) & \rightsquigarrow \end{pmatrix} \quad (iv) \quad \rightsquigarrow \neg\beta \right) \quad + \beta \text{ hence } \begin{Bmatrix} \text{cancel } \rightsquigarrow \neg\alpha \\ \text{or} \\ \text{cancel } \rightsquigarrow \text{CS} \end{Bmatrix} \tag{5}$$

Case 2: Backtracking Anderson-style examples (2) provides various pieces of information: (i) all the relevant situations in which John asked for help are necessarily situations in which there was no quarrel yesterday (epistemic analysis of backtrackers = $\alpha \to \Box\beta$); (ii) the inference that, given what we know, only in situations in which there had been no quarrel yesterday, would John have asked Jim for help today (notated: *only if*$_{EP}$ $\beta \to \alpha$, this follows from (i)); (iii) the antipresupposition (from the aspectual make-up). As a corollary from (i)-(iii): (iv) the implicature that the consequent is not true ($\rightsquigarrow \neg\beta$). The "strengthening" in (ii) is determined semantically by the epistemic resolution of the backtracker and not as a discourse-inference: given what is expected/normal/known, it is only in β-situations that α is true (an entailment, not really 'strengthening'). If we attempt Anderson-reasoning in backtrackers by claiming that β, the conclusion is that α is unequivocally true: since "strengthening" in backtrackers cannot be cancelled because it is semantically encoded, the only way to make sense of the fact that β is true is to cancel the aspect-based implicature $\rightsquigarrow \neg\alpha$ (i.e., we conclude that is true). (8) details cancellation options for *if* $\alpha, \Box\beta$ followed by β:

$$\left(\begin{pmatrix} (i) & \alpha \rightarrow \Box\beta \\ (ii) & (\text{only if}_{\text{EP}}\ \beta \rightarrow \alpha) \\ (iii) & \rightsquigarrow \neg\alpha \end{pmatrix} \quad (iv) \quad \rightsquigarrow \neg\beta\right) \quad + \beta \text{ hence } \alpha \tag{8}$$

Conclusion

If a backtracking Anderson-example if $\alpha, \Box\beta$ is followed by β, we are forced to conclude that α is true. The sequence is infelicitous for two reasons: (i) the speaker has available syntactically and semantically simpler constructions to claim that α is true (*pragmatic rationality*); (ii) the speaker is not justified in making the discourse move of using a perfect *would*-conditional (specialized in marking $\rightsquigarrow \neg\alpha$) to end up claiming that α is true (*discourse rationality*). The addressee does not agree to cancel aspect-triggered counterfactuality implicatures because that would amount to accepting a *pragmatic contradiction*: accepting first a move to mark α as false and then a move to mark α as true. But if a plain Anderson-example *if* α, β is followed by β, we need only conclude that is possible (we have the choice to cancel CS).

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Deriving the readings of French être en train de
Bridget Copley and Isabelle Roy

The traditional treatment of French *être en train de (êetd)* as a progressive faces two problems. The first (e.g. Binnick, 1991, Lachaux 2005) is that cross-linguistically "progressives" do not have exactly the same meaning: cf. English *It's raining* vs. *#Il est en train de pleuvoir*. Ongoing meanings in French are usually expressed with the French simple present. In this paper we address a second problem with the idea that *êetd* is a (mere) progressive: namely, the fact that *êetd* has a prominent reading (Lachaux 2005) with an apparent malefactive meaning, in which the speaker seems to express a negative attitude toward the described eventuality. Although the meaning of *êetd* is sometimes "bleached" (i.e., *êetd* contributes no meaning other than the progressive), where the simple present would be ambiguous, as in (1a), its usual meaning – as evidenced by the first 10 pages of Google search results (12/5/12) – is apparently malefactive, as in (1b-e), where the speaker is expressing a negative opinion about the eventuality in progress. The contrast in (2) makes the same point.

(1) a. Quand je pense moi, je cours. Je veux dire quand je m'imagine, je suis en train de courir. 'When I think about myself I run. I mean, when I imagine myself, I am running. `http://www.ktha.org/jsup.php`

b. Je suis prêt à donner de mon temps, mais je ne suis pas un magicien. La boxe française est en train de mourir. 'I'm ready to give of my time, but I am no magician. French boxing is dying.' `http://www.lemonde.fr/sport/article/2012/11/15/j-ai-hontepour-la-boxe_1791387_3242.html`

c. Comment Google est en train de changer nos villes 'How Google is changing our cities' `http://www.urbamedia.com/comment-google-est-en-train-de-changer-nos-villes`

d. Il est en train de pleuvoir sur les pistes et la route est vraiment mauvaise. 'It's raining on the roads and the route is really bad.' `http://lespradeaux.over-blog.com/article-mardi-20decembre-a-8h-93353168.html`

e. Le capitalisme est en train de s'auto-détruire 'Capitalism is self destructing' `http://www.alternatives-economiques.fr/le-capitalisme-est-en-train-de-s-autodetruire-par-patrick-artus-et-marie-paule-virard_fr_art_195_22478.html`

(2) a. Qu'est-ce que tu fais ?
what-is-it that you do
'What are you doing?'

b. Qu'est-ce que t'es en train de faire ?
what-is-it that you-are en train de do.inf
'What (the hell) are you doing (/ up to)"

Our proposal: The at-issue meaning of *êetd* applied to a proposition p is that of a progressive (leaving unspecified the exact nature of cross-linguistic aspectual variation amongst progressives). In addition it has a desiderative/expectation conventional implicature (CI; Potts, 2005). In this CI, a salient entity *wants* or *expects* a certain outcome q. The apparently "malefactive" flavor arises when a *wanted* outcome q is different from the natural outcome p of the eventuality in progress. These two outcomes p and q are not specified in the semantics to be different, but are usually assumed to be different (and indeed incompatible) through a Gricean assumption of maximal informativity. We adapt Portner's (1998) denotation of the progressive as the at-issue meaning of *êetd* ("NI" refers to "noninterruption"). We define **salient**(c) as a function that picks a salient entity in the context of utterance c; a CI of *êetd* then says that the salient entity *wants* or *expects* a proposition q:

(3) a. $[[\ \hat{e}etd\ ]]^{g,c} = \lambda p \lambda s\ .\ \forall w \in$ BEST(Circ, NI, s, p): [$\exists s'$ such that $s' \subset w$ and s is a non-final part of s' : [$p(s')$]]

b. CI: $\exists q$: [$\forall w \in$ BEST(desire/expect(**salient**(c)), NI, s, q) 's in s : [$\exists s'$ such that $s' \subset w$ and s is a non-final part of s' : [$q(s')$]]

The malefactive judgment thus does not reflect true malefactive semantics. To see this, note that the salient entity may have contradictory desires, wanting both p and q; in that case *êetd* is still felicitously used, but the malefactive flavor is not as prominent. For instance, suppose we tell Pierre to paint the walls even though we also want (incompatibly) to have lunch. *êetd* can still be used even though we want Pierre to paint the walls, where a real malefactive would not be expected to be used (cf. (4a,b)). As expected from this account, this "obstructed goal" construal also occurs when someone other than the speaker wants q, but q is impossible since p is ongoing (again, p is assumed incompatible with q) as in (4c):

(4) a. #We can't have lunch, Pierre is painting the walls on us.

b. On ne peut pas déjeuner, Pierre *est en train de* peindre les murs.
we neg can neg lunch.inf Pierre is en train de paint.inf the.pl walls

c. Est-ce que je peux parler à Jean? - Non, #il se brosse/est en train de
is-it that I can talk to Jean no he refl brush/is en train de se brosser les dents.
refl brush.inf the.pl teeth
'Can I talk to Jean? No, #he brushes/is brushing his teeth.'

The *bleached* reading, as in (1a) above, might *prima facie* look as though it lacks the CI. But if this were the case, we would expect *êetd* to be used quite regularly as a real progressive; it is only used in circumstances where the simple present would be ambiguous. We argue that the bleached reading retains the CI, but that in these cases the speaker has reason not to be maximally informative in introducing a new existentially-quantified proposition q; i.e., *p is allowed to equal q.* We defend the axioms in (5a,b) (calling on discussion in Heim, 1992 for (5a)) to motivate the paradigm in (6); also cf. Kearns, 1991 and Martin, 2006 for "interpretive" construals as in (6b,e), where q is a redescription of the situation described by p.

(5) a. if x wants p, x neither expects that p nor expects that not p

b. if x asserts *êetd* p, x expects that p

(6) a. CI = x wants q, p incompatible with q
malefactive/obstructed goal (cf. (1b-e, 2, 4)

b. CI = x wants q, p compatible with q but not equal to q
interpretive-want (cf. (7b))

c. CI = x wants q, p equal to q
impossible due to (5a) (cf. (7a))

d. CI = x expects q, p incompatible with q
impossible due to (5b)

e. CI = x expects q, p compatible with q but not equal to q
interpretive-expect (cf. (7c))

f. CI = x expects q, p equal to q

bleached (cf. 1a)

(7) a. #Dieu merci, il est enfin *en train de* pleuvoir sur mon jardin!
God thanks it is finally en train de rain.inf on my garden
'Thank God, it's finally raining on my garden!'

b. Si tu fais ça, tu es *en train de* me rendre un grand service.
if you do that, you are en train de me render a big favor
'If you (are) do(ing) that, you are doing me a big favor'

c. [Si tu fais ça,] tu es *en train de* croire au Père Noël. (Martin 2006)
if you do that you are en train de believe to.det Father Christmas
'[If you (are) do(ing) that,] you are believing in Santa Claus.'

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Event individuation by objects: Evidence from frequency adjectives

Berit Gehrke and Louise McNally

Though frequency adjectives (FAs) commonly distribute over events and specify the frequency with which an event of a particular type occurs (see Bolinger 1967, Stump 1981, Larson 1998, Zimmermann 2003, Schäfer 2007), Schäfer (2007) observes that the so-called adverbial reading, illustrated in (1), is not available across the board. In particular, in combination with non-event nouns it is only available for FAs that express infrequent distribution (e.g. *occasional*, (1a)) but not for those that express (absolute or relative) frequent distribution (e.g. *daily, frequent*, (1b)).

(1) a. An occasional sailor strolled by. = Occasionally, a sailor strolled by.
b. A frequent sailor strolled by. $\neq$ Frequently, a sailor strolled by.

However, Gehrke & McNally (2011) show that there is a systematic exception to this generalization when FAs like *frequent* combine with non-event nouns in certain argument positions of certain verbs, for example with the theme of verbs of creation and consumption ((2)), though not for incremental theme arguments more generally ((3)).

(2) a. She wrote me frequent letters. = Frequently, she wrote me letters.
b. She baked frequent batches of cookies. = Frequently, she baked batches of cookies.
c. She drank frequent cups of coffee. = Frequently, she drank a cup of coffee.

(3) a. ??She read frequent books to her mother.
b. ??She mowed frequent lawns.
c. ??She baked frequent potatoes.

(3c) provides striking evidence that some aspect of creation is relevant, as the transformation reading of *bake* does not license the adverbially used FA. Interestingly, additional conditions apply: Gehrke & McNally note the contrast between (2b) and (4a), which they suggest is due to the fact that cookies are stereotypically baked in batches, not individually – contrast (4b).

(4) a. ??She baked frequent cookies.
b. She baked frequent cakes.

However, they do not provide any analysis of these exceptional cases. The goal of this paper is to do exactly this. We propose that an adverbial reading with FAs expressing frequency is possible when the atoms in the denotation of the (plural) entity described by the DP containing the FA are strictly homomorphic to unique events describable by the verb.

In order for a strict homomorphism to be possible, the relevant argument of the verb has to satisfy the property that Krifka (1998) termed *strict incrementality* (SINC). According to Krifka, SINC only holds if mapping to subobjects, uniqueness of objects, mapping to subevents, and uniqueness of events hold ((5i), see Krifka 1998 for formal definitions); an additional condition has to be met to exclude situations in which both the object x and the event e are atomic ((5ii)). This condition excludes instantaneous events with no discernible subevents, such as *make a dot.*

(5) Θ is *strictly incremental*, SINC(Θ) iff
i) MSO(Θ) $\wedge$ UO(Θ) $\wedge$ MSE(Θ) $\wedge$ UE(Θ)
ii) $\exists x, y \in U_P \exists_e, e' \in U_E[y < x \wedge e' < e \wedge \Theta(x,e) \wedge \Theta(y,e')]$ (Krifka 1998: 213, ex. (51))

For example, the theme of the verbs in (2) can participate in such events only once (uniqueness of events and uniqueness of objects); this distinguishes these verbs from nonincremental themes in general (e.g. themes of verbs like *sell* or *see*). Mapping to subevents and mapping to subobjects distinguish created and consumed objects with other incremental themes, for example the theme of *read* in (3a): one can reread parts of a book, skip back and forth, and so on. Finally, the difference between *write letters* in (2a) and *bake cookies* in (4a) is that the theme in a letter writing event is usually atomic, whereas that in cookie baking is usually not atomic (that is, one usually bakes a plurality of cookies all at once). Just in case all of these conditions are met, distributing over the entities described by the nominal containing the FA automatically guarantees the proper temporal distribution over the corresponding events that the FA lexically entails.

Our analysis builds directly on Kennedy's (2012) treatment of incremental theme verbs like *eat* with DPs containing measure phrases (e.g. *eat half the cake*). Like him, we take verbs of creation and consumption to denote a plurality of events ((7a)). We take plural nouns to denote pluralities as well, and the FA to denote a property of pluralities of events ((7b-c), respectively).

Though the noun and the FA are not necessarily of the same semantic sort, we assume that, as with other cases of modification, their composition is mediated by a contextually-valued relation R between the entities described by the noun and the events described by the FA ((7d); cp. the mediating role of qualia in Pustejovsky 1995's Selective Binding). To guarantee that the free event argument in the DP representation is identified with that contributed by the verb, we again follow Kennedy and appeal to Kratzer's (1996:122) Event Identification rule ((6), where ϵ is the type of events).

(6) Event Identification: If α is a constituent with daughters β, γ, such that $[[\gamma]]$ is type $\langle e, \tau \rangle$, and $[[\gamma]]$ is type $\langle e, \langle \epsilon, \tau \rangle\rangle$, then $[[\alpha]] = \lambda x \lambda e [[\beta]](e) \wedge [[\gamma]](x)(e)$.

When R is valued as the thematic role borne by x in e and R satisfies SINC (**SINC**(**Th**) in (7e)), the adverbial reading will arise.

(7) a. bake: $\lambda e.\mathbf{bake}(e)$
b. cakes: $\lambda x.\mathbf{cakes}(x)$
c. frequent: $\lambda e.\mathbf{frequent}(e)$
d. frequent cakes: $\lambda x.[\mathbf{cakes}(x) \wedge \mathbf{frequent}(e) \wedge R(x, e)]$
e. bake frequent cakes: $\lambda x \lambda e.[\mathbf{bake}(e) \wedge \mathbf{cakes}(x) \wedge \mathbf{frequent}(e) \wedge (\mathbf{SINC}(\mathbf{Th}))(x, e)]$

Once the x argument is existentially closed, the event described in (7e) is a plurality with discernible subevents (of baking one cake) distinguished by the atomic subobjects of the plurality described by its theme (i.e. each cake). The distribution of this plurality of events is described by the FA. If R is given some other value or the event arguments are not identified, some other interpretation (perhaps as in *Sue read a daily newspaper*) or anomaly will result.

In addition to solving the puzzle raised by the data in (2)-(3), this work points to an interesting parallel between Kennedys analysis of the way measure phrases in DP can measure out events and the way in which FAs within DP can effect distribution over events, underscoring the generality of argument-mediated individuation of events.

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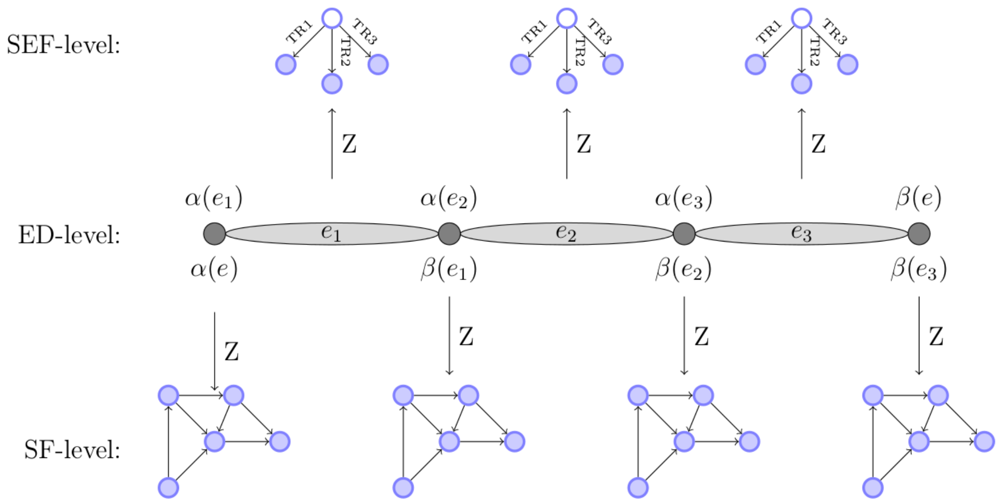
Towards a frame-based analysis of event structures and aspectual classes

Ralf Naumann and Wiebke Petersen

According to Barsalou, frames as recursive attribute-value structures are the fundamental format of conceptual knowledge in human cognition [1]. In [6] a formal account on frames for nominal concepts is given which represents them by graphs build up of attributes as transition functions between nodes. Nodes represent objects and their respective attribute values. The account has been proven useful in representing concepts belonging to different static concept types and in modeling their compositional semantics. Naumann extends this approach to capture dynamic concepts of actions and events [5]. In our talk we will demonstrate how the extended dynamic frame approach enables the modeling of event classes and aspectual composition. Closely following [5], we propose a multi-level model for event structures:



On the event decomposition level (ED-level), events can be decomposed into subevents which are not necessarily of the same event type. This level represents the temporal structure of the event and links it to the level of the described situation, the participating objects, and their thematic roles in the event. The linking between the ontologically different levels is formalized by a *zoom* function (Z) or bridge in the sense of Blackburn and De Rijke [2].

The model allows one to zoom into different aspects of an event: The static event frame (SEF) is closely related to Fillmores frames [4] and represents the links between the event and its participants by thematic role attributes. These links are usually static throughout the occurrence of a simple event. In order to analyze changes caused by an event one has to zoom into the situations at its beginning and end, i.e., at the left and right boundary of the event $(\alpha(e), \beta(e))$. These situations are like snapshots and therefore represented by static situation frames (SF), which are composed of object frames and relations between them. The object frames can be considered as refinements of the thematic-role-attributes in the SEF (not illustrated in the figure). Hence, each event or subevent is related by bridges to one static event frame (SEF) and two (static) situation frames (SF).

The conditions an event presupposes on these three static frames determine the type of the event. E.g., a *widening* event involves two situation frames which both contain an object frame as a subframe. The object frames differ with respect to the value of the attribute width. The boundary situation frames of an *arriving* event are composed of two object frames (one for the moving object and one for its destination). They differ with respect to the values of their attributes position which must be co-referential at the end of the event. For stative events or states like knowing the boundary events will be linked to identical situation frames.

In the talk we will elaborate our frame-based approach to event structures and demonstrate its benefits for the analysis of the semantics of aspect. We will focus on how Vendler's classical four aspectual classes, i.e., *states, activities, accomplishments*, and *achievements* (cf. [7]) and the class of *semelfactives* or *punctual verbs* (cf. [3]) can be analyzed by event types which are defined by zooming into the event up to the basis of the static event and situation frames.

Event types are defined in terms of liveness and safety properties from Temporal Logic. For example, dynamic events are characterized by liveness properties: a condition that does not hold at the beginning of the event comes to hold only at its end. By contrast, non-dynamic events are defined in terms of safety properties: a condition holding at the beginning of the event holds from some non-final subevent until the end of the event. Telicity corresponds to liveness-properties of sequential decompositions of events, while atelicity corresponds to safety-properties of such decompositions. In a fixed-point extension of Temporal Logic, liveness-properties can be defined in terms of least fixed points (LFP, LF-operator: μ) and safetey-properties in terms of greatest fixed points (GFP, GFoperator: ν):

$$\begin{aligned} \mu u(N\phi \vee N(\neg\phi \wedge u)) &\quad \text{liveness-property} \\ \nu u(N\phi \wedge Nu) &\quad \text{safety-property} \end{aligned}$$

The possibility to zoom in and out off the different representational levels in our model of event structures provides the flexibility that is needed to account for aspectual composition: object frames zoom into the inner structure of arguments which are linked to verbs by thematic roles in the SEFs. Therefore, compositional constraints and coercion effects can be expressed on their appropriate levels.

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Workshop on algebraic proof theory

Invited lectures

Reductive Algebraization
Matthias Baaz

The algebraization of first-order logic is one of the most important but also one of the most difficult problems in modern logic.In this lecture we relate this problem to compactness phenomena, in proof theoretic terms to the validity of (variants of) the Herbrand Theorem. Conversly, (variants of) the herbrand theorem can be used to reduce first-order problems to (then algebraic) propositional problems and provide interpolation problems as example.

Geometric Ideas in the Design of Efficient and Natural Proof Systems
Alessio Guglielmi

We aim at improving the representation of formal proofs by adopting very simple topological notions. Despite their simplicity, they seem to capture the essence of several proof theoretic notions.

The picture that emerges is that at least two of the fundamental compression mechanisms of proof systems, namely the cut and substitution, correspond to logic independent features of a certain class of graphs. This is interesting because a propositional formalism with those two mechanisms is so powerful that some conjecture it is an optimal proof system.

So, we are in a position to design a proof formalism, inspired by those topological notions, and whose proofs are as small as those in extended Frege, with minimal bureaucracy (in the sense of Girard) and with the usual proof-theoretic properties. I will illustrate the current candidate formalism, which is an extension of open deduction.

Herbrand's Theorem via Hypercanonical Extensions
Kazushige Terui

We study Herbrand's theorem for substructural logics from an algebraic perspective. The basic observation is that a substructural logic satisfies Herbrand's theorem for existential formulas if the corresponding variety is closed under compact completions. Although there are extensively studied compact completions in the literature, namely canonical extensions, they do not always work well for substructural logics. Motivated by this, we introduce new compact completions, called hypercanonical extensions. Our main result is that every variety of FLew algebras defined by P3 identities in the substructural hierarchy is closed under hypercanonical extensions, so that the corresponding logics satisfy Herbrand's theorem for existential formulas.

Contributed talks

Density elimination and the corresponding algebraic construction

Rostislav Horčík

The uninorm logic UL is the semilinear extension of the full Lambek calculus with exchange FL_e, i.e., it is a logic complete with respect to the class of all FL_e-chains (see [3]). Moreover, the logic UL is known to be standard complete, i.e., it is complete with respect to the class of all FL_e-chains whose universe is the real unit interval $[0, 1]$. Nevertheless, there is no algebraic proof of the above fact. The only proofs we have so far are based on a proof-theoretical elimination of the density rule [3, 1].

Interestingly, the proof-theoretical idea from [1] can be translated via residuated frames into an algebraic construction showing that UL is standard complete. This is possible since the residuated frames (introduced in [2] as a relational semantics for substructural logics) are tightly connected with the Gentzen sequent calculus. Nevertheless, the algebraic construction obtained via residuated frames is not very transparent. In this talk we will show how to describe this construction in a more transparent way using machinery of idempotent semirings and formal power series over them.

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One monadic predicate symbol

Norbert Preining

We will show that the logics based on well-ordered linear Kripke frames with constant domains can already be separated by the fragments of one monadic predicate symbol.

These investigations started from a very simple question:

> How much can we express with one monadic predicate symbol over linear order?

More specifically, considering logics over linear Kripke frames with constant domains, we asked ourselves how many separate fragments of one monadic predicate symbol exist. Very early guesses were as low as 4 ("What can we express more than infima and suprema and their order?"). It soon turned out that this was a over-simplification, but although we have now a much better view of the expressive power of monadic predicate symbols over linear oders, we are still far from a full understanding.

Separation is achieved by expressing infima of arbitrary finite degree, and their orders. While this method is currently restricted to the well-ordered case, it is straight forward to be extended to the case of dually well ordered Kripke frames in case the Baaz Delta operator is present. Without the Delta operator we expect difficulties.

Graded Natural Deduction Rules
Bartosz Wieckowski

Subatomic systems [3] make proofs of atomic sentences and the study of their component structure accessible to methods of structural proof theory (such as normalization) and allow to explain the proof-theoretic semantics of atomic sentences and their components by appeal to these methods. In this respect these systems arguably differ from atomic systems (or bases) used in [2]. We present a system of subatomic natural deduction which maintains graded introduction and elimination rules for atomic sentences and logical operators. Such rules determine, by appeal to algebraic methods (see, e.g., [1]), how the degree of derivability of a conclusion depends on the derivability degrees of the premisses.

Let $\mathcal{L}$ be a first-order language defined in the usual way. We take $\mathcal{C}$ and $\mathcal{P}$ to be the sets of nominal (or individual) constants and predicate symbols of $\mathcal{L}$, respectively. We let Atm be the set of atomic sentences of $\mathcal{L}$ and define $Atm(\alpha) = \{A \in Atm\colon A$ contains at least one occurrence of $\alpha\}$, for each $\alpha \in \mathcal{C}$; and $Atm(\varphi^n) = \{A \in Atm\colon A$ contains an occurrence of $\varphi^n\}$, for each $\varphi^n \in \mathcal{P}$. A *subatomic system* $\mathcal{S}$ is a pair $\langle \mathcal{I}, \mathcal{R}\rangle$ whose first element, the *subatomic base*, is a triple $\langle \mathcal{C}, \mathcal{P}, v\rangle$, with v a map such that $v : \mathcal{C} \to \wp(Atm)$ where $v(\alpha) \subseteq Atm(\alpha)$; and $v : \mathcal{P} \to \wp(Atm)$ where $v(\varphi^n) \subseteq Atm(\varphi^n)$. For any $\tau \in \mathcal{C} \cup \mathcal{P}$, we put $v(\tau) = \tau\Gamma$ and call $\tau\Gamma$ the *term assumption* for τ. The second member of $\mathcal{S}$ is a set of *rules for the introduction and elimination of atomic sentences*. Intuitively, the introduction rule allows to pass from term assumptions to an atomic sentence in case the sentence is contained in the intersection of the term assumptions of *all* the terms of which the sentence is composed. The elimination rule, by contrast, allows to eliminate an atomic sentence by means of a transition from that sentence to the term assumptions for the terms of which it is composed, where the assumptions are singletons which contain the eliminated atomic sentence. The *graded* introduction and elimination rules for atomic sentences to be included in the graded subatomic system relax the above restriction on intersection. The rules are:

$$\frac{\varphi^n\Gamma \quad \alpha_1\Gamma \quad \dots \quad \alpha_n\Gamma}{\varphi^n\alpha_1\dots\alpha_n}\, at\mathrm{I};\, k \qquad\qquad \frac{\dfrac{}{\varphi^n\alpha_1\dots\alpha_n}\, k}{\varphi^n\Delta \quad \alpha_1\Delta \quad \dots \quad \alpha_n\Delta}\, at\mathrm{E};\, k$$

where k is the number of term assumptions that contain $\varphi^n\alpha_1\dots\alpha_n$ and for each term $\tau \in \mathcal{C}\cup\mathcal{P}$, $\tau\Delta$ contains at most $\varphi^n\alpha_1\dots\alpha_n$. Given that there are $n+1$ (with $n \geqslant 1$) term assumptions for the atomic sentence A introduced by (atI; k), we may say that A is derivable to degree k with $0 \leqslant k \leqslant n+1$ in case it is contained in the intersection of k term assumptions for the terms of which A is composed. Accordingly, in case $k = n+1$, atom A will be derivable to the *maximal degree for* A and to the minimal degree for A in case $k = 0$. Let $D(\mathcal{S})$ be the set of atomic sentences derivable in the subatomic system $\mathcal{S}$ and let $d(C) = k$, for any $C \in D(\mathcal{S})$, be the degree of derivability of atom C. We call $d(A)$ such that $d(A) \geqslant d(C)$, for any $A, C \in D(\mathcal{S})$, the *maximal degree of derivability for* $\mathcal{S}$; and we call $d(A)$ such that $d(A) \leqslant d(C)$, for any $A, C \in D(\mathcal{S})$, the *minimal degree of derivability for* $\mathcal{S}$. We use $\top^\circ$ for the former and $\bot^\circ$ for the latter degree. The rule (atE; k) can be regarded as the *inverse* of (atI; k). This idea, expressed in the graded subatomic inversion principle, can be cast into a simple subatomic normal form theorem by adapting the observations in [3]. Graded introduction and elimination rules for the *logical operators* are set up in such a way that the derivability degree a formula has as a premiss in the introduction rule for an operator is exactly the degree the formula has as a conclusion of the elimination rule for that very operator. The rule for implication of the system to be considered, for instance, is:

$$\dfrac{\dfrac{\dfrac{}{[A]^{(i)}}\, l \quad \vdots \quad \dfrac{}{B}\, m}{A \supset B} \supset\mathrm{I},\ i\ ;\ k \qquad\qquad \dfrac{\dfrac{}{A \supset B}\, k \qquad \dfrac{}{A}\, l}{B} \supset\mathrm{E};\ m}{}$$

where $k = \top^{\circ}$ if $l \leqslant m$ and $k = m$ if $l > m$. Properties of the graded system will be discussed as well as applications to natural reasoning (e.g., with degrees of confidence, intensional transitive verbs).

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Conference abstracts

Tutorials

Contextual Semantics: From Quantum Mechanics to Logic, Databases, Constraints, Complexity, and Natural Language Semantics

Samson Abramsky

Quantum Mechanics presents a disturbingly different picture of physical reality to the classical world-view. These non-classical features also offer new resources and possibilities for information processing. At the heart of quantum non-classicality are the phenomena of non-locality, contextuality and entanglement. We shall describe recent work in which tools from Computer Science are used to shed new light on these phenomena. This has led to a number of developments, including a novel approach to classifying multipartite entangled states, and a unifying principle for Bell inequalities based on logical consistency conditions. At the same time, there are also striking and unexpected connections with a number of topics in classical computer science, including relational databases, constraint satisfaction, and natural language semantics. This is ongoing work, with a number of collaborators including Adam Brandenburger, Lucien Hardy, Shane Mansfield, Rui Soares Barbosa, Ray Lal, Phokion Kolaitis, Georg Gottlob, Jouko Vaananen and Mehrnoosh Sadrzadeh.

The lectures will present an introduction to contextual semantics, in a self-contained, tutorial fashion.

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Tutorial on Aspect

Daniel Altshuler

This tutorial examines two types of approaches to the study of aspect. The first is concerned with how grammatical aspect interacts with verbal meaning or *aktionsart.* This research program is important because it sheds light on how natural language is used to refer the completion (or lack thereof) of a given event. The second approach seeks to explain the context dependence of aspect. This research program is important because it sheds light on discourse dynamics and discourse coherence. The goal of the tutorial is to outline a new research program that seeks to unify these two approaches. The key idea will be that all aspectual operators make reference to events and their consequent states; aspectual operators differ in (i) whether they relate an event or its consequent state to temporal coordinates that are specified by tenses and temporal adverbs and (ii) the type of relation that holds between the given eventualities and the aforementioned temporal coordinates.

Tutorial on Admissible rules

Rosalie Iemhoff

Mathematical theories can in general be formalized in many different ways. This remains true even if only proof-theoretic formalizations, consisting of axioms and rules, are considered. In many cases theories allow inference steps that are not explicitly mentioned in their axiomatization. These so-called admissible rules can be added to the axiomatization without changing the set of theorems that can be derived from it.

As it turns out, many nonclassical theories such as intuitionistic and modal logic or Heyting arithmetic have nontrivial admissible rules. In recent years the variety of these rules has become much better understood and a close connection between such rules and unification theory has come to light.

In the tutorial I will give an overview of results in this area.

Invited lecture

Schema mappings and data examples
Balder ten Cate

The explosive growth of available data sources on the Web and elsewhere has created the need to develop new ways to access, store, and process data. In particular, information integration has become an important aspect of data management. How can we minimize the amount of human effort that goes into combining data from different sources to answer queries ("data integration"), or transforming data given structured under one schema into data structured under another schema ("data exchange")? These are active topics of research in the database literature. Schema mappings have emerged a key building block in formalizing and solving these tasks.

A schema mapping is a logical specification of the relationships between two database schemas. They are typically specified by sentences belonging to a fragment of first-order logic (or of second-order logic). Various such fragments have been proposed and are being used in practice. They are typically chosen with two criteria in mind: (a) expressive power sufficient to specify interesting data interoperability tasks and (b) desirable structural properties, such as query rewritability and existence of universal solutions, that, in turn, imply good algorithmic behavior when it comes to performing these interoperability tasks.

I will present some recent results concerning applications of schema mappings, schema mapping languages, and example-driven schema mapping design. This area of research is exciting not only for its practical relevance, but also because it draws on techniques and results from many other areas such as finite model theory, graph theory (in particular, homomorphism dualities) , and computational learning theory.

Proof Theory for Non-classical logics: The Baha'i method
Agata Ciabattoni

A central task of logic in computer science is to provide an automated introduction of analytic calculi for a wide range of non-classical logics. In this talk I will present a general method for doing that. The resulting calculi are obtained by transforming Hilbert axioms or semantic conditions into suitable structural rules. Our method applies to various formalisms, including hypersequent calculus, labelled deductive systems and display calculus, and sheds some light on their relations and expressive power.

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Regular Languages and Regular Cost Functions
Thomas Colcombet

In this talk, I will give a short panorama of some essential results concerning regular languages, namely the decidability results, and the numerous equivalent ways to present them (automata, algebra, logic, regular expressions, etc.). Then, I will introduce the more recent notion of cost functions that extends this theory with very mild quantitative capabilities. We consider there functions from words (or trees, or graphs, etc) to non-negative integers or infinity. A cost function is a class of such functions modulo the relation "being bounded over the same subsets". This allows to model quantitative phenomena through functions for which the exact values taken are not relevant, but rather their asymptotic behaviour matters. In particular, I will adopt the point of view of non-standard analysis, which sheds a bright light on the concepts and the objects used for solving regular cost functions.

Between-noun comparisons as contrast-based
Galit Sassoon

Within-predicate comparisons are constructions of the form "X is more A than y (is)". Adjectives are typically felicitous in these constructions, while nouns typically are not, as the oddity of examples (1-a)–(1-b) illustrates.

(1) a. Rubinstein is more #(of) a pianist than my son.
b. ??Rubinstein yoter psantran me-ha-ben sheli.

However, the situation is reversed in between-predicate comparisons-constructions of the form "X is more A than (y is) B". Whereas dimensional adjectives are often infelicitous in these constructions, nouns generally are perfectly acceptable, as the felicity of examples (2-a)–(2-b) illustrates.

(2) a. Rubinstein is more a pianist than a conductor.
b. Rubinstein yoter psantran me-menaceax.

The challenge is to account for the felicity of nouns in between-predicate comparisons, while capturing their infelicity in within-predicate comparisons. Existing analyses of between-predicate comparisons yield wrong predictions. Postulating either semantic gradability for nouns, or even only ad-hoc, contextual, meta-linguistic, last resort gradable interpretations for noun to capture the meaning of examples such as (2-a)–(2-b), results in wrong predictions for examples such as (1-a)–(1-b). The talk will present a solution to this problem, using the psychological notion of a contrast-set.

Algebra-Coalgebra Duality
Alexandra Silva

We give a new presentation of Brzozowski's algorithm to minimize finite automata, using elementary facts from universal algebra and coalgebra, and building on earlier work by Arbib and Manes on a categorical presentation of Kalman duality between reachability and observability. This leads to a simple proof of its correctness and opens the door to further generalizations.

Notably, we derive algorithms to obtain minimal, language equivalent automata from Moore, non-deterministic and weighted automata.

Perfectivity and Modality
Sergei Tatevosov

Perfectivity and Modality

The perfective has never been among those aspectual operators that are believed to deserve a modal analysis. Unlike what happens with the progressive (Dowty 1979, Landman 1992, Portner 1998, a.m.o.), the event described by the perfective occurs completely in the actual world, leaving no room for anything like the Imperfective Paradox to emerge. Unlike the perfect (Katz 2003, Portner 2003), the perfective does not give rise to modal presuppositions related to its current relevance and/or temporal orientation. However, in this paper I want to argue that properties of the perfective (in Slavic languages at least) are best accounted for if its semantics is endowed with a modal component, too. Specifically, I propose that the contribution of the Slavic perfective to the interpretation is two- fold. First, it introduces, as is commonly assumed, an operator mapping predicates of events to predicates of times in Klein's (1994) style. Secondly and crucially, the perfective indicates that the actual world is one of those where an event that falls under a given event description cannot have a continuation. To implement this idea, a circumstantial modal base and an event-maximizing ordering source are introduced, the former defining a set of worlds where a relevant event occurs, the latter imposing a strict partial order on this set. One good consequence of this analysis is that it allows to derive peculiar aspectual compositional effects characteristic of Slavic languages whereby undetermined plural and mass incremental arguments receive what Filip (2005) calls unique maximal interpretation.

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Contributed lectures

Searching for solutions
Martin Aher

Aim of the talk

We will present the deontic semantics MadRis,[1] which provides a uniform solution to prominent puzzles of deontic modals. MadRis stands for Modified Andersonian Deontic Radical Inquisitive Semantics. It is an extension of radical inquisitive semantics, and it modifies Andersonian deontic modals as it introduces quantification over alternatives.

Standard modal logic (SML)[19] and theories that extend it, such as Kratzer semantics[10, 15], express modals as quantification over possible worlds. There are a number of well known puzzles for standard theories (*Ross's paradox, dr. Procrastinate, free choice, conflicts of obligation, contrary to duty situations*) and some lesser known ones (*all or nothing*).

Our analysis is in the spirit of the current most prominent alternative to SML, which expresses modals as Andersonian[4] implications to violations, but this standardly suffers from puzzles of material implication, especially from *strengthening the antecedent.* Also, these theories share some puzzles of SML, for example, it does not account for the salient reading of $\neg\Diamond(p \vee q)$ nor for *contrary to duty* cases.

The current prominent theory on conditionals[13, 10] treats them as suppositional – the antecedent becomes the restrictor of a modal operator in the consequent. If no modal is found in the consequent, it is assumed to be a covert epistemic necessity operator. This approach avoids puzzles of material implication but not those of SML. Consider the new puzzle *all or nothing.*

(1) a. If the car passed its technical inspection and you have your license, then you may drive.
b. Hence, if the car passed its technical inspection, then you may drive.
c. $p \wedge q \rightarrow \Diamond r \models p \rightarrow \Diamond r$

According to Kratzer semantics, (1-c) holds, so one can weaken the antecedent of a conditional permission statement by removing conjuncts.

In MadRis, conditionals are treated suppositionally[2] but with a stronger clause for negation than in Kratzer semantics, but weaker than classical negation for material implication, to account for Ramsey's intuition that the conditional question *if p, then q?* has two contrary answers *if p, then q* and *if p, then not q*.

We will argue that although the semantics of deontic modals is related to the semantics of implications, they are basic operations that cannot be defined in terms of implication. We thereby also avoid the puzzles of material implication, especially from *strengthening the antecedent* which plague standard Andersonian approaches.

This account provides intuitive predictions for both modal sentences and their negations and solves the puzzles of SML.[3]

We will be focusing on the crucial feature of inquisitive semantics that its treatment of disjunction formalizes the intuition that *or* sentences serve to offer alternatives. This has been suggested in the literature as a solution to the *free choice* puzzle. For example, Aloni [3] proposed

[1] Based on work by Aher[1, 2].

[2] The treatment of conditionals will necessarily be brief. The radical framework, developed by Sano [18] and Groenendijk & Roelofsen [9], provides an intuitive basis for this treatment of deontic modals. An account of suppositional content is also in development [8].

[3] We are constrained to deontic modals. A paper on how to treat epistemic modals in a structurally similar manner is in preparation.

that with permission, every alternative satisfies permission such that we first quantify universally over alternatives and then existentially over accessible worlds within each alternative ($\forall\alpha\exists w$). Unlike Aloni, who quantified existentially over alternatives in case of obligation ($\exists\alpha\forall w$), we always quantify universally over alternatives.

Semantics

We consider a propositional language with negation ($\neg$), conjunction ($\wedge$) and implication ($\rightarrow$) as its basic connectives, to which we add a class of special atoms ($v_1, v_2, ...$) that state that a specific rule has been violated. We introduce deontic sentential operators ($\Diamond_{v1}\varphi, \Diamond_{v2}\varphi, ...$), read as permission. Depending on the rule, modals can refer to different violations, and we assume that each rule does generally refer to a different violation. When only one violation is pertinent, we will abbreviate $\Diamond_{v}\varphi := \Diamond\varphi$.

Disjunction is defined in the usual way: $\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$ and equally standardly a second deontic operator (*obligation*) is introduced: $\Box\varphi := \neg\Diamond\neg\varphi$. As in basic inquisitive semantics (See [5, 6, 7]), an interrogative sentential operator is introduced in the language by definition: $?\varphi := \varphi \vee \neg\varphi$.

A world is a binary valuation of the atomic sentences in the language, including the designated atoms that state that a specific rule has been violated. Let $\mathcal{A}$ be the set of atomic sentences. We represent a world w as a set which for each $a \in \mathcal{A}$ contains either a or $\overline{a}$, meaning that a holds in w, and that a doesn't hold in w, respectively. σ and τ are variables that range over states, which are sets of worlds, and we use ω to denote the set of all worlds, which corresponds to the ignorant state.

In our recursive semantics we define when a state supports ($\models^+$) and rejects ($\models^-$) a sentence.[4] We denote the set of states that supports a sentence by $[\varphi]^+$ and states that reject a sentence by $[\varphi]^-$. The recursive semantics that we will state guarantees that $[\varphi]^+$ and $[\varphi]^-$ are downward closed. I.e. if $\sigma \in [\varphi]^+$ and $\tau \subseteq \sigma$, then $\tau \in [\varphi]^+$ and same for $[\varphi]^-$. The meaning of a sentence is determined by the pair $\langle [\varphi]^+, [\varphi]^- \rangle$.

For the propositional case there are always one or more maximal supporting/rejecting states for a sentence denoted by $\textsc{alt}[\varphi]^+ := \{\sigma \in [\varphi]^+ \mid \neg\exists\tau \in [\varphi]^+ : \tau \supset \sigma\}$ and similarly for $\textsc{alt}[\varphi]^-$. When *the set of alternative possibilities for* φ, $\textsc{alt}[\varphi]^+$, contains more than one element then it is (support-) *inquisitive*, and similarly for rejection. This plays a crucial role in explaining *free choice* phenomena concerning deontic modals.

Since meanings are determined by the pair of supporting and rejecting states, entailment should also be stated relative to both components of meaning. Classically this would be a correct, but redundant formulation as the support and reject perspective on entailment would coincide.

Definition 1 (Entailment)**.**

Support-entailment: $\varphi \models_+ \psi$ iff $[\varphi]^+ \subseteq [\psi]^+$

Rejection-entailment: $\varphi \models_- \psi$ iff $[\psi]^- \subseteq [\phi]^-$

Entailment: $\varphi \models \psi$ iff φ support-entails ψ and φ rejection-entails ψ.

According to definition 1, a sentence φ support-entails the sentence ψ if every state that supports φ also supports ψ, and likewise for rejection. The dual nature of entailment plays an important role in the explanation of various deontic puzzles.[5]

The recursive statement of the semantics is as follows.

[4]There is a further extension of the system[8] which distinguishes a third relation between states and sentences which concerns a state dismissing a supposition of a sentence. In the semantics presented here, when a state rejects p, it both supports and rejects $p \rightarrow q$, and $\Diamond p$. In the suppositional extension such states are characterized as neither supporting nor rejecting them, but as dismissing a supposition of theirs.

[5]Equivalence can be defined as mutual entailment.

Definition 2 (MadRis).

1. $\sigma\,\text{sup}^+\,p$ iff $\forall w \in \sigma: p \in w$
   $\sigma\,\text{sup}^-\,p$ iff $\forall w \in \sigma: \overline{p} \in w$
2. $\sigma\,\text{sup}^+\,\neg\phi$ iff $\sigma\,\text{sup}^-\,\phi$
   $\sigma\,\text{sup}^-\,\neg\phi$ iff $\sigma\,\text{sup}^+\,\phi$
3. $\sigma\,\text{sup}^+\,\varphi \wedge \psi$ iff $\sigma\,\text{sup}^+\,\varphi$ and $\sigma\,\text{sup}^+\,\psi$
   $\sigma\,\text{sup}^-\,\varphi \wedge \psi$ iff $\sigma\,\text{sup}^-\,\varphi$ or $\sigma\,\text{sup}^-\,\psi$
4. $\sigma\,\text{sup}^+\,\varphi \to \psi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ : \tau \cap \sigma\,\text{sup}^+\,\psi$
   $\sigma\,\text{sup}^-\,\varphi \to \psi$ iff $\exists \tau \in \textsc{alt}[\phi]^+ : \tau \cap \sigma\,\text{sup}^-\,\psi$
5. $\sigma\,\text{sup}^+\,\Diamond_v\varphi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ : \tau \cap \sigma\,\text{sup}^-\,v$
   $\sigma\,\text{sup}^-\,\Diamond_v\varphi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ : \tau \cap \sigma\,\text{sup}^+\,v$

Inquisitiveness

Rejection can introduce inquisitiveness as $p \wedge q$ is rejection-inquisitive. $[p \wedge q]^-$ contains two maximal elements: $[p]^+$ and $[q]^+$.[6] $p \vee q$ is defined as $\neg(\neg p \wedge \neg q)$, so it is support-inquisitive as the maximal states that support it are $[p]^+$ and $[q]^+$. As standard, $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$, so $[(p \vee q) \to r]^-$ is rejection-inquisitive.

Violation-based deontic modals

Anderson is credited with the intuition that the meaning of deontic operators is connected to implication. In MadRis, $\Diamond\varphi \equiv \varphi \to \neg v$ as long as φ is not inquisitive. This holds e.g. when φ is the atom p. Figure 1 illustrates permission, prohibition and neutrality:

1. The state where p *is permitted* has no pv world in the maximal supporting state,[7] so looking at p worlds, $\neg v$ is also the case.
2. The state where p *is prohibited*, drawn dashed, has no $p\overline{v}$ world.
3. Both of these states are *deontically neutral towards* $\neg p$ as the maximal supporting states include both a $\overline{p}v$ and a $\overline{pv}$ world.

The dashed line in Figure 1 illustrates that $[\Diamond p]^- = [p \to \neg v]^-$.

Deontic modals are inquisitiveness resistant

Recall that $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$ so it is rejection-inquisitive (but not support-inquisitive). Under an Andersonian analysis, $\neg(\Diamond(p \vee q))$ is support-inquisitive. But intuitively, it is not. The drafters of a law or rule establish which permissions and obligations hold, which leaves no room for inquisitiveness. This leads to the standard non-inquisitiveness intuition regarding the interpretation of *free choice*[9] examples and their negation (see example (7)): $\neg\Diamond(p \vee q) \equiv \Box(\neg p \wedge \neg q)$, which the semantics predicts.

Unlike implication, both the support and reject clause has universal quantification scoping over the prejacent, guaranteeing that even with inquisitive prejacents $[\Diamond\varphi]^-$ is not rejection-inquisitive. So, $[\Diamond(p \vee q)]^-$ is stronger than (entails) $[(p \vee q) \to \neg v]^-$. As we

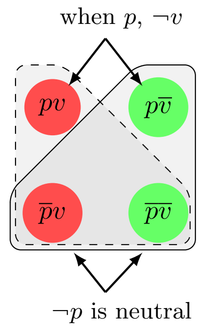

Figure 1: $\Diamond p / p \to \neg v$

[6] Unlike in $\mathsf{Inq_B}$[5] where $\neg(p \wedge q)$ contains only one element.
[7] SML treats permission as weaker, so $\Diamond p$ does not guarantee that when you bring about p, no violation occurs.

saw earlier, due to the existential quantifier in the rejection clause for implication, $[(p \vee q) \rightarrow \neg v]^-$ is rejection-inquisitive. But the only way modals can be inquisitive is when an inquisitive connective scopes over modals.

Solving puzzles

The above clauses provide solutions to all of the puzzles mentioned in the introduction, but due to space constraints, we will only illustrate the solution to the dr. Procrastinate[11] puzzle that combines an upward monotonicity puzzle with a contrary to duty puzzle.[8] This puzzle allows us to demonstrate the intuitiveness of this non-monotonic solution and its ability to introduce several violations.

Dr. Procrastinate

Dr. Procrastinate is an expert in her field but she never finishes her assignments. When she is asked to write a review, it's a fact that she will not write it, so, intuitively, the following two obligations hold.

(2) a. Dr. Procrastinate ought to accept and write the review.
b. Dr. Procrastinate ought not to accept.

According to the literature, there are two predictions to make: i) the conjunction of (2-a) and (2-b) is not intuitively absurd as they can be the case simultaneously; ii) we know that dr. Procrastinate will violate the obligation in (2-a) but could avoid violating the second violation in (2-b). And if dr. Procrastinate accepts, despite the fact that she will not finish writing the review, her behaviour is more reproachable than when she does not accept.

Upward monotonicity

(2-a)

is generally represented by an embedded conjunction. We will first assume the obligations refer to the same violation. In SML and Kratzer semantics obligation is upward monotonic, so the embedded conjunction (3-a) entails the embedded conjunct (4), which contradicts (3-b).

(3) a. $\Box(p \wedge q)$
b. $\Box\neg p$

(4) $\Box p$

MadRis captures that the obligations in (3-a) and (3-b) are not contradictory. Rejecting states for (4) include the state $\{\{p\bar{q}v\}, \{\overline{pqv}\}\}$ where *not writing* leads to a violation. This cannot reject (3-a), so, according to definition 1, (3-a) does not entail (4), making MadRis non-monotonic.

Contrary to duty

Intuition (ii) that needs to be covered concerns the possibility that dr. Procrastinate can avoid making the situation worse by fulfilling (2-b), despite violating (2-a). For contrary to duty examples, we separate the obligations so they refer to different violations.[9] Introducing multiple violations allows one to quantitatively determine states with less violations. We also need to add $\neg q$, the fact that dr. Procrastinate will no write, as the final piece of the formal picture.

[8] For full details on the free choice puzzle, see [1] or [2].

[9] This also guarantees that (5-a) and (5-b) are not contradictory but in a less interesting way, suggesting that we need a different example to showcase why $\Box(p \wedge q) \not\models \Box p$.

(5) a. $\boxed{v1}\,(p \wedge q)$
b. $\boxed{v2}\,\neg p$

(6) $\neg q$

As we know from the story, dr. Procrastinate will not write the book review. We will intersect (5-a), (5-b) and (6) and the maximal supporting state is shown in figure (6).

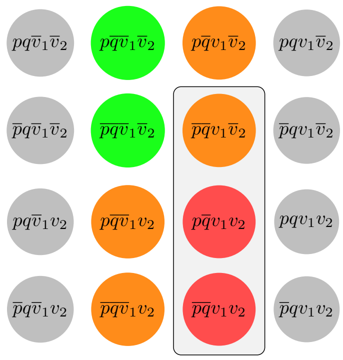

Figure 2: $(\boxed{v1}\,(p \wedge q)) \wedge (\boxed{v2}\,\neg p) \wedge (\neg q)$

The maximal supporting state for the story contains three worlds, so the situation is not absurd.[10] Each of the worlds is a v_1 world, which correctly captures the intuition that as long as dr. Procrastinate does not write the review, she is doing something wrong.
Furthermore, there remains only one p world and in that world v_2 occurs. This means that MadRis predicts that in case dr. Procrastinate does accept to write a review, despite not writing it, then she will incur a second violation on top of v_1. Yet, the two remaining $\neg p$ worlds differ in that one is a v_2 world and the other is a $\neg v_2$ world (coloured orange because it contains only one violation). When both a violation and a non-violation follows $\neg p$, we say that this state is deontically neutral with respect to $\neg p$. Thus, dr. Procrastinate - barring additional information - can avoid the second violation by not accepting to write the review. And this is the second intuition that MadRis had to cover.

All or nothing

The *all or nothing* puzzle from the introduction is straightforwardly solved in MadRis. $(p \wedge q) \rightarrow \Diamond r \not\models p \rightarrow \Diamond r$ because e.g the set of states $\{\{pqr\overline{v}\}, \{p\overline{q}rv\}\}$ supports $[(p \wedge q) \rightarrow \Diamond r]^+$ but not $[p \rightarrow \Diamond r]^+$. This is because the weakened antecedent can lead to a violation.

Counterarguments to non-monotonicity

von Fintel[7] criticized non-monotonic treatments of modals for not becoming downward monotone under negation. Consider (7).

(7) A country is not permitted to establish a research center or a laboratory.

The salient reading of a sentence such as (7) is that both disjuncts are prohibited. We refer to this as the *no choice* reading, in that choosing to establish either a research center or a laboratory

[10] The worlds factively eliminate by $\neg q$ are left grey. Green worlds contain no violations, orange worlds only one and red worlds two violations.

will break the rule in (7). Our non-monotonic semantics provides an intuitive account of *no choice* readings parallel to the free choice effect so that $\neg\Diamond(p \vee q) \equiv \Box(\neg p \wedge \neg q)$.

Furthermore, modals and conditionals both license NPI's such as "any" in their prejacent/antecedent, contrary to the prominent Fauconnier-Ladusaw theory which states that NPIs are licensed in downward entailing positions.

Standard accounts of conditionals, and also MadRis, do not predict that the antecedent of a conditional is downward entailing. If the antecedent were downward entailing, we would face the puzzle of strengthening the antecedent. But the fact that both antecedents of conditionals and prejacents of modals license NPIs reinforces that they are structurally similar. So an alternative explanation for NPI licensing will be needed in the future.

Pace von Fintel, one does not require a monotonic semantics for modals to account for the behaviour of modals under negation. Furthermore, the licensing of NPIs under negated modals is an expected result for an account of modality in the spirit of Anderson, as NPIs are also licensed in the antecedents of conditionals.

Alternative formulation of dr. Procrastinate

Consider a possibly more intuitive approach to the puzzle.

(8) a. $\Box p$
    b. $p \rightarrow \Box q$

(9) $\neg q$

As shown in figure (3), the factive information $\neg q$ eliminates the grey worlds and all the non-violation worlds are eliminated by the two obligations. We get a situation where one ought to accept to write a review. But when one does accept, and does not write, one is doing something worse. Dr. Procrastinate will not write, so, a semantics ought to capture that violating the obligation (8-a) is worse than violation (8-b) such that one ought to choose not to accept.

MadRis does not yet include the tools for such a characterization, but this opens a promising new avenue for future research.

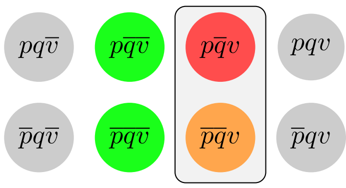

Figure 3: $\Box p \wedge (p \rightarrow \Box q) \wedge (\neg q)$

Conclusions

MadRis is an alternative semantics for deontic modals that provides a uniform semantic solution to puzzles of SML and Kratzer semantics. Not only does a non-monotonic semantics for modals solve monotonicity puzzles, it also provides an intuitive account of the *free choice effect*. MadRis also makes intuitive predictions concerning the behaviour of modals under negation.

Furthermore, *conflicts of obligation* and *contrary to duty* situations are common in deontic contexts. MadRis allows one to reason with several violations that not only avoid the problematic inferences in puzzles such as *dr. Procrastinate* but also give an intuitive characterizations of the situation.

Kratzer's account for modals and conditionals allows for weakening of permission as in the *all or nothing* puzzle. MadRis provides an intuitive solution to this puzzle as universal quantification is in both the support and reject clauses of modals.[11]

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Searching for directions

Martin Aher and Jeroen Groenendijk

Overview and aims

Groenendijk & Roelofsen have recently developed an extension ($\mathsf{Inq_{Sup}}$) [8] of basic inquisitive semantics ($\mathsf{Inq_B}$) [5, 6, 7], which adds suppositional content as a third component of meaning, next to informative and inquisitive content. Basic motivation is a treatment of implication which characterizes $p \to q$ and $p \to \neg q$ as *contradictory* answers to the conditional question $p \to ?q$, and characterizes $\neg p$ as a different type of response that signals *suppositional dismissal* of each of the other three sentences, thereby implying that the issue they jointly address is void.

$\mathsf{Inq_{Sup}}$ comes with an epistemic modality, where, e.g., $\Diamond_\epsilon p$ expresses that one's information state allows for consistently supposing that p is the case. In line with earlier work by Aher [1, 2], our aim in this paper is to show that there is a straightforward extension of $\mathsf{Inq_{Sup}}$ which can deal with deontic modalities in a novel way. The extension basically consists in enriching information states with *deontic information.*

We add deontic information via rulings that encompass the entire body of rules. When trying to establish which actions violate rules, and which do not, a deontic statement can either have a positive direction, where it is possible to avoid violations, or a negative direction, where it is no longer possible to avoid violations. For example, when $\Diamond p$ is supported, then there is a positive direction in the state, and when $\Diamond p$ is rejected, then there is a negative direction. When a state has a direction, it is no longer deontically ignorant.

The suppositional nature of the semantics is exploited by associating epistemic suppositions with deontic modalities. As in the conditional example above, whereas the deontic modalities $\Diamond p$ and $\neg\Diamond p$ (or $\Box\neg p$) are conflicting answers to the deontic question $?\Diamond p$, the assertion $\neg p$, and the epistemic modalities $\neg\Diamond_\epsilon p$ and $\Box_\epsilon \neg p$, render the deontic issue void by signalling suppositional dismissal.

Note that this implies that the suppositional content of $\Diamond p$ and $\neg\Diamond p$ (or $\Box\neg p$) is the same. In both cases the epistemic possibility of p is supposed. Likewise, in case of $\Box p$ the epistemic possibility of $\neg p$ is supposed. Expressing deontic obligation and prohibition supposes the epistemic possibility that the obligation or prohibition is violated.

In presenting the semantics, we will focus on two issues. First, modal auxiliaries have epistemic and deontic readings, which Kratzer semantics [10] captures by assigning two different modal bases while keeping the rest of the semantics identical. But Andersonian implication-based accounts [4] and Aher's modified account [1, 2] lack a similar account of epistemic modalities. We will show that in $\mathsf{Inq_{Sup}}$ epistemic and deontic modalities can be analyzed in structurally the same way. The difference is that where for epistemic modalities we call upon the basic property of (in)consistency of information states, for deontic modalities we introduce a further distinction between deontically safe and deontically fatal information states.

Secondly, we focus on the feature that the semantics deals in a straightforward and uniform way with the puzzles of deontic and epistemic free choice (See [9, 11]). The inquisitive analysis of disjunction that is a trademark of inquisitive semantics plays a crucial role in this.

Issues and information: factive and deontic

Inquisitive semantics provides an integrated account of informative and inquisitive semantic content. The basic semantic notion is that of an information state, standardly viewed as a set of possible worlds, those worlds that could be the actual world according to the information the state embodies.

In our deontic extension, we integrate an additional layer of information and issues which concerns the *deontic status* of the possible worlds in an information state. Our basic deontic notion is that of a *possible ruling.* A ruling comprises a set of, possibly conflicting, deontic rules. What the individual rules behind a ruling are are will remain implicit, but a ruling determines for each possible world whether it is or is not a world in which at least one of the rules involved is violated. Like factual information is information about which possible worlds could be the actual world, deontic information is information about which possible rulings could be the actual ruling.

Definition 3 (Worlds and rulings)**.**
A *world* is a function w such that for any atomic sentence $p\colon w(p) = 1$ or $w(p) = 0$.
A *ruling* is a function r such that for any possible world $w\colon r(w) = 1$ (no violation) or $r(w) = 0$ (violation).

We assume that deontic information is independent of factual information in the sense that if r is a ruling that could be the actual ruling according to our information, then r could be the actual ruling in any of the worlds that are compatible with our information. This means that a deontic information state σ could be represented by the pair of a set of worlds $\mathit{worlds}(\sigma)$ and a set of rulings $\mathit{rulings}(\sigma)$. But formally it is more handy to let states be a single set consisting of world-ruling pairs, and determine $\mathit{worlds}(\sigma)$ and $\mathit{rulings}(\sigma)$ in the following way:

Definition 4 (World-ruling pairs)**.** Let ω be the set of all worlds and ρ the set of all rulings, and let $\sigma \subseteq \omega \times \rho$.
$\mathit{worlds}(\sigma) = \{w \in \omega \mid \exists r \in \rho \mid \langle w, r\rangle \in \sigma\}$ and $\mathit{rulings}(\sigma) = \{r \in \rho \mid \exists w \in \omega \mid \langle w, r\rangle \in \sigma\}$.

We are now ready to define the core semantic notion of a *deontic information state*, where we stipulate the independence of deontic and factual information in a state.

Definition 5 (Deontic information states)**.**
σ is a *deontic information state* iff $\sigma \subseteq \omega \times \rho$ such that $\mathit{worlds}(\sigma) \times \mathit{rulings}(\sigma) = \sigma$.[1]

Any information-based semantics distinguishes between the absurd inconsistent state, and consistent information states. Adding deontic information gives rise to a more fine-grained distinction among consistent states between deontically fatal and safe states. The notion of consistency, and the distinction between safe and fatal states, will play a key role in characterizing epistemic and deontic modalities, respectively.

Definition 6 (Fatal and safe states)**.** Let σ be a deontic information state.

σ is a *safe state* iff $\sigma \neq \emptyset$ and $\exists w \in \mathit{worlds}(\sigma)\colon \forall r \in \mathit{rulings}(\sigma)\colon r(w) = 1$.

σ is a *fatal state* iff $\sigma \neq \emptyset$ and $\forall w \in \mathit{worlds}(\sigma)\colon \forall r \in \mathit{rulings}(\sigma)\colon r(w) = 0$.

Definition 7 (Deontic directions in states)**.** Let σ be a deontic information state.

There is a *positive direction* in σ iff $\exists \tau \subseteq \sigma\colon \mathit{rulings}(\tau) = \mathit{rulings}(\sigma)$ and τ is a safe state.

There is a *negative direction* in σ iff $\exists \tau \subseteq \sigma\colon \mathit{rulings}(\tau) = \mathit{rulings}(\sigma)$ and τ is a fatal state.

[1]Note that there could be many different rulings in a state that concide in the values they assign to the worlds in the state. Such multiplicity is redundant, but filtering it out is formally unhandy.

Definition 8 (Deontic ignorance)**.** Let σ be a deontic information state.
σ is a *deontically ignorant state* iff $\forall w \in worlds(\sigma)\colon \exists r, r' \in rulings(\sigma)\colon r(w) = 1$ and $r'(w) = 0$.

Fact 1 (Directions and deontic ignorance)**.**
If σ is a deontically ignorant state, then there is neither a positive nor a negative direction in σ.

Suppositions and suppositional dismissal

In $\mathsf{Inq}_{\mathsf{Sup}}$, the evaluation of a sentence ϕ in a state σ may involve making certain suppositions, hypothetically assuming certain pieces of information to hold in σ. For certain suppositions of ϕ that may not be possible in σ without ending up in the absurd state of inconsistent information. In this case we say that *dismissal of a supposition* of ϕ occurs in σ. Note that as information grows, ϕ runs a greater risk of suppositional dismissal. Our semantics assigns suppositional content to implication, and to epistemic and deontic modalities. In compound sentences, the suppositional content of its components is accumulated.

Language

The core language under consideration is a standard propositional language. As is usual in inquisitive semantics, we add an interrogative operator which is defined as $?\phi := \phi \vee \neg\phi$.

$\mathsf{Inq}_{\mathsf{Sup}}$ comes with an epistemic modality $\Diamond_\epsilon\phi$ (*might* ϕ), which, roughly speaking, signals epistemic consistency of ϕ with one's information state. To obtain sentences which carry deontic information, we add a basic deontic modality $\Diamond\phi$ (*may* ϕ), which, roughly speaking, signals that adding ϕ to our information leads to a deontically safe state.[2] Two operators $\Box\phi$ (*ought* ϕ) and $\Box_\epsilon\,\phi$ (*must* ϕ) are introduced by definition in the usual way: $\Box\phi := \neg\Diamond\neg\phi$ and $\Box_\epsilon\,\phi := \neg\Diamond_\epsilon\neg\phi$.

Semantics

Aloni [3] and other authors have suggested that the free choice effect arises because disjunction generates alternatives denoting the disjuncts. These treatments of deontic modals quantify existentially over alternatives ($\exists\alpha\forall w$) for obligation and universally for permission ($\forall\alpha\exists w$). Following such alternative-based solutions, we introduce an auxiliary notion:

Definition 9 (Alternatives)**.** The *alternatives* in a set of states $\mathcal{S}$ are its maximal elements: $\textsc{alt}(\mathcal{S}) = \{\sigma \in \mathcal{S} \mid \neg\exists\tau \in \mathcal{S}\colon \tau \supset \sigma\}$.

We will modify an Andersonian account of deontic modality by adding quantification over alternatives. Unlike Aloni, we always quantify universally over alternatives to capture the intuition that laws and rules do not allow for ignorance readings.

The semantics recursively characterizes three properties of states in relation to the sentences of the language: *support*, $\sigma\,\text{sup}^+\,\phi$; *rejection*, $\sigma\,\text{sup}^-\,\phi$; and *dismissing a supposition*, $\sigma\,\text{sup}^\circ\,\phi$. Whereas support and rejection are mutually exclusive semantic properties, dismissing a supposition is not, it only signals that *some* supposition of ϕ cannot be made consistently in σ, which does not necessarily exclude that ϕ is also supported in σ, or rejected (but not both). The set of states that support ϕ is $[\phi]^+ = \{\sigma \mid \sigma\,\text{sup}^+\,\phi\}$, and similarly for $[\phi]^-$ and $[\phi]^\circ$.[3]

[2] More precisely, $\Diamond_\epsilon\phi$ signals that *any possibility for* ϕ is consistent with one's state, and $\Diamond\phi$ signals that adding *any possibility for* ϕ to one's state results in a safe state. As compared to standard quantificational approaches to modality, the order of quantification is reversed. We do not quantify over accessible worlds and then inspect whether ϕ holds there, but we quantify over the possibilities for ϕ and inspect the epistemic or deontic status of the resulting states.

[3] The proposition a sentence expresses, is given by $[\phi] = \langle[\phi]^+, [\phi]^-, [\phi]^\circ\rangle$. In accordance with the general philosophy behind inquisitive semantics, where the meaning of ϕ is directly related to its role in the exchange of information, the three components are related to different ways of *responding to* ϕ.

When $\textsc{alt}[\phi]^+$, *the set of alternative possibilities for* ϕ, contains more than one element, this means that ϕ is (support-) *inquisitive.* Inquisitiveness plays a crucial role in the interpretation of implication and epistemic and deontic modalities.

We assume, for any sentence, that it supposes that the state of evaluation (the state of the responder) is consistent. The absurd state of inconsistent information neither supports nor rejects, but suppositionally dismisses any sentence.

Definition 10 (Deontic $\mathsf{Inq}_{\mathsf{Sup}}$)**.** 1. $\sigma \sup^+ p$ iff $\sigma \neq \varnothing$ and $\forall \langle w, r\rangle \in \sigma \colon w(p) = 1$

$\sigma \sup^- p$ iff $\sigma \neq \varnothing$ and $\forall \langle w, r\rangle \in \sigma \colon w(p) = 0$

$\sigma \sup^\circ p$ iff $\sigma = \varnothing$

2. $\sigma \sup^+ \neg\phi$ iff $\sigma \sup^- \phi$

$\sigma \sup^- \neg\phi$ iff $\sigma \sup^+ \phi$

$\sigma \sup^\circ \neg\phi$ iff $\sigma \sup^\circ \phi$

3. $\sigma \sup^+ \varphi \wedge \psi$ iff $\sigma \sup^+ \varphi$ and $\sigma \sup^+ \psi$

$\sigma \sup^- \varphi \wedge \psi$ iff $\sigma \sup^- \varphi$ or $\sigma \sup^- \psi$

$\sigma \sup^\circ \varphi \wedge \psi$ iff $\sigma \sup^\circ \varphi$ or $\sigma \sup^\circ \psi$

4. $\sigma \sup^+ \varphi \vee \psi$ iff $\sigma \sup^+ \varphi$ or $\sigma \sup^+ \psi$

$\sigma \sup^- \varphi \vee \psi$ iff $\sigma \sup^- \varphi$ and $\sigma \sup^- \psi$

$\sigma \sup^\circ \varphi \vee \psi$ iff $\sigma \sup^\circ \varphi$ or $\sigma \sup^\circ \psi$

5. $\sigma \sup^+ \varphi \to \psi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma \sup^+ \psi$

$\sigma \sup^- \varphi \to \psi$ iff $\exists \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma \sup^- \psi$

$\sigma \sup^\circ \varphi \to \psi$ iff $\sigma \sup^- \phi$ or $\sigma \sup^\circ \phi$ or

$\exists \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma \sup^\circ \psi$

6. $\sigma \sup^+ \Diamond_\epsilon \phi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma \neq \varnothing$

$\sigma \sup^- \Diamond_\epsilon \phi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma = \varnothing$, and $\sigma \neq \varnothing$

$\sigma \sup^\circ \Diamond_\epsilon \phi$ iff $\exists \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma = \varnothing$, or $\sigma \sup^\circ \phi$

7. $\sigma \sup^+ \Diamond \phi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma$ is a safe state

$\sigma \sup^- \Diamond \phi$ iff $\forall \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma$ is a fatal state

$\sigma \sup^\circ \Diamond \phi$ iff $\exists \tau \in \textsc{alt}[\phi]^+ \colon \tau \cap \sigma = \varnothing$, or $\sigma \sup^\circ \phi$

Illustrations of the semantics

Example 1 (Rejection by suppositional dismissal)**.** Consider $\Diamond_\epsilon p$. Atomic sentences are not inquisitive and do not provide deontic information: $\textsc{alt}[p]^+$ has a single element τ_p such that $\mathit{worlds}(\tau_p) = \{w \in \omega \mid w(p) = 1\}$ and $\mathit{rulings}(\tau_p) = \rho$. This means that rejection of $\Diamond_\epsilon p$ coincides with its suppositional dismissal (in a consistent state). $\Diamond_\epsilon p$ is suppositionally dismissed and hence rejected in σ in case $\tau_p \cap \sigma = \varnothing$, i.e., in case σ is inconsistent with the information that p is the case: $\forall w \in \mathit{worlds}(\sigma) \colon w(p) = 0$.

As long as $\exists w \in \mathit{worlds}(\sigma) \colon w(p) = 1$, it cannot fail to hold that σ supports $\Diamond_\epsilon p$. Note that this holds, e.g., if σ equals the totally ignorant state $\omega \times \rho$. This means that $\Diamond_\epsilon p$ is neither factively nor deontically informative.

Example 2 (Permission and prohibition suppose epistemic possibility)**.** Consider $\Diamond p$. The suppositional dismissal conditions are the same as for $\Diamond_\epsilon p$. So, $\Diamond p$ is not suppositionally dismissed in σ as long as $\exists w \in \mathit{worlds}(\sigma) \colon w(p) = 1$. Only when this is so, the support or reject conditions can apply. Then σ supports $\Diamond p$ if $\tau_p \cap \sigma$ is a safe state, i.e., $\exists w \in \mathit{worlds}(\sigma)$ such that $w(p) = 1$ and $\forall r \in \mathit{rulings}(\sigma) \colon r(w) = 1$. And then σ rejects $\Diamond p$ and hence supports $\Box\neg p$ (prohibits p), in case $\forall w \in \mathit{worlds}(\sigma)$ and $\forall r \in \mathit{rulings}(\sigma)$: if $w(p) = 1$, then $r(w) = 0$. Suppositional dismissal is preserved under negation. Like $\Diamond p$, $\neg\Diamond p$, and hence $\Box\neg p$, is dismissed by a state which is inconsistent with p. So, in prohibiting p one supposes the epistemic possibility of violating the prohibition.

Example 3 (Support by suppositional dismissal)**.** Consider $\Box_\epsilon p$. Since the dismissal clauses for deontic and epistemic modalities are the same, $\Box_\epsilon p$ is dismissed by a state which is inconsistent with $\neg p$, i.e., by a state that supports p, which is also a state that supports $\Box_\epsilon p$. Conversationally this means that when an addressee responds with p or $\Box_\epsilon p$ to an initiative $\Box_\epsilon p$, she establishes agreement by dismissing the supposition that p might not be the case according to her information.

Example 4 (Inquisitive disjunction)**.** Consider $p \vee q$. A state σ supports $p \vee q$ iff it either supports p or supports q. This means that $\textsc{alt}[p \vee q]^+$ has two elements τ_p and τ_q, where τ_p is as above, and τ_q is constructed in a similar way. So, $p \vee q$ is inquisitive, and it is also (factively) informative, which is signalled by the fact that $[p \vee q]^- \neq \varnothing$. $\textsc{alt}[p \vee q]^-$ consists of a single element τ such that $\mathit{worlds}(\tau) = \{w \in \omega \mid w(p) = w(q) = 0\}$ and $\mathit{rulings}(\tau) = \rho$. $p \vee q$ is not suppositional, only, like any sentence, in the inconsistent state $p \vee q$ is suppositionally dismissed.

Example 5 (Free choice effects)**.** Consider $\Diamond(p \vee q)$. As we saw in the previous example, $\textsc{alt}[p \vee q]^+$ has two elements τ_p and τ_q. Suppositional dismissal occurs in σ as soon as one of the two alternatives is inconsistent with σ, i.e., if $\forall w \in \mathit{worlds}(\sigma)\colon w(p) = 0$, or $\forall w \in \mathit{worlds}(\sigma)\colon w(q) = 0$. (Note that the same holds for $\Diamond_\epsilon(p \vee q)$.)

If no suppositional dismissal occurs, then σ supports $\Diamond(p \vee q)$ if both $\tau_p \cap \sigma$ and $\tau_q \cap \sigma$ are safe states, which means that $\exists w \in \mathit{worlds}(\sigma)$ such that $w(p) = 1$ and $\forall r \in \mathit{rulings}(\sigma)\colon r(w) = 1$; and $\exists w \in \mathit{worlds}(\sigma)$ such that $w(q) = 1$ and $\forall r \in \mathit{rulings}(\sigma)\colon r(w) = 1$. (In case of $\Diamond_\epsilon(p \vee q)$ support requires that $\exists w \in \mathit{worlds}(\sigma)\colon w(p) = 1$ and $\exists w \in \mathit{worlds}(\sigma)\colon w(q) = 1$). So, the semantics accounts in a very direct way for deontic (and epistemic) free choice effects.

Example 6 (Free choice does not imply both)**.** Continuing on the previous example, support of $\Diamond(p \vee q)$ allows for, but does not guarantee, that $\exists w \in \mathit{worlds}(\sigma)$ such that $w(p) = 1$ & $w(q) = 1$ and $\forall r \in \mathit{rulings}(\sigma)\colon r(w) = 1$, which would be a situation where $\Diamond(p \wedge q)$ is supported. The same holds for $\Diamond p \wedge \Diamond q$, which is 'almost equivalent' with $\Diamond(p \vee q)$. The only difference is that in a state σ that is inconsistent with p, $\Diamond(p \vee q)$ is dismissed and not rejected, whereas it could still be the case that $\Diamond p \wedge \Diamond q$, though also dismissed, is at the same time rejected because $\Diamond q$ is rejected.[4]

Example 7 (Free choice negation and no choice)**.** Continuing on the previous examples, σ rejects $\Diamond(p \vee q)$ —and hence supports $\neg\Diamond(p \vee q)$, $\Box\neg(p \vee q)$, and $\Box(\neg p \wedge \neg q)$— in case $\forall w \in \mathit{worlds}(\sigma)\colon$ if $w(p) = 1$ or $w(q) =$, then $\forall r \in \mathit{rulings}(\sigma)\colon r(w) = 0$. (In case of $\Diamond_\epsilon(p \vee q)$ rejection results when $\forall w \in \mathit{worlds}(\sigma)\colon w(p) = 0$ and $\forall w \in \mathit{worlds}(\sigma)\colon w(q) = 0$, which also means that $\Box_\epsilon(\neg p \wedge \neg q)$ and $\neg p \wedge \neg q$ are supported.)

Example 8 (Deontic inquisitiveness)**.** Consider $\Diamond p \vee \Diamond q$. A state supports a disjunction if it supports one of its disjuncts, and rejects it if it rejects both of them. This means that the states that reject $\Diamond p \vee \Diamond q$ are the same as those that reject $\Diamond(p \vee q)$, this also holds for the states where suppositional dismissal occurs. But $\Diamond p \vee \Diamond q$ is more easily supported: σ supports $\Diamond p \vee \Diamond q$ when either $\exists w \in \mathit{worlds}(\sigma)\colon$ such that $w(p) = 1$ and $\forall r \in \mathit{rulings}(\sigma)\colon r(w) = 1$ or $\exists w \in \mathit{worlds}(\sigma)\colon$ such that $w(q) = 1$ and $\forall r \in \mathit{rulings}(\sigma)\colon r(w) = 1$.[5] This means that $\textsc{alt}[\Diamond p \vee \Diamond q]^+$ contains two elements, i.e., the sentence is (support-) inquisitive. We assume that in free choice examples permission takes wide scope as in $\Diamond(p \vee q)$ but in cases of manifest ignorance, the modal takes narrow scope as in $\Diamond p \vee \Diamond q$, and likewise for epistemic cases.

Example 9 (Asking for directions)**.** Consider the following sequence of sentences, uttered by different speakers.

[4]The facts reported here mean that $\Diamond p \wedge \Diamond q \models \Diamond(p \vee q)$, and not the other way around.

[5]Continuing on the previous footnote, $\Diamond(p \vee q) \models \Diamond p \vee \Diamond q$, and not the other way around. Both this entailment fact and the one reported in the previous footnote also hold for the corresponding epistemic modalities.

1. $(p \vee q) \rightarrow ?\Diamond r$
2. $(p \rightarrow \Box\neg r) \wedge (q \rightarrow \Diamond r)$
3. $p \rightarrow \Diamond r$
4. $\neg\Diamond_{\epsilon} p \wedge (q \rightarrow \Diamond r)$
5. $\neg p \wedge \neg q$

The conditional question in (1) embodies an issue which concerns only deontic information. The conjunction in (2) completely resolves the denontic issue raised by (1). The deontic statement in (3) partially resolves the issue raised by (1), but contradicts (2). The sentence in (4) is partly an epistemic modality which signals dismissal of one of the suppositions of (1)-(3), and it also provides deontic information, compatible with the directions provided by (2) and (3). Finally, the factive assertion in (5) dismisses the whole issue raised by (1) and addressed by (2)-(4).

Conclusions

We have shown that $\mathsf{Inq}_{\mathsf{Sup}}$ can be straightforwardly extended to handle deontic and epistemic modals in a structurally similar manner – something Andersonian approaches have struggled to achieve. This approach provides intuitive semantic solutions to the free choice puzzles of both deontic and epistemic modals. Unlike most semantic approaches, it correctly predicts the behaviour of disjunction under negated permission and epistemic might.

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The Functional-Semantic Analysis of the Laz Verbal Vowel Prefixes

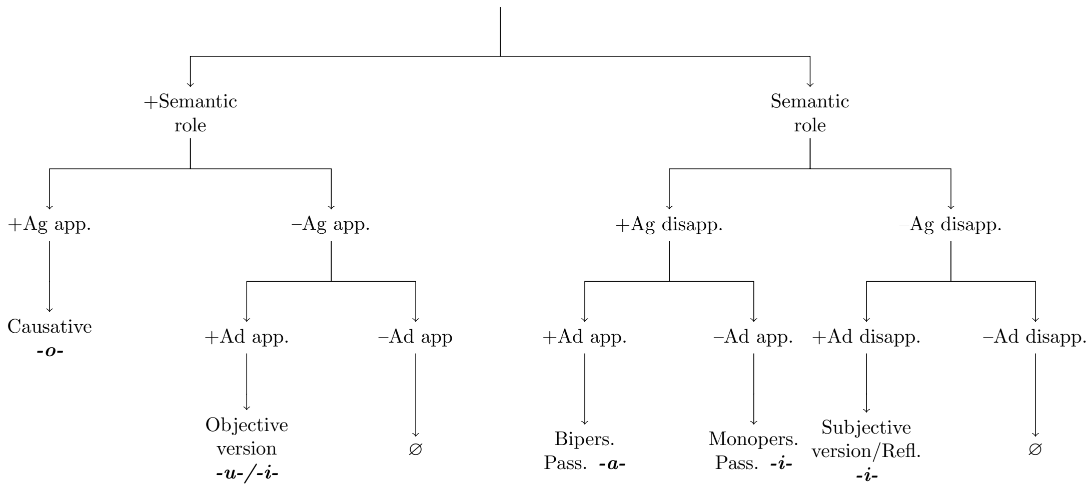
Rusiko Asatiani

Verbal vowel prefixes

There are four verbal vowel prefixes in Laz: **-a-**, **-i-**, **-o-**, **-u-**. The vowels are poly-functional and represent semantically different derivational verb forms – transitive, causative, contact, reflexive, passive, subjective version (resp. middle), and objective version.

Based on a semantic and functional analysis of vowel prefixes the following generalization could be suggested: *The main function of verbal vowel prefixes is formalization of conceptual changes raised as a result of either increasing or decreasing of the verb valency implying either appearance or disappearance of semantic roles (viz. Ag or Ad*[1]*) in the arguments structure of a verb.*

Various possibilities of verb valency variations in Laz can be summarized by the following scheme:



With regard to concretely which and what a semantic role disappears or appears in the defined by verb valency arguments structure the categories of voice (= *disappears an Agent*), version (= *either appears or disappears an Addressee*) and causation (= *appears an Agent*) have to be distinguished.

Voice

Voice is a functional verb category that reflects essential ways of the functional qualifications of semantic roles (Ag, Ad, P): If an Agent is realized as a Subject (Ag → S) and a Patient – as a Direct Object (P → DO), the verb is considered to be an active one; while, if a Patient

[1]Patient is a role closely connected with the verb semantics; its appearance or disappearance leads to a radical changes of the verb semantics; hence, in derivational models reflecting the increase or the decrease in the verb valency the Patient t take any part.doesn

is promoted to a Subject position (P → S) and an Agent demoted to a prepositional phrase (resp. PP) or sometimes completely disappears (that is, it does not take part in the argument structure of a verb and is not defined by the verb valency anymore) a verb is considered to be a passive one. According to such functional approach the active-passive semantics is actually distinguished just for bi- or tri-valentic verbs that are characterised by having the semantic role of P (and Ad); and as a rule the passive voice is regarded to be the conversive form of the active one:

(1) (i) if Ag → S and P → DO, the verb is active;
(ii) if P → S and Ag → PP// → ∅[2], the verb is passive (resp. conversive of an active one).

However, Ag is not an obligatory semantic role characteristic for all kinds of verbs' semantic structure (in contrast, the Subject is an obligatory function that must be realized with any kind of predication); e.g., with the verbs of passive semantics: *S is standing, is lying, is getting up* – an Ag is not even supposed. Therefore, for active-passive opposition its weakening and /or disappearing might not be taken as an essential distinguishing feature either. Through such an approach an explicit distinction between an active-passive opposition seems possible if some semantic (and not only functional) differences are taken into account as well. We can generalize the definition of active-passive opposition by comprising into analysis mono-personal verbs and including into functional qualifications the semantic role of an Addressee as well:

(2) (i) if Ag → S, a verb is active;
(ii) if P → S, a verb is passive;
(iii) if Ad → S, a verb is affective.

Analysis of the verb forms of Kartvelian languages seems simpler and more adequate through the second approach, since so called Active Morphosyntactic Model is followed not only by bi- or tri-personal active, transitive (resp. active voice) verbs but also by mono or bi-personal intransitive, active - semantically dynamic and atelic verbs (e.g.: S/he lives, works, resides, rolls, and etc.); and Passive Morphosyntaxic Model is supported not only by mono- or bi-personal agent-disappeared verbs but also by intransitive, inactive, yet, semantically dynamic and telic verbs (e.g.: S/he is getting up, is drying, is getting drunk, is whitening, and etc.).

On the basis of those morphosyntactic markers, four types of verb classes have been distinguished (comp. Shanidze 1973, with slight difference Harris 1991, Holisky 1991):

I-class verbs transitive, dynamic, telic (resp. active verbs);

II-class verbs intransitive, dynamic, telic (resp. passive verbs);

III-class verbs intransitive, active-dynamic, atelic (resp. medio-active verbs);

IV-class verbs intransitive, static[3], a prior atelic (resp. medio-passive verbs) and affective verbs[4].

These classes of verbs differ not only by syntactic relations but also by the verb morphology as well. II- and partially IV-class verbs' different morphological structure is considered to be the marked one expressing so called Passive Voice form and in Laz its morphological peculiarities are the following:

[2] As far as the agent may disappear and wouldn't be considered as even a "weakened" prepositional phrase.

[3] In the present tense forms of III- and IV-class verbs (in some speeches of Artashen) appears a new (different from the active or passive) model with an auxiliary verb conjugation (Kartozia 2005).

[4] Affective (or inversive) verbs create specific non-canonical syntactic constructions (with Dative Subject and Inverted person markers), yet, from the viewpoint of morphology they follow either active or passive models of representation.

(3) (i) In the present passive forms[5] of mono-personal verbs have either ending **-u** or the vowel prefix **i-** together with the ending **-e** (rarely **-u**). Out of them **-e** ending is mostly met in the verb forms without any series marker and **-u** in ones showing some series marker[6];

(ii) Bi-personal passive is distinguished by a vowel prefix **a-**;

(iii) Mono-personal, I/II person singular forms take a suffix **-r** (alike the active ones, after vowel-final form.

Example 1 (Examples)**. i-č'ar-e-n** (It's being written, It might be written), **i-č'k'om-e-n** (It could be eaten, It is eatable), **a-č'k'om-e-n/a-č'k'om-ap-u-n** (He may eat it, It is eatable for him), **p'-t'u-b-u-r** (I'm getting warm), **b-γur-u-r** (I'm dying to die), **a-č'ar-e-n** (It is being written to him, It might be written to him), **i-kt-ap-u-n** (S/he's coming back), **a-nt'al-ap-u-n** (It's getting mixed together), and etc.

Version

In Kartvelian languages a specific grammatical category is distinguished showing subject-object relationship regarding the orientation of an action:

(i) If (a) the subject's action is oriented towards an indirect object and/or (b) the subject's action is intended for an indirect object, the pre-root vowel-prefixes appear in a verb form. In (i)-case, if a verb is passive, in Laz appears the vowel-prefix **a-**, and in other cases the vowel-prefixes: **u-** (when the indirect object is the third person) and **i-** (when the indirect object is either the first or second person); Thus, an indirect object appears in the argument structure of a verb; hence, such an increase of a verb-valency formally is depicted in the verb morphological structure, it is possible to mark out the category of so called **Objective Version**;

(ii) If (a) a subject's action is oriented towards the subject itself (resp. reflexive) or (b) the object of an action is designated to the subject[7] (that is, the referent of a subject and a structurally indirect object are one and the same leading to disappearance of an indirect object and decreasing of verb valency), in the Laz verb form appears i- prefix; hence, it is possible to point out the category of so called **Subjective Version**;

(iii) If such kind of S/O relations is not reflected in an argument structure of a verb and, respectively, no changes of a verb valency takes place, the verb remains unchanged – no vowel-prefixes are observed; Yet through comparison it is possible to mark out a so called **Neutral Version**.

So, regarding the category of version there are three different forms: subjective, objective and neutral versions. The neutral version in Laz has no marker, the subjective version is expressed through the morpheme **i-** and the objective version - by the morphologically defined allomorphs: **u-, i-, a-**.

[5]Laz passive forms particularly derived by **a- -e** circumfix, mostly are used to express potencialis (in Megrelian the forms of passive and potencialis are different; yet, in other Kartvelian languages the semantics of potencialis is not as strongly differentiating as it is in Megrelian or Laz). Passive verb forms used in the function of potentialis show the inversion of person markers and the subject case (Kartozia, 2005).

[6]Some parallel forms are documented as well (Kartozia 2005).

[7](4)-case of subjective version in Laz is quite limited and unacceptable, especially for Sarphians (Chikobava 1936); that is, Laz speakers use subjective version form mostly for grammaticalization of reflexives – (3)-case.

Neutral version S, (IO), (DO)	Subjective version S→IO(=S)>Ø (S, DO→ IO(=S)>Ø)	Objective version			
		Transitive verbs S, DO→IO		Intransitive verb S→IO	
		IO=I/II	IO=III	active	passive
do-p'-č'k'iri (I have cut sth.)	***do-v-i-č'k'iri*** (I have cut my sth.)	***do-g-i-č'k'iri*** (I've cut you sth.)	***do-v-u-č'k'iri*** (I've cut sth. for him)	***v-u-bir*** (I am singing for you)	***m-a-giben*** (It is boiling for me)
bɣaps (S/he shaves sb./sth.)	***i-bɣaps*** (He shaves himself)	***m-i-bɣaps*** (S/he shaves sth. for me//my sth.)	***u-bɣaps*** (S/he shaves sth. for him//his sth.)	***do-v-i-q'uri*** (I shout to sb.)	***m-a-q'vilen*** (It is slaughtering) for me)
bonums (S/he washes sb.)	***i-bonums*** (S/he washes him/herself)	***m-i-bonums*** (S/he washes my sth.//sth. fot me)	***u-bonums*** (S/he washes his/her sth. for sb.)	***m-i-k'ank'al-s*** (My sth. shakes//It shakes for me)	***m-a-mbinen*** (It is binding for me)

Table 1: Examples of version

Causatives

If in the course of increasing of verb-valency an agent appears to bear the role of an initiator, "forcer"[8] of an action, not only the number of verb arguments increases but also changes the functional qualification of the arguments structure.

In Laz (similar to other Kartvelian languages) such a change is followed by the structural, morphological changes of a verb form: the vowel-prefix **o-** appears and a verb root becomes complicated either by the suffix **-in** or by a so called series marker **-ap** that further can't be qualified as a series marker any more, as far as it merges with a verb root and remains in all tense forms; at the same time in the present tense forms this **-ap** is followed by a formally similar **-ap** marker[9] which according to a common rule must be removed to the second series forms. Hence, the circumfixes **o- -in** and **o- -ap** might be suggested as causative markers. As for the changes of the verb arguments structure there are two options:

(i) A transitive verb transforms into a tri-personal one: the agent-initiator is actualized as a subject; the former subject transfers into an indirect object; an indirect object (if any, viz. the verb is initially tri-personal) loses its functional status of the argument and becomes an adverb[10]; the direct object's qualification remains unchanged;

(ii) An intransitive verb transforms into a bi-pesonal transitive verb: the agent- initiator is actualized as a subject; the former subject becomes a direct object, and the indirect object (if any, viz. the verb is initially bi-personal) loses its functional status of the argument and becomes an adverb.

[8] "Causation" in its wide sense may mean "giving permission" as well; usage of causative forms in this sense in Laz is quite frequent (Hollisky 1991).

[9] The **-ap** series marker is documented in Khopa; in other dialects of Laz it is substituted by **-am**.

[10] The former indirect object is given in Allative or Ablative cases and t govern verb agreement models any more.doesn

Functional changes	Present	Aorist
bi-personal transitive verb: Ag_{int}→S, S→IO, DO=DO	***č'ar-up-s*** → ***o-č'ar-ap-ap/am-s*** (S/he writes→S/he makes sb. write)	***o-č'ar-ap-u***
tri-personal transitive verb: Ag_{int}→S, S→IO, DO=DO, IO→ADV	***bon-um-s*** → ***o-bon-ap-ap/am-s*** (S/he washes sth. to sb. →S/he makes sb. wash sth. for him/her)	***do-(o)-bon-ap-u***
mono-personal intransitive verb: Ag_{int}→S, S→IO	***ɣur-u-n*** → ***o-ɣur-in-ap/am-s*** (S/he dies →S/he kills sb.)	***o-ɣur-in-u***
bi-personal intransitive verb: Ag_{int}→S, S→IO, IO→ADV	***u-bir-s*** → ***o-bir-ap-ap/am-s*** (S/he sings for sb. → S/he makes sb. sing for sb.)	***o-bir-ap-u***

Table 2: Examples of causative forms

Nearly all the vowel-prefixes are poly-functional and are used for various additional purposes as well; e.g.: **u-** appears in evidential forms, also to introduce an indirect object not expressing an objective version; **i-** appears in some verbs with concrete semantics representing neither voice nor version forms; etc. (Holisky 1991).

Conclusions

Above presented interpretation of vowel-prefixes as valency changing markers seems more adequate as far as it gives a possibility to generalize all specific cases and give a comprehensive analysis of the semantics and functions of pre-root vowel-prefixes:

(i) (i) **o-** marks out the increase of verb valency when Agent or Initiator of an action appears in the verb arguments structure (resp. causatives and equal to causative forms);

(ii) **u-/i-** marks out the increase of verb valency to introduce an Addressee-Possessor-Benefactive into the verb arguments structure (resp. objective version) or to add a pure Addressee that doesn't mean possession at all;

(iii) **i-** marks out a decrease of verb valency when in the verb arguments structure disappears either an Agent (resp. mono-personal passives often expressing the semantics of potentialis) or Addressee-Possessor-Benefactive (resp. subjective version mostly expressing the semantics of reflexive);

(iv) **a-** marks out the more complicated case: simultaneously, Agent's disappearance and Addressee's appearance mostly (but not always) showing possessive-benefactive relations (resp. bi-personal passives usually expressing the meaning of potentialis).

Admissibility and unifiability in contact logics

Philip Balbiani and Çiğdem Gencer

Introduction

The decision problem of unifiability in a logical system L can be formulated as follows: given a formula $\phi(X_1, \ldots, X_n)$, determine whether there exists formulas $\psi_1, \ldots, \psi_n$ such that $\phi(\psi_1, \ldots, \psi_n) \in L$. The research on unifiability was motivated by a more general decision problem, the admissibility problem: given an inference rule $(\{\phi_1(X_1, \ldots, X_n), \ldots, \phi_m(X_1, \ldots, X_n)\}, \psi(X_1, \ldots, X_n))$, determine whether for all formulas $\chi_1, \ldots, \chi_n$, if $\{\phi_1(\chi_1, \ldots, \chi_n), \ldots, \phi_m(\chi_1, \ldots, \chi_n)\} \subseteq L$, $\psi(\chi_1, \ldots, \chi_n) \in L$. Rybakov [9] proved that there exists a decision procedure for determining whether a given inference rule is admissible in intuitionistic propositional logic. See also [10]. Later on, Ghilardi [7, 8] proved that intuitionistic propositional logic has a finitary unification type. See also [5, 6].

Contact logics are logics for reasoning about the contact relations between regular subsets in a topological space [2]. They are based on the primitive notion of regular regions and on the Boolean operations (empty region, complement of a region and union of two regions) that allow to obtain new regular regions from given ones. The main semantics of contact logics are the contact algebras of the regular subsets in a topological space [3, 4]. But contact logics have also received a relational semantics that allow to use methods from modal logic for studying them [1]. In this setting, one important issue, of course, is the mechanization of reasoning in contact logics. Since admissible inference rules can be used to improve the performance of any algorithm that handles provability, it becomes natural to consider the admissibility problem and the unifiability problem within the context of contact logics.

In this paper, we prove that the admissibility problem and the unifiability problem are decidable in contact logics.

Syntax

Let AT be a countable set of atomic terms (with typical members denoted x, y, etc) and AF be a countable set of atomic formulas (with typical members denoted X, Y, etc). Terms (denoted a, b, etc) are inductively defined as follows:

$$a ::= x \mid 0 \mid -a \mid (a \sqcup b).$$

Reading terms as regions, the constructs 0, $-$ and $\sqcup$ should be regarded as the empty region, the complement operation and the union operation. Formulas (denoted ϕ, ψ, etc) are inductively defined as follows:

$$\phi ::= X \mid \bot \mid \neg\phi \mid (\phi \vee \psi) \mid C(a, b) \mid a \equiv b.$$

In contact logics, the language is used to reason about the contact relation between regions. Reading formulas as properties about regions, the constructs C and $\equiv$ should be regarded as the contact relation and the equality relation. Sets of formulas will be denoted Γ, Δ, etc. Terms, formulas and sets of formulas are also called "expressions" (denoted α, β, etc). We shall say that an expression α is weak iff no atomic formula occurs in α.

A substitution is a function s assigning to each atomic term x a term $s(x)$ and to each atomic formula X a formula $s(X)$. As usual, s induces a homomorphism $s(\cdot)$ assigning to each expression α an expression $s(\alpha)$.

Axiomatization

Formulas will also be called "axioms" and pairs of the form (Γ, ϕ) where Γ is a finite set of formulas and ϕ is a formula will also be called "inference rules". An axiomatic system consists of a collection of axioms and a collection of inference rules. Let λ_0 be the axiomatic system consisting of

- a complete set of axioms for Classical Propositional Calculus (i.e. $X \to (Y \to X)$, $(X \to (Y \to Z)) \to ((X \to Y) \to (X \to Z))$, etc),
- a complete set of axioms for non-degenerate Boolean algebras (i.e. $x \sqcup (y \sqcup z) \equiv (x \sqcup y) \sqcup z$, $x \sqcup y \equiv y \sqcup x$, etc),
- the following axioms: (i) $C(x,y) \to x \not\equiv 0$, (ii) $C(x,y) \to y \not\equiv 0$, (iii) $C(x,y) \wedge x \leqslant z \to C(z,y)$, (iv) $C(x,y) \wedge y \leqslant z \to C(x,z)$, (v) $C(x \sqcup y, z) \to C(x,z) \vee C(y,z)$, (vi) $C(x, y \sqcup z) \to C(x,y) \vee C(x,z)$,
- the inference rule of modus ponens (i.e. $(\{X, X \to Y\}, Y)$).

The extension of λ_0 with an arbitrary set A of axioms will be denoted $\lambda_0(A)$. The provable formulas of an extension λ of λ_0 will be called "theorems of λ". We will denote by $Th(\lambda)$ the set of all theorems of λ.

Admissibility

Let λ be an extension of λ_0. An inference rule (Γ, ϕ) is said to be admissible in λ iff for all substitutions s, if $s(\Gamma) \subseteq Th(\lambda)$, $s(\phi) \in Th(\lambda)$. In this respect, we will study the decidability of the following decision problem, called "weak admissibility problem in λ", in symbols $wADM(\lambda)$:

- input: a weak inference rule (Γ, ϕ),
- output: determine whether (Γ, ϕ) is admissible in λ.

It should be remarked that for all weak inference rules (Γ, ϕ), if (Γ, ϕ) is derivable in λ, (Γ, ϕ) is admissible in λ. Nevertheless, in the general case, it may happen that derivability and admissibility do not coincide. It suffices, for instance, to consider the weak inference rule $(\{C(x,y)\}, C(y,x))$:

- since for all substitutions s, $s(C(x,y)) \notin Th(\lambda_0)$, $(\{C(x,y)\}, C(y,x))$ is admissible in λ_0,
- since $C(x,y) \to C(y,x) \notin Th(\lambda_0)$, $(\{C(x,y)\}, C(y,x))$ is not derivable in λ_0.

Obviously, for all weak inference rules $(\Gamma(x_1, \ldots, x_n), \phi(x_1, \ldots, x_n))$, $(\Gamma(x_1, \ldots, x_n), \phi(x_1, \ldots, x_n))$ is not admissible in λ iff there exists terms $a_1, \ldots, a_n$ such that $\Gamma(a_1, \ldots, a_n) \subseteq Th(\lambda)$ and $\phi(a_1, \ldots, a_n) \notin Th(\lambda)$. Let n be a nonnegative integer. Let Φ_n be the set of all weak formulas with atomic terms in $x_1, \ldots, x_n$. We define on Φ_n the equivalence relation $\equiv_\lambda^n$ as follows: $\phi(x_1, \ldots, x_n) \equiv_\lambda^n \psi(x_1, \ldots, x_n)$ iff $\phi(x_1, \ldots, x_n) \leftrightarrow \psi(x_1, \ldots, x_n) \in Th(\lambda)$. Obviously, $\equiv_\lambda^n$ has finitely many equivalence classes on Φ_n. Let A_n be the set of all n-tuples of terms. Note that n-tuples of terms in A_n may contain occurrences of atomic terms distinct from x_1, $\ldots$, x_n. We define on A_n the equivalence relation $\cong_\lambda^n$ as follows: $(a_1, \ldots, a_n) \cong_\lambda^n (b_1, \ldots, b_n)$ iff for all weak formulas $\phi(x_1, \ldots, x_n)$ in Φ_n, $\phi(a_1, \ldots, a_n) \in Th(\lambda)$ iff $\phi(b_1, \ldots, b_n) \in Th(\lambda)$. Obviously, $\cong_\lambda^n$ has finitely many equivalence classes on A_n. It is of interest to consider the equivalence relation $\cong_\lambda^n$, seeing that, according to our definitions, if $(a_1, \ldots, a_n) \cong_\lambda^n (b_1, \ldots, b_n)$, for all weak inference rules $(\Gamma(x_1, \ldots, x_n), \phi(x_1, \ldots, x_n))$, $\Gamma(a_1, \ldots, a_n) \subseteq Th(\lambda)$ and $\phi(a_1, \ldots, a_n) \notin Th(\lambda)$ iff

$\Gamma(b_1, \ldots, b_n) \subseteq Th(\lambda)$ and $\phi(b_1, \ldots, b_n) \notin Th(\lambda)$. Now, we define on A_n the equivalence relation $\simeq_\lambda^n$ as follows: $(a_1, \ldots, a_n) \simeq_\lambda^n (b_1, \ldots, b_n)$ iff for all C-free weak formulas $\phi(x_1, \ldots, x_n)$ in Φ_n, $\phi(a_1, \ldots, a_n) \in Th(\lambda)$ iff $\phi(b_1, \ldots, b_n) \in Th(\lambda)$. Obviously, if $(a_1, \ldots, a_n) \cong_\lambda^n (b_1, \ldots, b_n)$, $(a_1, \ldots, a_n) \simeq_\lambda^n (b_1, \ldots, b_n)$. Moreover, $\simeq_\lambda^n$ has finitely many equivalence classes on A_n. In other respect, one can prove the following

Lemma 1. *If $(a_1, \ldots, a_n) \simeq_\lambda^n (b_1, \ldots, b_n)$, $(a_1, \ldots, a_n) \cong_\lambda^n (b_1, \ldots, b_n)$.*

By Lemma 1, $wADM(\lambda)$ would be decidable if $Th(\lambda)$ is decidable and a complete set of representatives for each class on A_n modulo $\simeq_\lambda^n$ could be computed. Let us prove the following

Proposition 1. *If there exists a finite set A of axioms such that $\lambda = \lambda_0(A)$, a complete set of representatives for each class on A_n modulo $\simeq_\lambda^n$ can be computed.*

As a result,

Proposition 2. *If there exists a finite set A of axioms such that $\lambda = \lambda_0(A)$, $wADM(\lambda)$ is decidable.*

Proof. Suppose there exists a finite set A of axioms such that $\lambda = \lambda_0(A)$. Hence, according to [2], $Th(\lambda)$ is decidable. We define an algorithm taking as input a weak inference rule $(\Gamma(x_1, \ldots, x_n), \phi(x_1, \ldots, x_n))$ and returning the value *true* iff $(\Gamma(x_1, \ldots, x_n), \phi(x_1, \ldots, x_n))$ is admissible in λ as follows: (i) compute a complete set $\{(a_1^1, \ldots, a_n^1), \ldots, (a_1^N, \ldots, a_n^N)\}$ of representatives for each class on A_n modulo $\simeq_\lambda^n$; (ii) if there exists a positive integer k such that $k \leqslant N$, $\Gamma(a_1^k, \ldots, a_n^k) \subseteq Th(\lambda)$ and $\phi(a_1^k, \ldots, a_n^k) \notin Th(\lambda)$ then return $false$ else return $true$. By Lemma 1, this algorithm is sound and complete with respect to $wADM(\lambda)$. Since there exists a finite set A of axioms such that $\lambda = \lambda_0(A)$, by Proposition 1, steps (i) and (ii) can be executed. $\square$

However, when there exists a finite set A of axioms such that $\lambda = \lambda_0(A)$, the exact complexity of $wADM(\lambda)$ is not known.

Unifiability

Let λ be an extension of λ_0. A formula ϕ is said to be unifiable in λ iff there exists a substitution s such that $s(\phi) \in Th(\lambda)$. It happens that if λ is consistent, ϕ is unifiable in λ iff $(\{\phi\}, \perp)$ is not admissible in λ. Now, let us consider the following decision problem, called "weak unifiability problem in λ", in symbols $wUNI(\lambda)$:

- input: a weak formula ϕ,
- output: determine whether ϕ is unifiable in λ.

Obviously, for all weak formulas $\phi(x_1, \ldots, x_n)$, ϕ is unifiable in λ iff there exists $(\epsilon_1, \ldots, \epsilon_n) \in \{0,1\}^n$ such that $\phi(\epsilon_1, \ldots, \epsilon_n) \in Th(\lambda)$. Hence, it is easy to check that if $Th(\lambda)$ is decidable, $wUNI(\lambda)$ is decidable. Now, remark that for all weak formulas $\phi(x_1, \ldots, x_n)$ and for all $(\epsilon_1, \ldots, \epsilon_n) \in \{0,1\}^n$, $\phi(\epsilon_1, \ldots, \epsilon_n)$ is equivalent modulo $\equiv_\lambda^n$ to one of the following formulas: $\perp$, $\top$, $C(1,1)$, $\bar{C}(1,1)$. Moreover, even if $Th(\lambda)$ is undecidable, the formula in $\{\perp, \top, C(1,1), \bar{C}(1,1)\}$ that is equivalent modulo $\equiv_\lambda^n$ to $\phi(\epsilon_1, \ldots, \epsilon_n)$ in λ can be computed. As a result, in all cases, i.e. whatever is the decidability status of $Th(\lambda)$,

Proposition 3. *$wUNI(\lambda)$ is decidable.*

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Completeness of Modal Logic Interpreted over Iterated Cantor Bendixon derivative operators

Philippe Balbiani and Levan Uridia

Introduction

The derived set $d(A)$ of a set $A \subseteq X$ of points is the set of all limit points of A with respect to a given topology τ on a nonempty set X. Introduced by Cantor, the derivative operator d possesses interesting properties. In particular, a set $A \subseteq X$ of points is closed if $d(A) \subseteq A$. A consequence of the entire description of the topology in terms of derived sets is the possibility to use derivative operators d as the primitive notion in topology. What happens if we iterate the derivative operator d, considering the sequence $d, d^2, d^3, ...$ of operators? If the topology τ satisfies T_D separation axiom then each element d^k of this sequence is again a derivative operator.

Now a question arises: what is the link between the topologies τ_α corresponding to the elements d^α of the sequence? The answer is simple: the topologies τ_α are getting finer when α increases. Since the lattice of all T_D topologies on a given nonempty set X is complete, this iteration process should stop. The Cantor-Bendixson rank of $(X; \tau)$ is then defined as the least ordinal α such that $d^\alpha(X) = dd^\alpha(X)$. A consequence of Tarski's fixpoint theorem [1] is that there exists an ordinal α^* such that $\alpha \leqslant \alpha^*$ and $d \circ d^{\alpha^*} = d^{\alpha^*}$, the greatest fixpoint of d.

After McKinsey and Tarski [2] suggested to treat modality as the derivative operator of a topological space, Esakia introduced **wK4**, the modal logic of all topological spaces, with the desired (derivative operator) interpretation of the $\Diamond$ - modality. **K4** is an extension of **wK4** and is characterized in this semantics by the class of all T_D-spaces [3]. It has been proved that the derived set topological semantics (also called d-semantics) is more expressive than C-semantics [4], [5]. In this paper we study derived set interpretation of multi modal language with two modalities $\Diamond$ and $\Diamond^*$ where we have special interconnection between the two modalities on the semantical level. In particular if d is the derivative operator for interpreting $\Diamond$, then the semantics of $\Diamond^*$ is provided by d^{α^*} described above. Similar approach has been considered in our previous paper [6] where the two modalities were interpretted over strict partial orders. The initial attempt of the mentioned paper was to prove topological completeness with actual interpretation of modalities, although example given by David Gabelaia showed that the logic from [6] is not complete with respect to topological semantics. In actual paper as an extension of the previous work as a main result we provide an axiomatisation of the multimodal language and prove the completeness theorem with respect to the class of all topological structures with T_D separation axiom.

The paper is organised as follows: In section 2 we provide preliminary material about topological spaces and Cantor-Bendixon sequences. In Section 3 we describe a modal logic L. In section 4 we provide topological semantics for L and prove soundness and completeness results.

Preliminaries

We define topologies in terms of derivative operators. Consequently we define some topological notions from general topology in these terms.

Definition 11. A **derivative operator** on X is a function d: $\mathcal{P}(X) \to \mathcal{P}(X)$ such that $d(\varnothing) = \varnothing$, for all $A, B \subseteq X$, $d(A \cup B) = d(A) \cup d(B)$, for all $A \subseteq X$, $d(d(A)) \subseteq d(A) \cup A$, for all $x \in X$, $x \notin d(\{x\})$.

- $A \subseteq X$ is said to be **d-closed** iff $d(A) \subseteq A$;
- We shall say that d is $\mathbf{T_D}$ iff for all $A \subseteq X$, $d(d(A)) \subseteq d(A)$;
- d is said to be **Alexandroff** iff for all $x \in X$, there exists a greatest $A \subseteq X$ such that A is d-closed and $x \notin A$;

Clearly one can define Kuratowski closure operator as $C(A) = A \cup d(A)$ and then closed sets as fixpoints of the operator. Then it is immediate that d-closed sets are nothing else but closed sets of the topology, d is $\mathbf{T_D}$ iff the corresponding topology satisfies $\mathbf{T_D}$ separation axiom and d is Alexandroff iff the corresponding topology is Alexandroff space. For these notions reader can refer to [7].

Let $\leqslant$ be the binary relation between derivative operators on X such that $d \leqslant d'$ iff for all $A \subseteq X$, $d(A) \subseteq d'(A)$. Remark that for all derivative operators d, d' on X if $d \leqslant d'$, then if d' is T_D, then d is T_D as well. Given a T_D derivative operator d on X let $\mathbf{L_d}$ be the set of all derivative operators d' on X such that $d' \leqslant d$. Remark that the least element of L_d is the derivative operator $d_\varnothing$: $\mathcal{P}(X) \to \mathcal{P}(X)$ such that for all $A \subseteq X$, $d_\varnothing(A) = \varnothing$. The greatest element of L_d is d itself. Hence $(L_d, \leqslant)$ is a complete lattice.

Given an Alexandroff T_D derivative operator d on X let $\theta_\mathbf{d}$ be the function θ_d: $L_d \to L_d$ such that for all $d' \in L_d$, $\theta_d(d') = d \circ d'$

Clearly θ_d is monotonic hence θ_d has a least fixpoint $\mathrm{lfp}(\theta_d)$ and a greatest fixpoint $\mathrm{gfp}(\theta_d)$. It is immediate that $\mathrm{lfp}(\theta_d) = d_\varnothing$ while $\mathrm{gfp}(\theta_d)$ is the least upper bound of the family $\{d'\colon d' \leqslant \theta_d(d')\}$ in L_d.

Cantor-Bendixson ranks of Alexandroff T_D derivative operators

For all ordinals α, we inductively define $\theta_\mathbf{d}{\downarrow}\alpha$ as follows

- $\theta_d{\downarrow}0$ is d;
- for all successor ordinals α, $\theta_d{\downarrow}\alpha$ is $\theta_d(\theta_d{\downarrow}(\alpha - 1))$;
- for all limit ordinals α, $\theta_d{\downarrow}\alpha$ is the greatest lower bound of the family $\{\theta_d{\downarrow}\beta\colon \beta \in \alpha\}$ in L_d;

From Tarski fixpoint theorem [1] it follows that there exists an ordinal α such that $\theta_d{\downarrow}\alpha = \mathrm{gfp}(\theta_d)$. The least ordinal α such that $\theta_d{\downarrow}\alpha = \mathrm{gfp}(\theta_d)$ is called the **Cantor-Bendixson rank** of d. Remark that standardly Cantor-Bendixon rank is defined in slightly different way and it just considers iterations of derivative operator on the entire space, while here iteration goes separately for an arbitrary subset of an entire space.

Example 1. Take $X = \mathbb{Z}$ and $d_{\mathbb{Z}}(A) = \{x\colon$ there exists $y \in A$ such that $x <_{\mathbb{Z}} y\}$. It is easy to check that $d_{\mathbb{Z}}(A)$ is a T_D derivative operator. Moreover $\theta_{d_{\mathbb{Z}}}(\theta_{d_{\mathbb{Z}}}{\downarrow}\omega) = \theta_{d_{\mathbb{Z}}}{\downarrow}\omega$ hence the Cantor-Bendixson rank of $d_{\mathbb{Z}}$ is ω.

Modal Logic

We consider multy-modal language with two modalities. Formulas are defined as follows: $\phi ::= p \mid \bot \mid \neg\phi \mid (\phi \vee \psi) \mid \Box\phi \mid \Box^\star\phi$. We have standard abbreviations for modalities $\Diamond\phi ::= \neg\Box\neg\phi$, $\Diamond^\star\phi ::= \neg\Box^\star\neg\phi$ and for the remaining Boolean operations.

Let L be the normal modal logic containing all propositional tautologies, axiom K for each box plus the following set of axioms:

1. $\Box\phi \to \Box\Box\phi$;
2. $\Box^\star\phi \to \Box^\star\Box^\star\phi$;
3. $\Box\phi \to \Box^\star\phi$;
4. $\Box^\star\phi \to \Box\Box^\star\phi$;
5. $\Box^\star\phi \to \Box^\star\Box\phi$;
6. $\Box\Box^\star\phi \to \Box^\star\phi$;
7. $\Box(p \to \Diamond p) \to (\Diamond p \to \Diamond^\star p)$;

The rules of inference are Modus-Ponens and generalisation for each box operator.

Topological semantics

Now we define topological semantics for the modal language described in previous section. The topological semantics for the diamond modalities is provided by derivative operators. In the literature this semantics is often referred as d-semantics.

Definition 12. A **topological frame** is a structure of the form $\mathcal{F} = (X, d, e)$ such that X is a nonempty set, d is a T_D derivative operator on X and e is the greatest fixpoint of the function θ_d in L_d

Lemma 1. *If $\mathcal{F} = (X, d, e)$ is a topological frame, then $d \circ d \leqslant d$, $e \circ e \leqslant e$, $e \leqslant d$, $d \circ e \leqslant e$, $e \circ d \leqslant e$ and $e \leqslant d \circ e$.*

A **topological model** is a structure of the form $\mathcal{M} = (X, d, e, V)$ such that (X, d, e) is a topological frame and V is a valuation on X. An interpretation of a formula φ in a topological model $\mathcal{M}$, in symbols $\parallel \varphi \parallel_{\mathcal{M}}$ is defined in the following way:

- $\| p \|_{\mathcal{M}} = V(p)$
- $\| \perp \|_{\mathcal{M}} = \varnothing$
- $\| \neg\phi \|_{\mathcal{M}} = X\backslash \| \phi \|_{\mathcal{M}}$
- $\| \phi \vee \psi \|_{\mathcal{M}} = \| \phi \|_{\mathcal{M}} \cup \| \psi \|_{\mathcal{M}}$
- $\| \Box\phi \|_{\mathcal{M}} = X\backslash d(X\backslash \| \phi \|_{\mathcal{M}})$
- $\| \Box^{\star}\phi \|_{\mathcal{M}} = X\backslash e(X\backslash \| \phi \|_{\mathcal{M}})$

We shall say that a formula φ is valid in a topological frame $\mathcal{F} = (X, d, e)$ in symbols $F \models \varphi$, if for all topological models $\mathcal{M} = (X, d, e, V)$ on $\mathcal{F}$, $\| \varphi \|_{\mathcal{M}} = X$.

Now we prove soundness of the logic L with respect to the class of all topological frames. It is easy to notice the connection between the six properties of Lemma 1 and Axioms 1-6 of the logic L. The main idea for showing that Axiom 7 is also valid is given in [8] section 8.5.

Lemma 2. *The logic L is sound w.r.t. the class of all topological frames.*

Next we prove completeness of the logic L with respect to the class of all topological frames. This result is the main contribution of the paper. The first part of the proof follows well adopted technique to reduce topological completeness to Kripke completeness. Proving Kripke completeness itself is not trivial and we use tableau method for this part. For space limitations we do not present details.

Theorem 1. *If a formula φ is valid in the class of all topological frames then it is provable in the logic L.*

Conclusion and future work

The paper presents modal logic with two modal operators interpreted on topological spaces where the first modality is interpreted as a derivative of a space while the second modality is interpreted as an iterated Cantor-Bendixon derivative of the first derivative operator. There are several questions to be considered as a future work. Firstly definability issues are not studies. Secondly finite model property (fmp) or decidability of the logic is also unknown. Moreover it is interesting to consider some extensions of the logic with well known operators such as difference modality or universal modality.

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LTAG semantics for issues around focusing
Kata Balogh

The main purpose of the presented work is twofold: (1) it proposes an analysis of the syntax-semantics interface missing in the preceding work [2], which provides a context-based approach of focusing, that gives a logical-semantic analysis of focus constructions within the framework of Inquisitive Semantics [4]; and (2) it wants to extend the system to broad focus constructions, intonation and focus marking, and the interaction of quantifier scope and focusing. I propose here an analysis in Lexicalized Tree-Adjoining Grammar [6], deriving the semantic representations of various focused sentences based on their syntactic structure and focus marking. I extend the LTAG focus analysis in Balogh [3], based on the LTAG semantics by Kallmeyer & Romero [8].

One of the main aims is the analysis of the syntax-semantics interface, where the semantic representations of sentences with different focus structures are derived in a compositional and uniform way. The analysis is provided within LTAG with a semantic component as developed by Kallmeyer & Joshi [7] and Kallmeyer & Romero [8]. The derived semantic representations are given and can be further interpreted in the system of Inquisitive Semantics [5]. The choose and combination of the two frameworks – LTAG and InqS – is motivated in several ways. One of the main claims in favor of InqS is – above the classical focus semantics approaches[1] –, that it provides a logical semantics together with a dialogue management system, so that it offers a straightforward and elegant way to analyze various discourse-related phenomena involving focus such as: focusing in answers, question-answer relations, contrast in denial and specification by focusing (see [2]). However, the analysis of [2] lacks the important component of the syntax-semantics interface. To fill this gap, I adapt the LTAG semantics as introduced by Kallmeyer & Romero [8]. One of the main motivations using this system is the elegant way it deals with quantifier scope, that offers a promising solution for the issues around the interaction of focus and quantification.

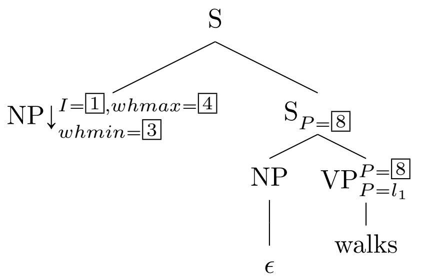
Background

Inquisitive Semantics. The derived semantic representations are given in the logical language of Inquisitive Semantics [5], serving the broader purpose to integrate the analysis with a component of semantics-pragmatics interface and modeling question-answer relations. In InqS, all utterances are claimed to be divided into a *theme* and a *rheme*, where the rheme corresponds to the information content of the given utterance and the theme to the issue behind. In (Balogh 2009; 2012) an analysis of focused sentences is proposed, claiming that focusing leads to a special theme-rheme division. Next to the parallelisms with the distinction of new and old information in the generative view, an important difference is that in this analysis the sentences itself are not split into two parts, but the way is defined how to signal the inherent issue (theme) of the utterance and the data it provides (rheme). The theme of an utterance is always inquisitive (and non-informative), introducing two or more *alternatives.* For example, the semantic

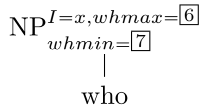
[1] See e.g. Babko-Malaya [1], that integrates LTAG with Alternative Semantics.

translation of the question *Who walks?* is $?\exists x.walks(x)$ that leads to the set of *alternatives*: $walk(a), walks(b), ..., \neg\exists x.walks(x)$ depending on the given domain of individuals.[2]

LTAG semantics. In this approach each elementary tree comes with a feature-structure and a (flat) semantic representation, each of them consisting of a set of labelled propositions and a set of scope constraints. These propositions and constraints contain meta-variables of individuals, propositions or situations, all of them given by boxed numbers. The feature structures are all linked to a semantic representation and by substitution and adjunction of the trees, feature structures get unified and the meta-variables get values. Also the semantic representation of the resulting tree is calculated by taking the union of the representations of the participating trees. For an illustration, see the following example, the LTAG derivation of the question *Who walks?*.



$l_0 : ?\boxed{4},\ l_1 : walk(\boxed{1})$
$\boxed{4} \geqslant \boxed{3},\ \boxed{3} \geqslant \boxed{8}$



$l_2 : \exists x \boxed{5}$
$\boxed{6} \geqslant l_2,\ \boxed{5} \geqslant \boxed{7}$

The features WHMAX and WHMIN indicate the scope window for (wh-)questions and for focus, inspired by the MAXS and MINS features introduced by Kallmeyer & Romero [8] that indicate the scope window (maximum/minimum scope) of a given quantificational phrase. Next to the propositions l_0 and l_1, the scope constrains $\boxed{4} \geqslant \boxed{3}$, $\boxed{3} \geqslant \boxed{8}$ are defined that determine the scope relations between the given propositions. The NP-tree of the wh-phrase gets substituted into the S-tree of 'walks' resulting in the equations $\boxed{1} = x, \boxed{4} = \boxed{6}, \boxed{3} = \boxed{7}$ and since nothing is adjoined at the VP node, we have $\boxed{8} = l_1$. After these equations the combination of the semantic representations results in:

$l_0 : ?\boxed{4},\ l_1 : walk(x),\ l_2 : \exists x \boxed{5}$
$\boxed{4} \geqslant \boxed{3},\ \boxed{3} \geqslant l_1,\ \boxed{4} \geqslant l_2,\ \boxed{5} \geqslant \boxed{3}$

Following these scope constraints, the possible plugging is: $\boxed{4} \mapsto l_2, \boxed{5} \mapsto l_1, \boxed{3} \mapsto l_1$, resulting in the InqS-style semantic representation: $?\exists x.walks(x)$.

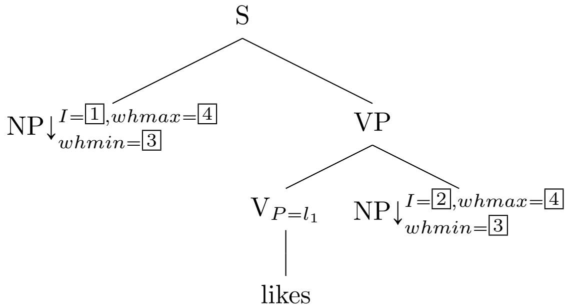
Proposal: representing focus in LTAG

As claimed before, a focused utterance gives rise to a special theme-rheme division. The focus marking effects the way, how the issue behind the utterance is derived. In [3] I adapt this idea in the LTAG approach, so the semantic representations of utterances always consist of two components: one that represents the theme and one that represents the rheme. Accordingly, each S-tree comes with a semantic representation as illustrated below: the above part represents the theme, and the below one represents of the rheme. Hence, in the semantics of all S-trees come the theme leads to a question, the *issue behind*, and the rheme to the data provided, the *semantic content*.

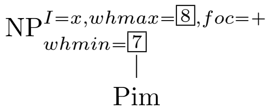
[2]By the space limits, I skip the details of the language of InqS; see [5].

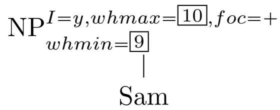
$$\left\langle \begin{array}{l} \boxed{\begin{array}{l} l_0 : ?\boxed{i},\ l_1 : R^n(\boxed{t_1}, ..., \boxed{t_n}) \\ \{\boxed{i} \geqslant \boxed{j}, \boxed{j} \geqslant l_1, ...\} \end{array}} \\ \boxed{\begin{array}{l} l_1 : R^n(\boxed{t_1}, ..., \boxed{t_n}) \\ \{\text{constraints}\} \end{array}} \end{array} \right\rangle$$

The analysis derives the special theme and rheme of different focus constructions including multiple foci and focusing in questions. The advantage of this approach is that sentences with different focus structures bear the same propositional content (rheme), while they lead to different inherent issues (themes) indicating that these sentences are felicitous in different contexts. Consequently, they relate to different wh-questions, which offers a straightforward way to deal with the basic cases of question-answer congruence. This analysis follows the core ideas of the context-based approach of [2], that concentrates merely on the interpretation of different focus constructions. The proposed analysis in LTAG provides an extension with the syntax-semantics interface and derives in a straightforward, intuitive and compositional way the intended semantic representations. For a quick illustration, see the LTAG derivation of multiple focus constructions below. In multiple focus constructions, such as *PIM_F likes SAM_F*, both arguments are focused, that leads to the underlying issue (theme) as the multiple wh-question *Who likes whom?* and to the information content (rheme) as the proposition *Pim likes Sam*. The analysis derives the correct theme-rheme division, by substituting the NP-trees of the focused arguments into the S-tree of 'likes'.



$$\left\langle \begin{array}{l} \boxed{\begin{array}{l} l_0 : ?\boxed{4},\ l_1 : like(\boxed{1}, \boxed{2}) \\ \boxed{4} \geqslant \boxed{3}, \boxed{3} \geqslant l_1 \end{array}} \\ \boxed{l_1 : like(\boxed{1}, \boxed{2})} \end{array} \right\rangle$$





$$\left\langle \begin{array}{l} \boxed{l_2 : \exists x \boxed{11}, \boxed{8} \geqslant l_2, \boxed{11} \geqslant \boxed{7}} \\ \boxed{l_2 : pim(x)} \end{array} \right\rangle \left\langle \begin{array}{l} \boxed{l_3 : \exists y \boxed{12}, \boxed{10} \geqslant l_3, \boxed{12} \geqslant \boxed{9}} \\ \boxed{l_3 : sam(y)} \end{array} \right\rangle$$

The substitutions of the trees of the focused subject (PIM_F) and object (SAM_F) into the tree of 'like' lead to the semantic representation:

$$\left\langle \begin{array}{l} \boxed{\begin{array}{l} l_0 : ?\boxed{4},\ l_1 : like(x, y),\ l_2 : \exists x \boxed{11},\ l_3 : \exists y \boxed{12}, \\ \boxed{4} \geqslant \boxed{3}, \boxed{3} \geqslant l_1, \boxed{4} \geqslant l_2,\ \boxed{11} \geqslant \boxed{3}, \boxed{4} \geqslant l_3, \boxed{12} \geqslant \boxed{3} \end{array}} \\ \boxed{l_1 : like(x, y),\ l_2 : pim(x),\ l_3 : sam(y)} \end{array} \right\rangle$$

For the disambiguation, two different pluggings are possible: (i) $\boxed{4} \mapsto l_2, \boxed{11} \mapsto l_3, \boxed{12} \mapsto l_1, \boxed{3} \mapsto l_1$ and (ii) $\boxed{4} \mapsto l_3, \boxed{12} \mapsto l_2, \boxed{11} \mapsto l_1, \boxed{3} \mapsto l_1$, yielding the logically equivalent semantic representations: $\exists x \exists y.like(x,y)$ and $\exists y \exists x.like(x,y)$ that both correspond to the question *Who likes whom?*

New issues in the analysis

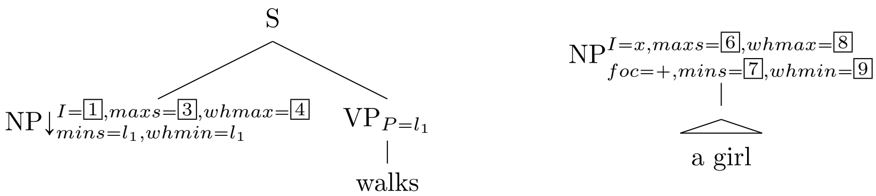
In my presentation I address further issues of the LTAG-analysis (based on Balogh [3]) of more focus related phenomena, such as the relation of accent placement and focus, and the relation of focusing and quantifier scope.

Focus marking and accent

An important issue for the current approach is, how to analyze the relation between the placement of the pitch accent and the marking of the focused constituent. For this we need to introduce two features FOC and PITCH that stand for focus marking and accenting respectively. The placement of the pitch accent is given by the feature *pitch* = + coming from the lexicon together with the lexical anchor. The value of the pitch accent is then passed to the FOC feature that appears on some nodes of the elementary tree of the noun phrase. As for the focus projection, the + value of the FOC feature can be optionally passed up from the rightmost NP argument to the higher VP node marking the possible focus projection. This is not possible from the subject position (or from the not right-most argument), the focused NP in that position gets narrow focus interpretation. However, the picture is more complex, since by focusing we have to deal with the issues: (i) the information structure of the sentence: which part of the content is the Focus/Topic/Background; (ii) the coherent discourse: what is "given/retrievable" and "non-given/no-retrievable" information.

Focus and quantifier scope

I introduced the scope window for focus and questions by the features WHMAX/WHMIN based on MAXS/MINS by Kallmeyer & Romero [8]. In case we have both a quantifier and focus in the sentence, the distinction of the two scope windows gets relevant and important. Consider the sentence *[A GIRL]$_F$ walks*, having a quantificational NP in focus. Its theme refers to the focus/question-window by the features WHMAX/WHIMN, while its rheme makes use of the scope window by MAXS/MINS.



$$\left\langle \begin{array}{c} \boxed{\begin{array}{l} l_0 : ?\boxed{4},\ l_1 : walk(\boxed{1}) \\ \boxed{4} \geqslant l_1,\ \boxed{4} \geqslant \boxed{3} \end{array}} \\ \boxed{\begin{array}{l} l_1 : walk(\boxed{1}) \\ \boxed{3} \geqslant l_1 \end{array}} \end{array} \right\rangle \qquad \left\langle \begin{array}{c} \boxed{\begin{array}{l} l_2 : \exists x.\boxed{12} \\ \boxed{8} \geqslant l_2, \boxed{12} \geqslant \boxed{9} \end{array}} \\ \boxed{\begin{array}{l} l_2 : \exists x.\boxed{10} \wedge \boxed{11}, l_3 : girl(x) \\ \boxed{10} \geqslant l_3, \boxed{6} \geqslant \boxed{11}, \boxed{11} \geqslant \boxed{7} \end{array}} \end{array} \right\rangle$$

After substitution of the NP-tree of *A GIRL$_F$* into the S-tree of 'walk', the underspecified semantic representation of the sentence *[A GIRL]$_F$ walks* is as below, that – after plugging – correctly derives the theme of the sentence as $?\exists x.walk(x)$ corresponding to the question *Who walks?* while the rheme as $\exists x.girl(x) \wedge walk(x)$ corresponding to the proposition *a girl walks.*

$$\left\langle \begin{array}{|l|}\hline l_0 : ?\boxed{4}, l_1 : walk(x), l_2 : \exists x\boxed{12} \\ \boxed{4} \geqslant l_1,\ \boxed{4} \geqslant \boxed{3},\ \boxed{4} \geqslant l_2,\ \boxed{12} \geqslant l_1 \\ \hline \\ \hline l_1 : walk(x), l_2 : \exists x\boxed{10} \wedge \boxed{11}, l_3 : girl(x) \\ \boxed{3} \geqslant l_1, \boxed{10} \geqslant l_3, \boxed{3} \geqslant \boxed{11}, \boxed{11} \geqslant\ l_1 \\ \hline \end{array} \right\rangle \rightsquigarrow \left\langle \begin{array}{|l|}\hline ?\exists x.walk(x) \\ \hline \\ \hline \exists x.person(x) \wedge walk(x) \\ \hline \end{array} \right\rangle$$

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Dialect Dictionaries with the Functions of Representativeness and Morphological Annotationin Georgian Dialect Corpus

Marina Beridze, Liana Lortkipanidze and David Nadaraia

Introduction

The Georgian Dialect Corpus is being developed within the framework of the larger project *Linguistic Portrait of Georgia*. Presently, the GDC is available for researchers on the web at `http://mygeorgia.ge/gdc/`.

> The GDC consists of: 1 453 261 tokens; 301 203 word forms; 199861 contexts; 3017 texts.

The material included in the GDC has been recorded from 2703 informants at 812 villages in Georgia, Turkey, Iran, and Azerbaijan. The earliest data date back to the early 20^{th} century, while the latest ones have been recorded in 2012. The working team of the GDC processes the collection of dialect texts. It carries out the entire technological procedure, beginning from field work to the inclusion of a text into the corpus; hence, the requirements of representativeness have been taken into account at the very moment of text recording. The corpus incorporates samples of 17 sub-varieties of Georgian and of the Laz dialect of Zan. Currently, the corpus can be queried for entire words and for word subsequences (beginning, inner part, ending). Query hits are shown as KWIC concordances. It is also possible to view a text as a whole, which turns the corpus into a library of dialect texts.

New texts are continuously being added to the corpus, and at the same time, the morphological annotation of the material is being worked on; therefore, so far, the corpus can only be queried according to the following meta-textual (non-linguistic) features:

- language and dialect
- place of recording
- the informant's identity
- thematic and chronological features of a text
- text type (narrative, poetry, conversation ...)

We plan to provide other query options (according to non-linguistic features) like text title, scientific author of a text, demographic data about an informant, etc.

Naturally enough, after the morphological annotation is in effect, querying for parts of speech and grammatical categories will be added.

To facilitate the morphological annotation of the corpus, we allocated a significant place to equipping dialect dictionaries with grammatical information and to applying them in the process of lemmatization and linguistic annotation. We decided to use the data of Georgian dialect lexicography to increase the lexical base (textual base) of the corpus as well.

The Problem of Representativeness in the GDC and Dialect Dictionaries

To achieve representativeness has been one of the primary tasks in corpus-building. Although the main principles of representativeness have long been established, a specific corpus project nevertheless faces the necessity of stating, 'defending' and implementing its own concept of representativeness. For instance, we expect any more or less important corpus initiative to be familiar with and to consider Douglas Biber's seminal paper [3]; however, truly enough, designers of individual corpora do not always follow Biber's canonical rules. Here is what G. Leech has to say about this: "A seminal article by Biber has frequently been cited, but no attempt (to my knowledge) has been made to implement Biber's plan for building a representative corpus" [6].

Clearly, whenever universal rules of representativeness are concerned, it should be borne in mind that the rules differ for general and special corpora [9]. It is, for instance, a necessary condition for a national corpus to be balanced according to genres and registers in order to function as a micro-model of a given language that is as accurate as possible, whereas for any other special corpus, dimensions of representativeness may be rather distinct.

Balance is conceived as a near synonym of representativeness in contemporary corpus linguistics; however, it should be admitted that corpus designers are more and more frequently facing conditions that oblige them 'to revise' this synonymic pair.

A dialect corpus is somehow a special corpus; as is known, the concept of representativeness is different in such a corpus [6, 7].

We chose an approach to corpus documentation of dialect data which implies that a dialect corpus should become a scientific source of a new type, one which would facilitate the representation and study not only of a language, but also of a linguistic-communicative model [4].

The representativeness concept of the Georgian Dialect Corpus was established with the national historico-cultural reality in mind, by considering the role and the place of this resource within the Georgian national scientific and cultural paradigm. The principal requirements according to this concept are as follows:

- completeness of lexical data
- completeness of linguistic annotation
- complete representation of dialect and intra-dialect strata
- completeness of age, gender, social variation and other metadata
- representation of sectoral, economic, folklore variation
- representationof mosaic variation caused by migration
- representation of features connected to chronological factors
- representation of speech features of smaller or marginal groups (for instance, speech of Georgian Jews ...)

We have already dealt with the composition of the textual base of the corpus in our other papers as well [1, 2]. We have also noted that, in order to increase the degree of representativeness of the corpus, we include non-linguistic components, that is, dictionaries.

The illustration block of a dictionary will be incorporated in the common concordance of the corpus in which a head word acts as a key word.

The decision was prompted by the following considerations: first, in accordance with the tradition established in Georgian dialectology, dialect data were described mainly for the sake of illustrating of scientific research and not for the creation of scientific publications of texts; hence, the data are fragments of valuable linguistic information, collected by qualified dialectologists, and they should be necessarily included in the common corpus 'context.'

Most Georgian dialect dictionaries are compiled by scholars who were native speakers of those dialects and had a thorough knowledge of both a dialect and the cultural, social, and economic space within which that dialect was spoken. Now, when this space is at the brink of extinction and referents of many words, as cultural and historical phenomena, no longer exist, these dictionaries preserve very valuable information.

Dictionaries are said to be a world arranged alphabetically. We want to create the most accurate dialect 'reflection' of this world by means of a corpus. So why shouldn't we be able to apply a dictionary to achieve this? Why shouldn't we use a head word as a key word and an illustration as a context? As they say, a concordance is 'a cut-off text.' A dictionary, like a concordance, cannot be used to reconstruct a primary text; however, it is a valuable material to represent a so-called 'cultural text'; moreover, the tissue of a dialectal 'cultural text' is destroyed on a daily basis, and a corpus, amalgamating the existing lexical repository as dialect texts, dialect dictionaries (and as other non-textual components), in the only way to preserve this cultural text.

Dictionaries and the Problem of Morphological Annotation in the Corpus

This paper describes the process of morphological annotation of the Upper Imeretian collection of the GDC based on the *Upper Imeretian Dictionary* [5].The annotation is based on **GeoTrans** (see

[8]), an automated morphological dictionary of Standard Georgian. At the present experimental stage of the morphological annotation of the GDC, the following has been achieved:

- Formatting of the dictionary: development of the digital version of the dictionary and creation of a list of lemmas (totally 5671 lemmas)
- Automated selection and part-of-speech tagging of the forms from the list of lemmas of the dialect dictionary, coinciding with those of the standard. Totally 784 such lemmas were detected; a list of homonyms was identified, totaling 27 items. After the operation, 4860 'unidentified' elements were spotted in the list of lemmas, which were manually ascribed part-of-speech tags.
- By means of the received marked lists, the knowledge base of the automated morphological dictionary of Standard Georgian was enriched.This implies that a subsystem for morphological modeling of a given dialect variety was added to **GeoTrans**. In this system, each dialect form will be tagged in accordance with a respective part of speech and, frequently, marked in accordance with an inflectional pattern by means of which word forms are lemmatized.
- At the next stage, the **GeoTrans** standard language-analyzer enabled us **to select the lemmas** from the textual data of the corpus, **coinciding with those of the standard language**, amounting to3331 lemmas.
- The GeoTrans dialect analyzer specific dialect lemmas were selected and tagged, 472 **lemmas** in total.
- By means of the standard language analyzer, all the **word forms** underwent complete morphological analyses, coinciding with those of the standard, which amounted to 9285 forms. Here too, homonymous (528) and non-homonymous (8757) forms will be similarly distinguished.
- Following that, by means of ***lemmas and standard inflectional patterns***, lemmatization was performed and dialect (specific) word forms were morphologically tagged.

Conclusion: Equipping dialect dictionaries with morphological information and in such a way enriching the morphological knowledge base by means of the automated standard analyzer is an optimistic perspective for the automation of dialect corpus analysis. The concept of morphological annotation of the GDC envisages a differentiated approach to text data: to present dialect (specific) vocabulary, vocabulary common with the standard language, inflectional and derivational patterns common with the standard language, dialect-specific inflectional and derivational patterns as separate 'sectors' and then, to undertake the annotation strategy accordingly.

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A coalgebraic logic for preordered coalgebras

Marta Bílková and Jiří Velebil

Moss' coalgebraic logic for Set coalgebras, introduced in Moss' pioneering paper [8], has become a lively field of study. Its finitary version has been explored by various authors, resulting in almost a full picture, including (among others) the axiomatization and completeness, nice proof theory, applications in automata theory, and applications in fixpoint logics. The Moss' language extends the boolean logic with a single modality whose arity is given by the coalgebra functor and whose semantics is given by a lifting of the satisfaction relation with the coalgebra functor. Despite its nonstandard syntax, the language is easy to deal with. In particular, it allows for a simple proof of the Hennessy-Milner property - it is expressive for *bisimilarity*.

It is natural to investigate possibilities of extending the results beyond the category of sets. We make a preliminary step in this direction, building on results obtained in [2], namely existence of a functorial *relation lifting* for functors preserving exact squares in the category Pre of preorders and monotone maps, providing the technical background for applications of the Moss' idea to *preordered coalgebras*. Examples of preordered coalgebras include e.g. ordered automata or frames for substructural logics. We present a finitary version of Moss' coalgebraic language based on the logic of distributive lattices equipped with a single cover modality nabla ∇, its semantics, and a sound axiomatics. As expected, axiomatics consists of certain distributive laws which are quite straightforward analogues of those known from the case of Set providing a complete axiomatics of Moss' logic for Set coalgebras, see [6]. Formally, the laws look similar as in the case of Set, yet the techniques used have to be more subtle and they reveal the hidden symmetries.

We prove that, for certain class of finitary functors (namely those preserving exact squares and admitting a base), the resulting finitary Moss' logic has the Hennessy-Milner property - it is expressive for a notion of *similarity* based on the relation lifting, which coincides with the notion of similarity given in literature, see e.g. [11, 4, 1, 7], and it also coincides with the preorder on the final coalgebra, if it exists. The result matches the similar result for Moss' logic for Set coalgebras, and it can also be seen as a counterpart to the result proved in [5] for positive coalgebraic logics in the category of posets, stating that for any finitary, locally-monotone, and embedding preserving poset-functor, the logic of all monotone predicate liftings is expressive.

Base

For the finitary setting to work, we need to develop, for a functor T, a notion of a *base*. It enables us to define, for each object in TX, a finite preorder of its "support" or "generators". In the case of Set, a base for a finitary standard set functor T preserving weak pullbacks is captured by a natural transformation $\mathsf{base}_X : TX \to \mathcal{P}_\omega X$, assigning to each element α of TX the smallest finite subset Y of X such that α is an element of TY. We want to develop an appropriate notion of base in the case of Pre, and prove it has properties analogous to the Set case. Base is later used to produce for each element of TX a finite preorder of its "generators", e.g. subformulas of a formula of arity T, or successors of a state in a T-coalgebra.

The powerset functor from the Set case is replaced with the subobject functor Sub_ω. For a preorder X, $\mathsf{Sub}_\omega(X)$ is the preorder of all finite subobjects of X, a finite subobject of X being an order-embedding of a finite preorder into X. Given a monotone map $h : X \to Y$, we define $\mathsf{Sub}_\omega(h) : \mathsf{Sub}_\omega(X) \to \mathsf{Sub}_\omega(Y)$ by sending $f : Z \rightarrowtail X$ to the $\mathcal{M}$-part $h^*(f) : h^*(Z) \rightarrowtail TX$ of the $(\mathcal{E}, \mathcal{M})$-factorisation of Tf ($(\mathcal{E}, \mathcal{M})$ being monotone surjections and order embeddings). Every T preserving order embeddings induces a functor $T^*_X : \mathsf{Sub}_\omega(X) \to \mathsf{Sub}(TX)$ that sends $f : Z \rightarrowtail X$ to $Tf : TZ \rightarrowtail TX$. We say that T *admits a base* , if for every X a free object $\mathsf{base}_X(\alpha) : \widetilde{Z} \rightarrowtail X$ w.r.t. T^*_X exists on every $\alpha : Z \rightarrowtail TX$, Z finite. The definition results in $\mathsf{base}_X : \mathsf{Sub}_\omega(TX) \to \mathsf{Sub}_\omega(X)$ which is a lax natural transformation. We cannot yet offer a characterization of functors admitting such a notion of base. However, the polynomial functors from the following section do so. For example for the finitary lowerset functor $\mathbb{L}_\omega$, the base of a finitely generated lowerset is the finite preorder of its generators.

Functors, coalgebras and relation lifting in Pre

The 2-category Pre has preorders as objects and monotone maps as arrows. The two-dimensional structure is given by the pointwise preorder of monotone maps. A functor is called 2-functor, or a locally monotone functor, if it preserves the two-dimensional structure. Examples of such functors include the Kripke polynomial functors defined by the following grammar:

$$T ::= const_X \mid Id \mid T^\partial \mid T + T \mid T \times T \mid T^A \mid \mathbb{L}T$$

where T^∂ is the *dual* of T, defined by putting $T^\partial A = (TA^{op})^{op}$ and $\mathbb{L}X = [X^{op}, \mathbb{2}]$ are the lowersets on X, ordered by inclusion. $\mathbb{U}X$, the uppersets on X, ordered by reversed inclusion, are obtained as $\mathbb{L}^\partial X = [X, \mathbb{2}]^{op}$. The finitary functor $\mathbb{L}_\omega$ is defined as follows: $\mathbb{L}_\omega X$ consists of finitely generated lowersets on X. A preordered T-coalgebra is a monotone map $c : X \to TX$. Examples of preordered coalgebras include e.g. ordered automata or frames for distributive substructural logics:

Example 1. A *deterministic ordered automaton*, [9], is a Büchi automaton with a partial order Q of states, the discrete alphabet A, and monotone transitions: if $x \leqslant y$ and $x.a$ is defined, then $y.a$ is defined as well and $x.a \leqslant y.a$. Such automata are coalgebras for the functor $Id^A \times \mathbb{2}$, i.e. coalgebras of the form $c : Q \to Q^A \times \mathbb{2}$, where $\mathbb{2}$ is the two-element preorder used to encode the final states. The nondeterministic ordered automata with the set E of transitions can be modelled as a coalgebra $c : Q \to (PQ)^A \times \mathbb{2}$, where the functor P assigns to any preorder the preorder of its subsets preordered by the first half of the Egli-Milner lifting.

Example 2. *Frames for distributive substructural logics* [10] are frames consisting of a preorder of states X, together with a ternary relation on X, satisfying the following monotonicity condition: $R(x, y, z)$ & $x' \leqslant x$ & $y' \leqslant y$ & $z \leqslant z'$ implies $R(x', y', z')$. It has been shown in [3], that they can be treated as coalgebras, so that the coalgebraic morphisms precisely coincide with the frame morphisms. The coalgebraic functor (more precisely the one of the three functors defined in

[3] which is locally monotone, and corresponds to the part of the frame interpreting the fusion connective) is $\mathbb{L}(Id \times Id)$.

A *monotone relation* R between preorders X and Y is, roughly speaking, a relation satisfying the condition: $R(x, y)$ and $x' \leqslant_X x$ and $y \leqslant_Y y'$ implies $R(x', y')$. Preorders together with monotone relation constitute a 2-category. If T is a locally monotone endofunctor on Pre which preserves certain squares, called *exact squares*, we can lift it to a locally monotone functor $\overline{T}$ on the category of preorders and monotone relations, see [2] for details. The lifting $\overline{T}$ thus preserves relation composition, relation inclusion, and it satisfies additional properties: e.g. it commutes with graphs, and $\overline{T^\partial} R = (\overline{T} R^{op})^{op}$, and restricts to bases. For example, the lifting of the lowerset functor is given by the first half of the Egli-Millner lifting: $\overline{\mathbb{L}} R(u, v)$ iff $(\forall x \in u)(\exists y \in v) R(x, y)$.

Moss' logic for preordered coalgebras

We fix a finitary functor $T : \mathsf{Pre} \to \mathsf{Pre}$ which preserves exact squares and admits a base, thus the 2-functors $\overline{T}$ and $\overline{T^\partial}$ exist, satisfying the required properties. We introduce a *finitary* coalgebraic language for T-coalgebras using the language of *distributive lattices* — finitary conjunction and disjunction operators — extended with the so called *cover* modality ∇_{T^∂}, whose arity is given by the *dual* of the coalgebra functor T, i.e. T^∂.

$$\bigwedge : \mathbb{U}_\omega \mathcal{L} \to \mathcal{L} \qquad \bigvee : \mathbb{L}_\omega \mathcal{L} \to \mathcal{L} \qquad \nabla_{T^\partial} : T^\partial \mathcal{L} \to \mathcal{L}$$

The language $\mathcal{L}$ is given by the finitary functor $\mathbb{U}_\omega + \mathbb{L}_\omega + T^\partial$: it is obtained as the (underlying preorder of the) free algebra of the functor over the preorder At. Given a coalgebra $c : X \to TX$ and a monotone valuation relation of propositional atoms, we obtain the semantics as follows:

$$x \Vdash \bigwedge \phi \text{ iff } (\forall a \in \phi)\ x \Vdash a \qquad x \Vdash \bigvee \phi \text{ iff } (\exists a \in \phi)\ x \Vdash a$$

$$x \Vdash \nabla_{T^\partial} \alpha \text{ iff } c(x)\ \overline{T^\partial}(\Vdash)\ \alpha$$

We can define a preorder on formulas, putting $a \leqslant_{\mathcal{L}} b$ iff for all coalgebras c, states x and all valuations $x \Vdash a$ implies $x \Vdash b$ (this preorder extends the one given by the construction of $\mathcal{L}$). It is easy to see that ∇_{T^∂} is monotone:

$$\alpha \leqslant_{T^\partial \mathcal{L}} \beta \text{ entails } \nabla_{T^\partial} \alpha \leqslant_{\mathcal{L}} \nabla_{T^\partial} \beta.$$

Example 3. Consider frames for distributive substructural logics from the previous example. Such frames, equipped with a monotone valuation, enable one to interpret a conjunction, disjunction, and a fusion connective:

$$x \Vdash a_0 \otimes a_1 \text{ iff } (\exists x_0, x_1)(R(x_0, x_1; x)\ \&\ x_0 \Vdash a_0\ \&\ x_1 \Vdash a_1).$$

A frame is a coalgebra $c : X \to \mathbb{L}(X \times X)$. Nabla for (the dual of) the coalgebra functor $\mathbb{L}(Id \times Id)$, which is $\mathbb{U}(Id \times Id)$, works as follows:

$$x \Vdash \nabla \alpha \text{ iff } (\forall (a_0, a_1) \in \alpha)(\exists (x_0, x_1) \in c(x)\ (x_0 \Vdash a_0\ \&\ x_1 \Vdash a_1).$$

Therefore

$$\nabla \alpha \equiv \bigwedge_{\alpha \ni (a_0, a_1)} (a_0 \otimes a_1).$$

To be able to state the modal distributive law capturing an interaction of the nabla modality with conjunction, we need to define a certain way of redistributing elements of an object A in $\mathbb{U} T^\partial \mathcal{L}$. A *slim redistribution* of A is any Φ in $T^\partial \mathbb{U}\mathsf{base}(A)$, such that for each α with $A \ni \alpha$ it holds that $\Phi \ \overline{T^\partial}(\ni) \ \alpha$. The slim redistribution of A form a lowerset $\mathsf{srd}(A)$. The following two modal distributive laws hold for the nabla modality:

$$\bigvee_{\alpha \overline{T^\partial}(\in)\Psi} \nabla\alpha \equiv \nabla(T^\partial \bigvee)\Psi \qquad \bigwedge_{A \ni \alpha} \nabla\alpha \equiv \bigvee_{\Phi \in \mathsf{srd}(A)} \nabla(T^\partial \bigwedge)\Phi$$

where Ψ is in $T^\partial \mathbb{L}\mathcal{L}$, Φ is in $T^\partial \mathbb{U}\mathcal{L}$, and α in $T^\partial \mathcal{L}$. We expect that the laws, together with the rule of monotonicity of the nabla modality constitute a complete axiomatization of the logic, but we have no proof of completeness yet.

Expressivity

Using the relation lifting, we can define a notion of *simulation* that coincides with the one given by Worrell in [11] in the setting of $\mathscr{V}$-categories (of which preorders are a special case). The resulting notion of similarity coincides with the preorder on the final coalgebra (if it exists). Let $c : X \to TX$ and $d : Y \to TY$ be given. We call a monotone relation R a T-*simulation* of c by d iff

$$R(x, y) \text{ implies } \overline{T}R(c(x), d(y)).$$

We say that a pointed coalgebra $(c, x, \Vdash_c)$ T-*simulates* $(d, y, \Vdash_d)$ if there exists a T-simulation R with $R(x, y)$. We say that $(c, x, \Vdash_c)$ is *modally stronger* than $(d, y, \Vdash_d)$ if $x \Vdash_c a$ implies $y \Vdash_d a$, for each formula a in $\mathcal{L}$. We can prove that the logic has the Hennessy-Milner property — it is adequate and expressive:

Lemma 1. $(c, x, \Vdash_c)$ T-simulates $(d, y, \Vdash_d)$ *iff* $(c, x, \Vdash_c)$ *is* modally stronger *than* $(d, y, \Vdash_d)$. *In particular, the relation of being modally stronger is a simulation.*

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On Models of the modal logic KD45-O

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Introduction

Modal logic is a useful language to study knowledge and belief of human agents which has been a main issue of concern to philosophers as well as computer scientists. This line of research has been initiated by von Wright and possible world semantics has been used to model knowledge as well as belief [2]. In this work we are concernedd with the KD45-O model of belief. Plausibility models are more general in nature in the sense that one can always build a KD45 Kripke structure from them, as described in [1]. That the plausibility ordering between worlds can be used for comparing beliefs has been first studied in the work of Lewis, and Grove introduced an order on the set of all interpretations of a belief set [5].

Explicit ordering of belief in the logical language is introduced and shown to be sound and complete with respect to KD45-O models in [1]. In this paper we prove the completeness of KD45-O by the filtration method and show models of KD45-O that are obtained by the two different methods are isomorphic. For that purpose we use modal completeness and canonical model of KD45-O.

Syntax

For formulas φ and ψ, ordering of formulas $B\varphi > B\psi$ intuitively means that belief in φ is stronger than belief in ψ. We write $\varphi \geqslant_B \psi$ instead of $B\varphi > B\psi$. Ordering of worlds $w \leqslant w'$ stands for w-world is more plausible than w'-world. This ordering can be used for comparing belief ordering. Ordering of belief sets $\Gamma \geqslant_B \Gamma'$ is decribed as Γ is more plausible than Γ'.

To introduce the comparison of strenghts of belief explicitly in the logical language, a new relation symbols is added to the existing modal language of belief to form the language of belief logic with explicit ordering KD45-O together with universal modality U. This modality is definable in terms of the ordering. Let Φ be a countable set of atomic propositions. Formulas φ are defined inductively where $\mathbf{p} \in \Phi$:

$$\varphi := \mathbf{p} \mid \bot \mid \neg\varphi \mid \varphi \vee \varphi \mid B\varphi \mid U\varphi \mid \varphi \geqslant_B \varphi \mid \varphi \succ_B \varphi$$

$B\varphi$ and $U\varphi$ can be defined by $\varphi \geqslant_B \neg\bot$, $\bot \geqslant_B \neg\varphi$, respectively.

Definition 13. A KD45-O model is a structure $\mathfrak{M} = \langle S, \leqslant, \geqslant_B, V\rangle$ where S is a finite set of states, V is valuation function, $\leqslant$ is a quasi linear order relation (plausibility ordering) over S, and $\geqslant_B$ is quasi linear order relation over $\mathcal{P}(S)$ such that

(i) If $X \subseteq Y$, then $Y \geqslant_B X$

(ii) If $\mathcal{B} = \{s \in S \mid \text{for all } t \in S, s \leqslant t\}$ is the set of all $\leqslant$-minimal states (the set of most plausible worlds), $\mathcal{B} \subseteq X$ and $\mathcal{B} \nsubseteq Y$, then $X >_B Y$.

(iii) If $X \neq \emptyset$, then $X >_B \emptyset$.

The truth definition for formulas φ in a KD45-O model $\mathfrak{M}$ is as usual with the following clauses for the belief and ordering modalities:

$\mathfrak{M}, s \models B\varphi$ iff for all $\leqslant$-minimal states t, $\mathfrak{M}, t \models \varphi$.

$\mathfrak{M}, s \models \varphi \geqslant_B \psi$ iff $\{t \mid \mathfrak{M}, t \models \varphi\} \geqslant_B \{t \mid \mathfrak{M}, t \models \psi\}$.

Axiomatization

An axiomatic system for KD45-O consist of axioms and inference rules of KD45 together with following axioms and rules :

Axioms:	**Inference Rule:**
(i) $\varphi \geqslant_B \varphi$	(R1) The inference rules for B
(ii) $(\varphi \geqslant_B \psi) \wedge (\psi \geqslant_B \chi) \rightarrow (\varphi \geqslant_B \chi)$	(R2) $\frac{\varphi \rightarrow \psi}{\psi \geqslant_B \varphi}$
(iii) $(\varphi \succ_B \psi) \leftrightarrow (\varphi \geqslant_B \psi) \wedge \neg(\psi \geqslant_B \varphi)$	
(iv) $(\varphi \geqslant_B \psi) \vee (\psi \geqslant_B \varphi)$	
(v) $(B\varphi \wedge \neg B\psi) \rightarrow (\varphi \succ_B \psi)$	
(vi) $(\varphi \geqslant_B \psi) \rightarrow B(\varphi \geqslant_B \psi)$	
(vii) $(\varphi \succ_B \psi) \rightarrow B(\varphi \succ_B \psi)$	
(viii) $(\bot \geqslant_B \neg(\varphi \rightarrow \psi)) \rightarrow (\psi \geqslant_B \varphi)$	
(ix) $\varphi \rightarrow (\varphi \succ_B \bot)$	
(x) $(B\varphi \succ_B \bot) \rightarrow B\varphi$	

Completeness

It is shown that the system KD45-O is sound and complete wrt KD45-O models in [1]. Completeness proof is given finite by the Henkin method using maximal KD45-O consistent set of states. We give the completeness proof of KD45-O by filtration method and show that the models obtained for the KD45-O by finite Henkin method and filtration method are isomorphic.

Definition 14. The canonical Kripke model for the logic KD45-O is defined as $\mathfrak{M}^c = \langle S^c, R_B, R_U, V^c \rangle$ with

(i) $S^c = \{\Gamma \mid \Gamma \text{ a maximal consistent set}\}$,

(ii) $R_B = \{(\Gamma, \Gamma') \mid B\varphi \in \Gamma \Rightarrow \varphi \in \Gamma'\}$, $R_U = \{(\Gamma, \Gamma') \mid U\varphi \in \Gamma \Rightarrow \varphi \in \Gamma'\}$

(iii) $V^c(p) = \{\Gamma \mid p \in \Gamma\}$.

Lemma 1. *The canonical model $\mathfrak{M}^c$ for the logic KD45-O satisfies (i) R_U is an equivalence relation and (ii) R_B is a euclidean subrelation of R_U.*

Since the relation R_U is not universal in canonical model $\mathfrak{M}^c$, we define a generated submodel of the canonical model using an adequate set to obtain a right KD45-O model. A finite set of formulas Φ is said to be adequate wrt φ iff Φ is closed under subformulas and Φ satisfied the following conditions: (i) If $\psi \in \Phi$, Φ contains a formula equivalent to $\neg\psi$; (ii) If $B\psi, B\chi \in \Phi$, Φ contains a formula equivalent to $B(\psi \wedge \chi)$; (iii) If $B\psi, B\chi \in \Phi$, Φ contains a formula equivalent to $B(\psi \vee \chi)$; (iv) $B\psi \in \Phi$, Φ contains $UB\psi$; (v) Φ contains $B\top$ and $B\bot$.

Now, we define a representable subset of the canonical model S^c.

Definition 15. Let X be a subset of S^c and φ be a formula. If X is the set of states where φ is true, then φ represent X. It is showed as $V(\varphi) = \text{X}$. Given an adequate set Φ, for some $B\varphi \in \Phi$, if φ represent X, X is called representable.

Definition 16. Let φ be a consistent formula, and let Φ be a finite adequate set wrt φ. Suppose Φ_0 is a maximal consistent subset of Φ containing φ. The model $\mathfrak{M}_{\Phi_0} = \langle S_{\Phi_0}, T_B, T_U, V_{\Phi_0} \rangle$ is the submodel of the canonical model $\mathfrak{M}^c$ such that it is generated by R_U from Φ_0 as follows:

(i) $S_{\Phi_0} = \{\Gamma \in S^c \mid (\Phi_0, \Gamma) \in R_U\}$,

(ii) $T_B = R_B \cap (S_{\Phi_0} \times S_{\Phi_0})$, $T_U = R_U \cap (S_{\Phi_0} \times S_{\Phi_0})$,

(iii) $V_{\Phi_0}(p) = V(p) \cap S_{\Phi_0}$

Now we define a filtration of a KD45-O model.

Definition 17. Let $\mathfrak{M}_{\Phi_0}$ be a model for $KD45 - O$ given in definition 4. The filtration $\mathfrak{M}^\Phi = \langle S^\Phi, T_B^\Phi, T_U^\Phi, V^\Phi \rangle$ of $\mathfrak{M}_{\Phi_0}$ is defined for a finite adequate set Φ wrt φ as the following:

(i) $S^\Phi = \{[\Gamma]_\Phi : \Gamma \in S^c\}$, where $[\Gamma]_\Phi$ is $\Gamma \cap \Phi$.

(ii) $T_U^\Phi = \{([\Gamma]_\Phi, [\Gamma']_\Phi) \mid \text{for all } U\psi \in \Phi, (U\psi \in [\Gamma]_\Phi \text{ iff } U\psi \in [\Gamma']_\Phi)\}$,
$T_B^\Phi = \{([\Gamma]_\Phi, [\Gamma']_\Phi) \mid$ for all $B\psi \in \Phi$, (if $B\psi \in [\Gamma]_\Phi$, then $B\psi, \psi \in [\Gamma']_\Phi$) and (if $\neg B\psi \in [\Gamma]_\Phi$, then $\neg B\psi \in [\Gamma']_\Phi)\}$,

(iii) $V^\Phi(p) = \{[\Gamma]_\Phi : \Gamma \in V_{\Phi_0}(p)\}$.

Lemma 2. *The model* $\mathfrak{M}^\Phi$ *is a filtration of* $\mathfrak{M}_{\Phi_0}$.

Proof. It is enough to show that the relations T_U^Φ and T_B^Φ are filtrations of T_U and T_B [3]. □

Lemma 3. *The model* $\mathfrak{M}^\Phi$ *is a KD45-O model.*

Proof. Let us define the relations $\leqslant$ and $\geqslant_B$ over S^Φ and $\mathscr{P}(\mathcal{S}^\Phi)$: $\leqslant$ is the plausibility ordering on the states of S^Φ defined by any state in $\mathfrak{B}$ is more plausible than any $S^\Phi - \mathfrak{B}$, and within these two sets, the worlds are equi-plausible. To define the relation $\geqslant_B$ on $\mathscr{P}(S^\Phi)$ we use representable subset of S^Φ, as $\psi \geqslant_B \chi$ if and only if $V(\psi) \geqslant_1 V(\chi)$. Since the relation $\geqslant$ satisfies the axioms (i)-(iii), it is a quasi linear relation, and so the relation $\geqslant_1$ is as well. We need to ensure that $\geqslant_1$ provides the conditions of the definition 1.

(i) If $V(\psi) \subseteq V(\chi)$, then $U(\psi \to \chi)$. Using the axiom $U(\psi \to \chi) \to \chi \geqslant_B \psi$, we obtain $V(\psi) \geqslant_1 V(\chi)$. Thus, the subset condition is satisfied.

(ii) $V(\psi)$ contains $\mathcal{B}$ but $V(\chi)$ does not. Then, $B(\psi) \wedge \neg B(\chi)$ is hold. Using $B(\psi) \wedge \neg B(\chi) \to \psi \succ \chi$ and MP, we have $V(\psi) \geqslant_1 V(\chi)$, i.e. $V(\psi) >_B V(\chi)$.

(iii) If $\varphi \neq \perp$, then $V(\varphi) \neq V(\perp)$. Using the axiom $\varphi \to (\varphi >_B \perp)$, we have $V(\varphi) >_B V(\perp) = \varnothing$.

Therefore, the representable elements of $\mathscr{P}(S^\Phi)$ are ordered by the relation $\geqslant_1$. Let $R(X)$ denotes the greatest set of representable sets of $X \subseteq S^\Phi$. Since the model is finite and representable sets are closed under unions, there is a such set. Now we can define $\geqslant_B$ as follows: $X \geqslant_B Y$ iff $R(X) \geqslant_1 R(Y)$. We show that the relation $\geqslant_B$ satisfies the conditions of definition 1. Since $\geqslant_1$ is quasi-linear so, $\geqslant_B$ as well. If $X \subseteq Y$, then $R(X) \subseteq R(Y)$. So, the subset condition holds. To verify the belief condition we have to show that $\mathcal{B} = R(\mathcal{B})$. Suppose w is a state such that $w \notin B$. Since $S^\Phi - \mathfrak{B}$ is not reflexive, $(w, w) \notin T_B$. Thus, for a formula $B\psi \in \Phi$, $B(\psi_w)$ is in true w, ψ_w is not. ψ_w denotes the formula that is true at w. So, $B(\psi_w)$ is true in all elements of $\mathcal{B}$. For all states w in the complement of $\mathcal{B}$, let us consider the conjunction ψ of all ψ_w. While ψ is true in all elements of $\mathcal{B}$, $B(\varphi) \in \Phi$. Since $\psi \to \psi_u$ and ψ_u is falsified in the complement of $\mathcal{B}$, $B(\psi)$ is falsified in this states. So, $V(\psi)$=$\mathcal{B}$. Then, we have $\mathcal{B} = R(\mathcal{B})$, belief condition is ensured. Thus, if $\mathcal{B} \subseteq X$, $\mathcal{B} \nsubseteq Y$, then $\mathcal{B} \subseteq R(X)$ and $\mathcal{B} \nsubseteq R(Y)$. By definiton of R, $R(X) \geqslant_1 R(Y)$, i.e. $X \geqslant Y$. Therefore, $\mathfrak{M}^\Phi$ is a KD45-O model. □

Using a similar argument in [3] and [4] we give main result.

Theorem 1. *The models obtained for the modal logic KD45-O by the finite Henkin method are isomorphic to the ones obtained by a filtration of the canonical model for KD45-O if both are defined wrt the same finite adequate set* Φ*.*

Sketch of proof. Let $\mathfrak{M}_H$ be a KD45-O model obtained by finite Henkin method. We construct an isomorphism between the models $\mathfrak{M}_H$ and $\mathfrak{M}^\Phi$ as the following : $f : \mathfrak{M}_H \to \mathfrak{M}^\Phi$, $\Gamma \mapsto [\Gamma]$ where $\Gamma = [\Gamma] \cap \Phi$. It can be shown that f is an isomorphism. $\square$

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Unless and Until: A Compositional Analysis

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Until and unless you discover that money is the root of all good,
you ask for your own destruction.

— Ayn Rand, *Atlas Shrugged*

The logic of *unless* crops up in papers about quantification [6], conditionals [3], and anaphora [4]. Our analysis of *unless* will make use of Geis's analysis of *if* as involving quantification over the relevant cases indicated by the topic of the sentence. When *unless* interacts with anaphoric reference in such constructions as

(1) John doesn't own a donkey, *unless* he hides it well ,

the pronoun it appears to refer to a [possibly] non-existent donkey.

At least on the surface, *unless* and *until* are morphologically similar. To our knowledge, no one has linked *unless* and *until* in a formal analysis. In some contexts, they even appear interchangeable: I will not leave *unless/until* you have a replacement'. This phenomenon is also found in Russian and Hungarian:

(2) Ja budu žit' na Long Islande **esli** ja **ne/poka** ja **ne** najdu kvartiru v gorode
I will live on Long Island if I NEG/until I NEG find apartment in city
"I will live on Long Island, *unless/until* I find an apartment in the city." (Russian)

(3) Long Island-en fog-ok él-ni **ha nem/amíg nem** talál-ok egy lakást a
Long Island-on will-I live-to if NEG/until NEG find-I an apartment the
város-ban.
city-in.

"I will live on Long Island, *unless/until* I find an apartment in the city."(Hungarian)

Moreover, *unless* in these languages literally translates as *if (only) not*, providing strong evidence that the treatment of *unless* may be represented as *if not.* Is it possible to have a compositional analysis that shows a deeper correlation between the two connectives?

Our linguistic analysis of *until* makes it equivalent to *unless* by adding a temporal frame and temporal particles such as *yet*, *already*, and *still.* This analysis builds upon Löbner's analysis [7] of how temporal particles in German interact with a two-dimensional adverbial temporal frame.

Three apparent pitfalls potentially block a systematic compositional analysis.

Quine argues that *unless* is symbolized by the logician's *or.* Suppose you're deciding whether to make an offer on Smith's house, and the broker tells you,

(4) Smith will sell *unless* he hears from you .

Quine argues that this statement can be paraphrased by "*if* Smith does *not* hear from you, he will sell," which is, in turn, equivalent to Quine's preferred translation:

(5) Smith hears from you *or* he will sell .

Despite the oddity of tense, Quine concludes that *unless* should be symbolized by the logician's *or*, which, like *unless*, has an inclusive and exclusive sense. However, disjunctions built from *or* commute, whereas constructions built with *unless* do not. Sentence (4) is clearly not equivalent to:

(6) ?Smith hears from you *unless* he will sell .

The second issue concerns Negative Polarity Items (henceforth NPIs) words that can only appear in a negative environment. In the following example, *if not* and *never* are negative environments where the NPI *any* is licensed.

(7) *If* you do*n't* do *any*thing, you'll *never* get *any*where

If we analyze unless as if not, then the two connectives should behave similarly with respect to NPIs. However, replacing *if not* with *unless* results in an ungrammatical construction:

(8) ??*Unless* you do *any*thing, you'll never get anywhere .

The third pitfall occurs when we assimilate *until* to *unless* adding time parameters. Consider the sentence:

(9) The baby didn't sleep *until* 2 a.m.[1]

A naïve way of analyzing *until* as *if not* yields an ungrammatical construction:

(10) **If* it's *not* 2 a.m., then the baby *didn't* sleep.

Despite these pitfalls, there are strong arguments for treating *unless* compositionally as *if not.*

Suppose you hate John but love Mary. You might express your willingness to go to a party by using the following complex sentence:

(11) I will go to the party, *unless* John does *or* Mary doesn't .

Suppose, following Quine, we treat *unless* simply as *or*—in either the inclusive or exclusive sense. The above sentence is clearly not equivalent to:[2]

[1] This is ambiguous between the *continuous action* reading "the baby wasn't sleeping [continuously] until 2 a.m." and the *completed achievement* reading "the baby didn't [achieve] fall[ing] asleep until 2 a.m."

[2] (13) is equivalent to (13a) but not (13b):

(13a) I will go to the party *unless* John goes, *and* I will go to the party *unless* Mary doesn't go.

(13b) I will go to the party *if* John goes and Mary doesn't.

(12) I will go to the party, *or* John does *or* Mary doesn't

However, treating *unless* as *if not* results in the intended meaning:

(13) I will go to the party if John does not go *and* I will go to the party if Mary does .

Another argument in favor of treating *unless* as *if not* comes from the meaning of *unless.* Etymologically, *unless* was derived as a mid-15th century Old English contraction of *on a less condition than.* The original contraction of this phrase was *onlesse* which then became *unless* because of its lack of phonological stress and its negative connotation.[3] What is important for our argument here is that the negative connotation of *onlesse* coincides with the negative environment created by *unless.*

Along similar linguistic lines, in other languages, such as Russian and Hungarian, *unless* literally translates as *if (only) not*, again providing evidence that *unless* is logically and linguistically represented as *if not.*

Finally, we shall argue that analyzing *unless* as *if not* allows for a compositional treatment of *until* built upon unless combined with temporal parameters.

Contrary to Quine's admonition to ignore the shifting tenses in the unless clause, these shifts turn out to be evidence for the constructiveness, if not the completeness, of a Reichenbachian analysis.

Despite the three pitfalls in section I, we show that a proper analysis of these objections in fact supports the compositional analysis of *unless* and *until* that we are proposing.

We shall argue that Quine's analysis is based on a false methodological assumption. Contrary to ignoring linguistic phenomena such as tense and topic, we pay attention to these cues to obtain a more accurate, and elegant, semantic analysis of *unless* and *until.*

The objection based on NPIs in Negative Polarity environments can be accommodated by paying attention to interactions of scope and the laws of quantifier confinement.

Future avenues of research follow naturally from our compositional analysis—pursuits such as quantification in Donkey sentences, branching analyses of time, and cross-linguistic explorations of *unless* and *until* in languages like Russian and Hungarian.

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This equivalence is based on: $(\sim J \rightarrow G) \wedge (M \rightarrow G) \leftrightarrow (J \vee \sim M \rightarrow G)$ (see [5, T50, p. 77]

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Universal models for intuitionistic logic and its fragments: a duality based approach

Dion Coumans and Sam van Gool

In this work we investigate universal models for the intuitionistic propositional calculus (IPC) and some of its fragments. Our methods are based on generalizations of Esakia duality [7, 9, 2, 1] and the step-by-step construction for free Heyting algebras [8, 4].

Heyting algebras form the algebraic counterpart of intuitionistic propositional logic. Indeed, the raison d'être of Heyting algebras lies in the fact that *the free n-generated Heyting algebra is the Lindenbaum algebra for* IPC *on n variables*. Esakia duality for Heyting algebras [7] is the mathematical content of the intimate link between the syntactic and semantic approach to intuitionistic logic. In particular, the dual space of the free n-generated Heyting algebra is isomorphic to the canonical model for IPC on n variables, $C(n)$. The n-universal model for IPC, $U(n)$, is the image-finite part of $C(n)$, i.e., the points of the canonical model whose upward closure is finite.

There is also a recursive construction of the universal model $U(n)$, starting from its 2^n maximal nodes and then constructing its lower layers consecutively; cf., e.g., [3] for details. We thus have a concrete description of the universal model, and it is significantly smaller than the complete canonical model (when $n \geqslant 2$). The universal model is valuable in the study of intuitionistic logic because it still contains enough points to distinguish any two non-equivalent formulas of IPC on n variables. To state this fact more precisely, let us denote by $[[\phi]]$ the set of points in $U(n)$ in which the formula ϕ holds under the canonical valuation. This function $[[\cdot]]$ from the free n-generated Heyting algebra to the collection of upsets of $U(n)$ is injective; in symbols, we have $[[\cdot]] : F_{\mathsf{HA}}(n) \hookrightarrow \mathrm{Up}(U(n))$, where $\mathrm{Up}(P)$ denotes the collection of upsets of a poset P. It is known that the function $[[\cdot]]$ is not surjective, as a cardinality argument shows [3, Thm 3.2.19(2)]. An upset of $U(n)$ is called *definable* if it lies in the image of the function $[[\cdot]]$. With this terminology, we can now phrase an interesting open problem posed by De Jongh:

Problem 1 (De Jongh)**.** Provide an intrinsic characterization of the definable upsets of $U(n)$.

It is immediate from Esakia duality that the definable upsets of $U(n)$ are exactly the intersections of clopen upsets of $C(n)$ with $U(n)$. Therefore, for this problem to be non-trivial, ‘intrinsic’ is understood to mean: ‘without referring to the topology on $C(n)$’.

A satisfactory answer to this problem would give a necessary and sufficient condition for an upset of $U(n)$ to be definable that only refers to the accessibility relation and valuation on the universal model, which are given by the recursive definition already referred to above.

In Theorem 2 below, we will provide an answer to Problem 1 for the upsets which are $(\wedge, \rightarrow)$-*definable*, that is, the upsets lying in the image of the restriction of the map V to the $(\wedge, \rightarrow)$-fragment of IPC. To do so, we will first recall the definition of inductive points, also see

[10, 11]. We refer to the introductory section of [10] for a thorough historical overview of the study of fragments of IPC and their universal models, which are also known as 'exact models'.

Let M be a Kripke model for IPC on n propositional variables, $p_1, \ldots, p_n$. Recall that the *colour* of a point $x \in M$ is the element $c(x) \in \{0,1\}^n$ which is the characteristic function of the set $\{p_i \mid M, x \models p_i\}$. Because valuations are persistent, c is order-preserving from M to $\{0,1\}^n$, when the latter is viewed as a poset with the product order of $\{0 < 1\}$. For the benefit of the duality-minded reader, we note that this order-preserving function $c : M \to \{0,1\}^n$ is the Priestley dual of the natural distributive lattice homomorphism $F_{\mathsf{DL}}(n) \to \mathrm{Up}(M)$. A point $x \in M$ is called *inductive*[1] if $c(x) = \inf\{c(y) \mid x < y\}$. Note that, with this definition, a maximal point is inductive iff its colour is $(1, \ldots, 1)$. We call a point *separated* if it is not inductive, i.e., if there exists a propositional variable which holds at all successors of x but not at x itself. Inductive and separated nodes can be used to semantically characterize the $(\wedge, \to)$-fragment of IPC, by the following elementary lemma.

Lemma 1 ([10]). *Let M be a Kripke model, and $x \in M$. Then x is separated if, and only if, there exists a formula ϕ in the $(\wedge, \to)$-fragment of* IPC *such that, for all $y > x$, $y \models \phi$, and $x \not\models \phi$.*

In the rest of this abstract, we will outline a novel approach towards obtaining an answer to Problem 1. The novelty of our approach lies in the use of *duality for implicative semilattices* [9, 2], as well as *step-by-step constructions* of free Heyting algebras [8, 4]. We will briefly review both of these techniques and indicate how they shed new light on the universal model.

The algebraic structures corresponding to the $(\wedge, \to)$-fragment of IPC are *implicative semilattices*: these are structures of type $(S, \wedge, \to)$ where $(S, \wedge)$ is a semilattice and $\to$ is the upper residual of $\wedge$. A duality for finite implicative semilattices was given in [9] and generalized to arbitrary implicative semilattices in [2]. A particular case of this duality, which will suffice for our purposes here, says that *the category of Heyting algebras with $(\wedge, \to)$-preserving maps is dually equivalent to the category of Esakia spaces with partial Esakia morphisms* [1]. We will now indicate how this duality provides a perspective on Problem 1.

Firstly, the free n-generated implicative semilattice, $F_{\wedge,\to}(n)$, is isomorphic to the $(\wedge, \to)$-subalgebra of the free Heyting algebra $F_{\mathsf{HA}}(n)$ which is generated by the propositional variables. That is, we have a $(\wedge, \to)$-preserving embedding $h : F_{\wedge,\to}(n) \hookrightarrow F_{\mathsf{HA}}(n)$ whose image is $\langle p_1, \ldots, p_n \rangle_{\wedge,\to}$. A classical theorem by Diego [6] says that the variety of implicative semilattices is locally finite, i.e., finitely generated implicative semilattices are finite. This theorem can be proved easily using Lemma 1 above, along with the fact that there are only finitely many separated points in $U(n)$ (also cf. [11]). In particular, $F_{\wedge,\to}(n)$ is a finite implicative semilattice, and therefore a Heyting algebra. Thus, by finite Esakia duality, $F_{\wedge,\to}(n)$ is isomorphic to $\mathrm{Up}(V(n))$, where $V(n)$ is the poset of join-irreducibles of $F_{\wedge,\to}(n)$, also known as the *exact* or *universal model* for the $(\wedge, \to)$-fragment.

We will now describe the $(\wedge, \to)$-embedding $h : F_{\wedge,\to}(n) \hookrightarrow F_{\mathsf{HA}}(n)$ using duality for $(\wedge, \to)$-morphisms. First note that the set $U(n)^s$ of separated points of $U(n)$ is a finite (non-generated) submodel of $U(n)$. Therefore, by the fact that any finite model admits a unique p-morphism to $U(n)$, we have a unique p-morphism $f : U(n)^s \to U(n)$. The function f is not injective, because there are distinct bisimilar points in the model $U(n)^s$ (when $n > 1$).

Lemma 2. *The image of the p-morphism $f : U(n)^s \to U(n)$ is isomorphic to $V(n)$.*

Now, viewing f as a partial map $U(n) \to V(n)$ whose domain is $U(n)^s$, it is almost a partial Esakia morphism; it only fails to satisfy Condition 3 of [1, Def. 3.3]. However, we may easily

[1] Our notion of "inductive point" would be called "full or inductive point" in [10]. This is due to the fact that, for the purposes of this abstract, we only discuss the $(\wedge, \to)$-fragment, whereas the separate notion of "full" is needed when one also studies the $(\wedge, \to, \bot)$-fragment, cf. [10] for details.

extend the domain of f to obtain an actual partial Esakia morphism g.[2]

Denoting by $()^*$ the dual equivalence functor going from spaces to algebras in the duality in [1], we then have:

Theorem 1. *The partial Esakia morphism g is, up to isomorphism, dual to the embedding $h : F_{\wedge,\to}(n) \hookrightarrow F_{\mathsf{HA}}(n)$, i.e., the following diagram commutes:*

$$\begin{tikzcd} F_{\wedge,\to}(n) \arrow[r, tail, "h"] \arrow[d, "\cong"'] & F_{\mathsf{HA}}(n) \arrow[d, "\cong"'] \\ \mathrm{Up}(V(n)) \arrow[r, tail, "g^*"'] & \mathrm{DefUp}(U(n)) \end{tikzcd}$$

Let us call a subset S of a model M *inductively closed* if, whenever x is inductive and for all $y > x$ we have $y \in S$, we also have $x \in S$. Moreover, we call a subset S of $U(n)$ *g-saturated* if, for all $x \in S$ and $x' \in U(n)$ such that $g(x) = g(x')$, we have $x' \in S$. One may now show that g^* maps an upset U of $V(n)$ to its 'inductive g-saturation' in $U(n)$, that is, the smallest inductively closed and g-saturated set containing U. We can now state a first partial answer to Problem 1.

Theorem 2. *Let U be an upset of $U(n)$. Then U is $(\wedge, \to)$-definable if, and only if, U is inductively closed and g-saturated.*

Note that a result along the same lines could have been stated for $(\wedge, \to, \bot)$-definable upsets; we omit this slight generalization here. Let us also emphasize that this result relies heavily on Lemma 1, which goes back to [10] and the references therein.

We will now connect this duality perspective with the step-by-step construction of free Heyting algebras. The step-by-step construction of the free algebra for a variety relative to a locally finite reduct was introduced by Ghilardi [8], and was revived in recent years, starting with the paper [4]. Let us briefly recall how this construction works in the case of Heyting algebras, specializing the general view from [5]. Observe that the lattice reduct of the free Heyting algebra $F_{\mathsf{HA}}(n)$ is the colimit of a chain of approximating finite distributive sublattices $(D_k)_{k\in\mathbb{N}}$. Here, D_k consists of precisely those elements of $F_{\mathsf{HA}}(n)$ which are equivalence classes of formulas of implication rank at most k, i.e., with no more than k nestings of $\to$ in it. Note that, for each $k \geqslant 0$, the distributive lattice D_k is indeed finite since the variety of distributive lattices is locally finite. Moreover, D_{k+1} has a partially defined implication on it, whose domain is the sublattice D_k, and thus forms a 'partial Heyting algebra'. Now, the key result from [8] is that *the partial Heyting algebra D_{k+1} can be constructed from the partial Heyting algebra D_k in a uniform way.* In fact, it can be seen from the results in [4, 5] that there exists an endofunctor K on the category of partial Heyting algebras such that $D_{k+1} = K(D_k)$. Moreover, the functor K can be described concretely using duality for finite distributive lattices, giving a functorial construction on finite posets. Repeatedly applying K to D_0, the free distributive lattice on n generators, yields a construction of the approximating chain, and, taking the colimit, of the free Heyting algebra.

An alternative step-by-step construction of the free Heyting algebra can be given by using the implicative semilattice reduct instead of the lattice reduct. The process is completely analogous to the one outlined above. To be precise, we start from the subalgebra $A_0 := \langle p_1, \ldots, p_n \rangle_{\wedge,\to}$ of $F_{\mathsf{HA}}(n)$. We now let B_k be the join-semilattice $\langle A_k \rangle_{\vee,0}$ generated by A_k in $F_{\mathsf{HA}}(n)$, and define $A_{k+1} := \langle B_k \rangle_{\wedge,\to}$.

[2]To be more precise, $U(n)$ is not itself an Esakia space, but it is a dense subspace of $C(n)$ and the map $g \colon U(n) \to V(n)$ corresponds (uniquely) to a partial Esakia morphism $C(n) \to V(n)$.

It is clear that $F_{\mathsf{HA}}(n)$ is the union of the increasing chain of subalgebras $(A_k)_{k \in \mathbb{N}}$. However, in contrast with the case of the approximating chain $(D_k)_{k \in \mathbb{N}}$, no functorial construction of A_{k+1} from A_k is known so far. This also seems an important problem to investigate further.

We note that there is a similarity between the step-by-step construction and the recursive construction of the universal model mentioned above, but the two constructions are not the same. In fact, we expect it will be crucial to understand these constructions relative to each other. We have obtained some preliminary results in this direction, which we omit here for reasons of space.

Coming back to our initial Problem 1, we can now outline our approach to characterizing the definable upsets of the universal model $U(n)$. Theorem 2 says that the image of the subalgebra A_0 under the embedding $V : F_{\mathsf{HA}}(n) \hookrightarrow \mathrm{Up}(U(n))$ consists of the inductively closed sets. It is obvious that, for $k \geqslant 0$, the image of B_k under this embedding consists of finite (possibly empty) unions of sets from A_k. The hard part is then to characterize the image of A_{k+1} under the embedding, i.e., to characterize the upsets that may be described by formulas in the subalgebra of $F_{\mathsf{HA}}(n)$ which is $(\wedge, \to)$-generated by B_k. To do so, we generalize Theorem 2 to a notion of *level k inductively closed*, using a new colouring of the points of $U(n)$ induced by the inductively closed sets at level $k-1$. Using these (countably many) new colourings, we are able to characterize all definable upsets. However, this answer to Problem 1 is not yet completely satisfactory, as the number of colours that is needed in step k is equal to the size of the algebra A_{k-1}, which is known to grow very quickly. We therefore leave it as an important direction for future work to find a more tractable definition of the level k inductively closed sets, using the methods outlined here.

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A Method Overcoming Induction During Cut-elimination

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Gentzen's cut-elimination theorem is one of the most important theorems of logic. Removing cuts from formal proofs corresponds to the elimination of intermediary lemmas from mathematical proofs. In cut-free proofs all statements are subformulas of the end-sequent. Cut-free proofs are analytic in sense that they allow extraction of the "hidden" knowledge, such as Herbrand disjunctions and interpolants. Systems, that have a cut-elimination theorem, are easily proved to be consistent.

Gentzen's original proof of his cut-elimination theorem is constructive. He gave a reductive cut-elimination method, which can be interpreted as a proof rewriting system, which is terminating, and normal forms of this system are cut-free proofs. Such kind of systems usually are nondeterministic, i.e. no explicit algorithm is specified which selects the reduction rule to be applied next. Another reductive method was given also by Tait and Schütte. These two methods differ in the selection of the cut that has to be eliminated first. These methods analyze only small parts of proofs (the derivation of the uppermost logical operator of a cut-formula) and leave other parts of the proof unchanged, which may lead to major redundancy.

A cut-elimination method, called CERES, was developed by Matthias Baaz and Alexander Leitsch in [5], which radically differs from reductive cut-elimination methods. CERES is a cut-elimination method by resolution, reducing cut-elimination to a theorem proving problem. In this method not only the derivation of the uppermost logical operator of a cut-formula is analyzed, but all derivations of cut-formulas at once. This analysis leads to the construction of an unsatisfiable set of clauses, called characteristic clause set. Then a resolution refutation of the characteristic clause set is used as a skeleton of a proof with atomic cuts only. The method was originally developed for first-order logic, but later it was extended to second- and higher-order logics (see [12, 11]). The CERES method was implemented and real mathematical proofs were analyzed using the `CERES`-system (see [4, 10]).

Another very important tool in computer science and mathematics in general is induction. But it adds complexity to proofs, because the rule corresponds to an infinitary modus ponens rule. Also, on the syntactic side, while logical rules change the complexity of formulas, the induction rule, although it may change formulas, not necessarily changes the complexity of formulas. Therefore, reducing the complexity of formulas cannot be used to show the termination of cut-elimination in the presence of induction.

Gentzen's procedure, and reductive cut-elimination methods in general, shift cuts upwards until they are eliminated. When an induction rule occurs in the proof, such kind of shift over it is not always possible, therefore cut-elimination is not possible in general in the presence of induction.

This issue first was investigated in [17]. From the Gentzen's proof of consistency of Peano arithmetic it is shown that cut-elimination is possible when induction is grounded; in this case the induction term can be evaluated to numeral and can be replaced by a finite number of cuts.

Later, in [15] a reductive cut-elimination method was given for intuitionistic proof systems with induction. In [14] a cut-elimination theorem for subclass of inductive proofs of weakly quantified theorems was proved.

In [6] the author presented a so-called cyclic proof system and showed that such kind of systems subsume the use of the induction rule. He also mentioned the problem that there is no proof of the cut-elimination theorem for classical cyclic proof systems in the literature. The point is that reductive cut-elimination methods cannot be extended to proofs with cycles (shifting cuts over cycles is a major problem).

Schemata are widely used in mathematics on a meta-level as an alternative to induction.

Since cut-elimination in the presence of induction is so problematic, there was an attempt in [4] to use schemata in practice, instead of induction, to analyze interesting mathematical proofs using the CERES method. In this paper the `CERES`-system was applied to (a formalization of) a mathematical proof: Fürstenberg's proof of the infinity of primes [1]. The proof was formalized as a sequence of proofs $(\pi_k)_{k \in \mathbb{N}}$ showing that the assumption that there exist exactly k primes is contradictory. The analysis was performed in a semi-automated way: $\mathrm{CL}(\pi_k)$ (the characteristic clause set of π_k) was computed for some small values of k and from this, a general scheme $\mathrm{CL}(\pi_n)$ was constructed and refuted by hand. Even the analysis of this proof was very interesting: from Fürstenberg's proof, which makes use of topological concepts, Euclid's elementary proof was obtained by cut-elimination. However, it was unsatisfactory that the fact, that $\mathrm{CL}(\pi_n)$ is really the correct schema for all $n \in \mathbb{N}$, could not be verified, and that the analysis of $\mathrm{CL}(\pi_n)$ could not be performed in a computer-aided way.

The aim of this work was to define a language for treating schemata on the object level and to develop a cut-elimination method $CERES_s$ for it. In such a way we avoid the explicit use of induction and get strong tool to represent and reason on proofs with some kind of cycles. The method $CERES_s$ overcomes the shortcoming of reductive cut-elimination methods on such proofs by analyzing the proof as a whole and eliminating all cuts together. Hence we obtain a cut-elimination method for cyclic proof systems. It will be shown that there is a translation from a proof with an ordinary induction rule into our system, and since cut-elimination is possible in our settings, hence showing that our system has an advantage over usual systems with induction rule.

To achieve this aim, we extended the notion of formula schemata [2, 3] and defined a schematic version of sequent calculus, called $\mathbf{LK}_s$ [9, 8], for it. A schematic proof is a tuple of pairs of $\mathbf{LK}_s$-proofs (corresponding to the base and step cases of an inductive definition). The $CERES_s$ method is based on the notion of a schematic characteristic clause set, which is extracted from a schematic proof and is always unsatisfiable. This will close the gap in the application described above (and in future ones) by automatically computing the correct schema $\mathrm{CL}(\pi_n)$. We use a resolution refutation of this clause set as a skeleton for a proof schema with atomic cuts only. This is achieved by replacing clauses from the resolution refutation by the corresponding projection schemata of the original proof schema. We show that there is a correspondence between the schematic and the standard CERES methods: when given a proof schema, one can instantiate it for some specific number n, get an $\mathbf{LK}$-proof and apply the standard CERES method on it, or apply $CERES_s$ directly to the given proof schema and then instantiate the ACNF schema for the number n. Of course these two normal forms cannot be the same in general, since cut-elimination is not confluent in classical logics.

The $CERES_s$ method, besides its theoretical importance, has also very useful practical applications. Therefore the method was implemented under the GAPT[1] framework and applied to some schematic problems in a semi-automated way (see [7, 16]). Of course it cannot be fully automatized since the resolution calculus for reasoning on clause set schemata is undecidable.

By presenting a cut-elimination method $CERES_s$ for some kind of cyclic proofs, which we call proof schemata, we have shown that cut-elimination is possible also in proofs with cycles (some kind of cut-elimination theorem). To the best of our knowledge no other proof of cut-elimination theorem exists for cyclic proofs.

There are several important lines left for the future work. The first is to extract valuable information from cut-free proofs, such as Herbrand sequent and interpolants. Note that such kind of procedures exists for usual $\mathbf{LK}$ (see [13]), but their extension to proof schemata are not straightforward. Considering the fact, that $CERES_s$ does not produce a cut-free sequence of proofs in usual notation, it is worthy to have such algorithms for proof schemata as well, which

[1]General Architecture for Proof Theory, `http://www.logic.at/gapt`

will allow the extraction of valuable information form the ACNF schema.

Another interesting extension of the method is to allow multiple parameters. Having only one parameter is a major restriction of our language, since nested inductions cannot be handled. Therefore extending the calculus and the method to multiple parameters is of major importance for handling a full power of induction.

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Estimating the Impact of Variables in Bayesian Belief Networks

Sicco Pier van Gosliga and Frans Groen

Introduction

Bayesian belief networks (BBNs) are often designed to aid decision making. A BBN models uncertainty and enables to compute posterior probabilities given prior information and current observations [8]. In this paper we focus on a solution to two practical problems that arise with the application of BBNs: First, in real world applications observations are associated with costs. To keep these costs within acceptable limits we would like to prioritize observations most relevant to our decision. Second, models can grow too large for feasible computations [2]. Rather than restricting the design of a BBN for decision making, we pursue an ad-hoc sub-model tailored to its relevance for the decision maker. For these reasons, we propose an efficient approximation algorithm to compute the maximum impact of observing a variable in respect to the posteriors of other variables in a BBN. The algorithm is guaranteed to never underestimate the real impact. First the impact of the variables within the markov blanket of a variable are calculated, followed by a message passing algorithm to include other variables in the network. The method is closely related the field of sensitivity analysis [3][5], which mostly focuses on aiding the design of a BBN.

Methodology

Distance measure. To quantify belief changes in posterior probability distributions we will use the maximum absolute distance as defined by the Chebyshev distance function $D_{Ch}(P, Q)$ which takes the largest difference between pairs of probability values in two discrete probability distributions P and Q:

$$D_{Ch}(P,Q) = \max_i \Big(\big| p_i - q_i \big| \Big) \tag{1}$$

Distance functions to quantify differences in probability distributions are often based on entropy [1][6][7]. Entropy based distance functions are relative distance measures. As a result, small absolute differences can be valued equally important as a large difference when both span an equal order of magnitude. Also, entropy based distance functions evaluate the general difference between two discrete probability distributions, while the Chebyshev distance focuses on the maximum difference. Since decisions are based upon the absolute posterior probability of a specific outcome, the Chebyshev distance is the measure that directly relates to the decision making. Van Engelen [9] introduced a method to compute an absolute upper bound for the maximum absolute error based on the K-L divergence. However, its reliance on computing prior marginals in advance limits its applicability for pruning BBNs. We base our method directly on the Chebyshev distance and local prior conditional distributions rather than prior marginals.

Aim. We aim to get a safe estimate of the real maximum impact. Given a BBN $\mathcal{G}$ containing variables V_1 and V_2 we first define, in Eq. 2, the real maximum impact of V_2 on V_1 in the context of evidence $\mathbf{e}$. We then define, in Eq. 3, the generalized real maximum impact of V_2 on V_1 as the maximum absolute difference that two different instantiations of V_2 can cause in the posterior probability for any state of V_1 given any possible combination of evidence $\mathbf{e}$ for other variables in $\mathcal{G}$, where set E holds all evidence configurations for $\mathcal{G}$.

$$\delta_{\mathbf{e}}(V_1|V_2) = \max_{v_{2i}\in V_2; v_{2j}\in V_2} D_{Ch}\Big(P(V_1|v_{2i},\mathbf{e}), P(V_1|v_{2j},\mathbf{e})\Big) \tag{2}$$

$$\delta(V_1|V_2) = \max_{\mathbf{e}\in\mathsf{E}} \Big(\delta_{\mathbf{e}}(V_1|V_2)\Big) \tag{3}$$

Algorithm

First Phase. We first calculate the maximum impact a variable may potentially have on other variables within its markov blanket, if the BBN beyond the markov blanket could take any form. Figure 1 shows V_1's markov blanket, a subgraph of $\mathcal{G}$ that contains V_1's parents, children and parents of children. Suppose $\mathbf{e}$ is a set of evidence, and we want to compute the posteriors for V_1 given this evidence set: $P(V_1|\mathbf{e})$. Each edge to V_1 can be considered to partition $\mathcal{G}$ in subgraphs: edge $V_2 \to V_1$ divides $\mathcal{G}$ in an upper subgraph $\mathcal{G}^-_{V_2}$ and a lower subgraph $\mathcal{G}^+_{V_2}$. Let $\mathbf{e}^-_{V_2}$ be the subset of $\mathbf{e}$ that concerns the variables in $\mathcal{G}^-_{V_2}$. Likewise, edges $V_3 \to V_1$, $V_1 \to V_4$ and $V_1 \to V_5$ create the following subgraphs end evidence sets: $\mathcal{G}^-_{V_3}$ with $\mathbf{e}^-_{V_3}$, $\mathcal{G}^+_{V_4}$ with $\mathbf{e}^+_{V_4}$ and $\mathcal{G}^+_{V_5}$ with $\mathbf{e}^+_{V_5}$. Applying Bayes' rule, the posterior probability distribution for V_1 given $\mathbf{e}$ can then be computed as follows:

$$P(V_1|\mathbf{e}) = P(V_1|\mathbf{e}^-_{V_2},\mathbf{e}^-_{V_3},\mathbf{e}^+_{V_4},\mathbf{e}^+_{V_5}) \tag{4}$$

$$= \eta\, P(V_1|\mathbf{e}^-_{V_2},\mathbf{e}^-_{V_3})\, P(\mathbf{e}^+_{V_4}|V_1)\, P(\mathbf{e}^+_{V_5}|V_1)$$

$$= \eta\Big(\sum_{V_2 V_3} P(V_1|V_2,V_3)P(V_2|\mathbf{e}^-_{V_2})P(V_3|\mathbf{e}^-_{V_3})\Big)\lambda(V_1) \tag{5}$$

In Eq. 4 and 5, η is a normalizing constant and $\lambda(V_1) = P(\mathbf{e}^+_{V_4}|V_1)P(\mathbf{e}^+_{V_5}|V_1)$. The posteriors of V_1 are computed by combining prior information $P(V_1|V_2,V_3)$ and current observations in evidence set $\mathbf{e}$. Each parent and child variable of V_1 contributes a parameter conditioned by a subset of $\mathbf{e}$. The maximizing causal parameters for V_1's parents can be derived from the conditional probability table $P(V_1|V_2,V_3)$. For computing the local potential maximum impact of V_2 on V_1, $\Delta_{V_1}(V_1|V_2)$, we set these as $p = P(v_{1k}|v_{2i},v_{3c})$ and $q = P(v_{1k}|v_{2j},v_{3c})$. For the diagnostic parameters of V_1's children, we set $\lambda = \lambda(v_{1k})$. The λ value that maximizes $\Delta_{V_1}(V_1|V_2)$ can be calculated in closed form with Eq. 7. The value $\Delta_{V_1}(V_1|V_2)$ can then be computed with Eq. 6.

$$\Delta_{V_1}(V_1|V_2) = \max_{i,j,k,c}\left|\frac{1}{2\,\lambda\,p - \lambda - p + 1}\,\lambda\,p - \frac{1}{2\,\lambda\,q - \lambda - q + 1}\,\lambda\,q\right|$$

$$\text{unless } p \text{ or } q \text{ is 0, or } p \text{ or } q \text{ is 1, then } \Delta_{V_1}(V_1|V_2) = 1 \tag{6}$$

$$\lambda = \frac{\sqrt{(p-1)p(q^2-q)} + p(-q) + p + q - 1}{p+q-1}$$

$$\text{unless }\; p+q-1=0\;,\text{ then: }\; \lambda = \frac{1}{2} \tag{7}$$

Second Phase. The second phase assesses the maximum potential impact between nodes further apart. A propagation algorithm is used to investigate all d-connecting paths between each pair of nodes. Multiple d-connecting paths may exist between a pair of variables, and are likely to partly overlap. For a single path segment, we take the product of all local potential impact values along intermediate edges as a safe overestimate of the real maximum impact. When parallel path segments converge we take their sum to approximate their joint influence.

Experiments

To assess the quality of our approximation of the real maximum impact values, the algorithm was tested on randomly generated BBNs. The BBNs were generated by using an algorithm designed and implemented by [4]. The $\Delta(V_a|V_b)$ values were compared to the corresponding $\delta(V_a|V_b)$ values, based on simulated evidence. In each BBN, all possible constellations of evidence were taken into account to discover the real maximum impact. In total 48000 pairs of variables were evaluated in singly connected graphs, and 90000 pairs of variables in multiply connected graphs. The experiments show that accuracy of the $\Delta(V_a|V_b)$ values decreases with the number of states a variable may have, the maximum degree of the graph, and the number of edges between V_a and V_b. The algorithm never returned an underestimate of the real maximum impact. Figure 2 gives the estimated impact as function of the real impact in these experiments.

Discussion

The method was successfully used to estimate the impact of variables in a BBN. Instead of getting an optimally accurate assessment of the real maximum impact, we have chosen an approach that is guaranteed never to return an underestimate of the real impact. Not underestimating a variable's impact in the context of decision making can be crucial. To improve the method's accuracy and applicability, it could be extended to respect existing evidence and to get the joint impact of multiple variables to a single goal variable.

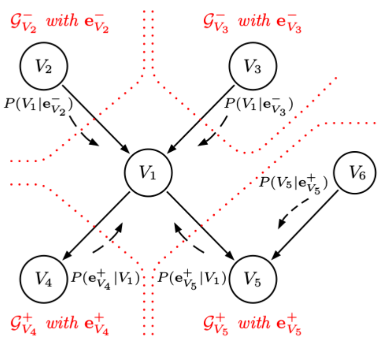

Fig. 1. The markov blanket of variable V_1.

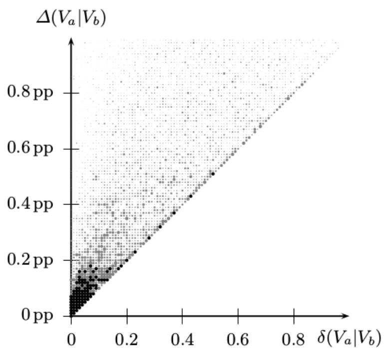

Fig. 2. Estimated impact as function of the real impact.

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A syntactic characterisation of the Gabbay–de Jongh logics

Jeroen Goudsmit

[11] introduced refutation systems as a formal means of reasoning non-derivability. One can think of a refutation system as a proof system for non-derivability. The "proofs" in this system are formal inferences of the refutability of a given formula ϕ, written $\dashv \phi$, and these inferences are called refutations. When every refutable formula is non-derivable the refutation system is sound, and when the converse holds it is said to be complete.

[12] proposed a refutation system for intuitionistic propositional logic (IPC), which consisted of the rules listed below for $\mathsf{L} = \mathsf{IPC}$. In stating that this system is sound and complete for IPC he, in essence, conjectured that IPC is the sole intermediate logic with the disjunction property. The situation however turned out more subtle, see [2] for a survey.

$$\frac{}{\dashv p}\,\mathsf{Ax} \qquad \frac{\dashv \sigma(\phi)}{\dashv \phi}\,\mathsf{Subs} \qquad \frac{\dashv \psi \qquad \phi \vdash_{\mathsf{L}} \psi}{\dashv \phi}\,\mathsf{MT} \qquad \frac{\dashv \phi \qquad \dashv \psi}{\dashv \phi \vee \psi}\,\mathsf{DP}$$

The axiom Ax states that no propositional variable is derivable, which is a valid statement for any intermediate logic. The reverse substitution rule Subs is also sound for any intermediate logic, and the same goes for *modes tollens*, MT. Finally, the rule DP is basically the counter-positive of the disjunction property, which states that if $\phi \vee \psi$ is derivable, then one amongst ϕ and ψ must be derivable. To summarise, Ax, Subs and MT form a sounds refutation system for each intermediate logic L. When we let L be CPC, classical propositional logic, it is complete. The latter is clear when one realises that a counter model of a formula ϕ gives rise to a substitution σ such that $\sigma(\phi) \equiv \bot$. Note that, although each refutation system can be sound for many logics, there is but one for which it is both sound and complete. It is in this sense that a refutation system syntactically characterises a logic.

[13] proved that replacing the rule DP by the rules GDP_n for all $n \geqslant 2$ as below suffices to refute all non-theorems of IPC. That is to say, the refutation system containing Ax, Subs, MT and

GDP_n for all $n \geqslant 2$ is both sound and complete for IPC. There is a striking resemblance between the rules GDP_n and the Visser rules, an important scheme of admissible rules as described in detail by [9].[1]

$$\frac{\dashv \bigwedge_{i=1}^{n} (\phi_i \to \psi_i) \to \phi_j \text{ for all } j = 1, \ldots, n}{\dashv \bigwedge_{i=1}^{n} (\phi_i \to \psi_i) \to \bigvee_{i=1}^{n} \phi_i} \, \mathsf{GDP}_n$$

We will present a characterisation of the intermediate logics introduced by [5] in terms of refutation systems. These Gabbay–de Jongh logics play an interesting role in the study of admissibility of intermediate logics. [6] proved that these logics have finitary unification, and that admissibility of these logics is axiomatised by a restriction of the above mentioned Visser rules. We demonstrate conditions under which one obtains an intimate connection between admissibility and refutability. In particular, the Gabbay–de Jongh logics satisfy these conditions.

Let us get a bit more technical. Consider any intermediate logic, that is to say, a consistent axiomatic extension of IPC, and let $\vdash$ denote its derivability relation from here onwards. A rule is a pair, written Γ/Δ where Γ and Δ are both finite sets of formulae. Such a rule is said to be *admissible*, denoted $\Gamma \mathrel{|\!\!\sim} \Delta$, when for all substitutions σ one has that

$$\text{if } \vdash \sigma(\phi) \text{ for all } \phi \in \Gamma \text{ then } \vdash \sigma(\chi) \text{ for some } \chi \in \Delta.$$

This multi-conclusion form of admissibility is taken from [10]. As an example, see that the rule $p \vee q/\{p, q\}$ is admissible precisely if the intermediate logic at hand enjoys the disjunction property.

When χ is classically non-derivable we know that $\nvdash \chi$. Consequently, if $\phi \mathrel{|\!\!\sim} \Delta$ for some set of clasically non-derivable formulae Δ, then ϕ is non-derivable. These observations motivate the following theorem connecting refutation systems and admissibility.

Theorem 1. *Suppose that for all formulae ϕ one has $\nvdash \phi$ if and only if there is a set* CPC*-non-derivable formulae Δ and a substitution σ such that $\sigma(\phi) \mathrel{|\!\!\sim} \Delta$. The refutation system defined as below is sound and complete, that is to say, it satisfies $\nvdash\, = \,\dashv$.*

$$\frac{}{\dashv p} \qquad \frac{\dashv \sigma(\phi)}{\dashv \phi} \qquad \frac{\dashv \psi \qquad \phi \vdash \psi}{\dashv \phi} \qquad \frac{\dashv \chi \text{ for all } \chi \in \Delta \qquad \phi \mathrel{|\!\!\sim} \Delta}{\dashv \phi}$$

A rule Γ/Δ is said to be derivable when $\Gamma \vdash \chi$ for some $\chi \in \Delta$. All derivable rules are admissible. Moreover, a rule that is admissible in any intermediate logic is in fact derivable in CPC.[2] In particular, all admissible rules of CPC are derivable in CPC. From these observations it is not hard to see that CPC is the sole intermediate logics whose admissible rules are derivable that satisfies the hypothesis of Theorem 1.

[5] introduced a sequence of intermediate logics stratified over the natural numbers. For each $n \geqslant 1$ they considered the logic complete with respect to finite rooted trees with branching up until degree n. This logic later also became known as BB_n ([1]). We remark that BB_0 is simply CPC, and that BB_1 is known as the Gödel–[3] logic. The admissible rules of this intermediate logic are derivable as per [4], so by the above we know it to not satisfy the hypothesis of Theorem 1.

We can prove that BB_n satisfies the hypothesis of Theorem 1 for all $n \neq 1$. As an immediate consequence we thus obtain a complete refutation system for this infinite chain of intermediate logics. The result can furthermore be strengthened by restricting the amount of rules included in the thusly constructed refutation system.

[1]The Visser rules appear in the context of arithmetic in [14, Theorem 9.1], see Section 1.4 for some historical background.

[2]This can be proven by an argument similar to that of [9, Fact 3.12].

[8] proved that all admissible rules of IPC follow from the Visser rules. A similar result for the Gabbay–de Jongh logics was obtained by [6]. This suggests that in the above definition of $\dashv (-)$ it suffices to restrict the right-most clause to those rules arising from Visser rules. This turns out to be the case, culminating in the following theorem which gives a syntactic characterisation of all Gabbay–de Jongh logics, except for BB_1. Future work would be to see if Theorem 1 can be generalised to have a hypothesis that is both necessary and sufficient.

Theorem 2. *Let $n \geqslant 2$ be arbitrary. The refutation system containing the rules* Ax*,* Subs*,* MT *and* GDP_n *is sound and complete for* BB_n*.*

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Suppositional inquisitive semantics: support, reject, dismiss

Jeroen Groenendijk and Floris Roelofsen

Aim

In inquisitive semantics, a sentence is taken to express a *proposal* to update the common ground of the conversation in one or more ways. The aim here is to develop a more fine-grained semantics than the standard implementation of inquisitive semantics, which does not only characterize the responses that *support* the proposal expressed by a sentence, but also the responses that *reject* it, and responses that *dismiss a supposition* of it. In fact, we will end up with a logical characterization of a wider catalogue of response types than the three mentioned above, but only these three figure in the recursive definition of the semantics. We argue that such a more fine-grained approach considerably broadens the empirical scope of the inquisitive framework, especially in the domain of conditional sentences.

Background: from truth to support

Classically, the meaning of a sentence resides in its *truth-conditions.* The underlying idea is that one knows the meaning of a sentence, or at least a core aspect thereof, if one knows under which circumstances the sentence is true and under which it is false. Clearly, this notion of meaning is only suitable for a particular type of sentences, namely declarative statements. There are other types of sentences, for instance interrogatives, which are not naturally thought of as being either true or false in a given situation.

One way to obtain a notion of meaning that is suitable for both declarative and interrogative sentences is to move from truth-conditions to *support-conditions.* The underlying idea, then, is that one knows the meaning of a sentence just in case one knows which information states support the given sentence, and which don't. For instance, an information state s, modeled as a set of possible worlds, supports an atomic declarative sentence p just in case every world in s makes p true, it supports $\neg p$ if every world in s makes p false, and finally, it supports the interrogative sentence $?p$ just in case it supports either p or $\neg p$. A support-based semantics for a propositional language, which deals with declaratives and interrogatives in a uniform way, has been developed in recent work on *inquisitive semantics* (e.g. [2, 3, 2]).

Given a support-based semantics, it is natural to think of a speaker who utters a sentence ϕ as *proposing* to enhance the common ground of the conversation, modeled as an information state, in such a way that it comes to support ϕ. Thus, in uttering p a speaker proposes to enhance the common ground in such a way that it comes to support p, and in uttering $?p$ a speaker proposes to enhance the common ground in such a way that it comes to support either p or $\neg p$.

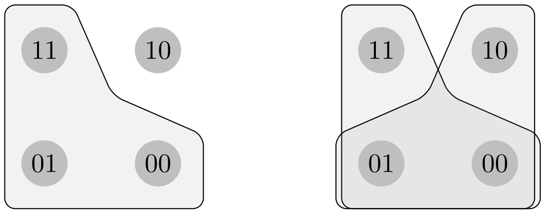
Prima facie it is natural to assume that support is *persistent*: if an information state s supports a sentence ϕ, then we may assume that every more informed information state $t \subset s$ also supports ϕ. This property is indeed assumed in the work on inquisitive semantics cited above, and determines to a large extent how the developed system behaves.

Support for conditionals

Let us now zoom in on conditional sentences, since this is where we would like to argue that a more refined picture is ultimately needed. Consider the following two sentences, a declarative and an interrogative conditional, respectively:

(1) If Peter goes to the party, Maria will go as well. $p \to q$

(2) If Peter goes to the party, will Maria go as well? $p \to ?q$



(a) Meaning of (1) (b) Meaning of (2)

Figure 4: Support for declarative and interrogative conditionals.

The meanings of these sentences in standard support-based inquisitive semantics are depicted in figures 4(a) and 4(b), respectively. In these figures, 11 is a world where p and q are both true, 10 a world where p is true but q is false, etcetera. We have only depicted the *maximal* states that support each sentence. Since support is persistent, all substates of these maximal supporting states also support the given sentences.

A state s supports a conditional sentence $\phi \rightarrow \psi$ (whether declarative or interrogative) just in case every state $t \subseteq s$ that supports ϕ also supports ψ. For instance, the state $s = \{11, 01, 00\}$ supports $p \rightarrow q$, because any substate $t \subseteq s$ that supports p (there are only two such states, namely $\{11\}$ and $\emptyset$) also support q. Similarly, one can verify that the states $\{11, 01, 00\}$ and $\{11, 01, 00\}$ both support $p \rightarrow ?q$. For convenience, we will henceforth use $|\phi|$ to denote the state consisting of all worlds where ϕ is classically true. So the states $\{11, 01, 00\}$ and $\{10, 01, 00\}$ can be denoted more perspicuously as $|p \rightarrow q|$ and $|p \rightarrow \neg q|$, respectively.

Support and reject

As mentioned, the support-conditions for a sentence ϕ are naturally thought of as capturing the *proposal* that a speaker makes in uttering ϕ. There are at least two ways in which other conversational participants may react to such a proposal: they may go along with it, or reject it. Now, what does it mean to reject the proposal made by an utterance of ϕ? Can this be explicated in terms of the support-conditions for ϕ?

At first sight, it indeed seems possible to do this in a very natural way. Suppose that a speaker A utters a sentence ϕ, and a responder B reacts with ψ. A proposes to enhance the common ground to a state that supports ϕ, while B proposes to enhance the common ground to a state that supports ψ. Then we could say that B *rejects* A's initial proposal just in case any state s that supports ψ is such that no consistent substate $t \subseteq s$ supports ϕ. After all, if this is the case, then any way of satisfying B's counterproposal leads to a common ground which does not support ϕ and which cannot be further enhanced in any way such that it comes to support ϕ while remaining consistent.

For many basic cases, this characterization seems adequate. For instance, if A utters p and B responds with $\neg p$, then according to the given characterization, B rejects A's initial proposal, which accords with pre-theoretical intuitions. However, in the case of conditionals, the given characterization is problematic. Intuitively, the proposal expressed by (1) above can be rejected with (3).

(3) No, if Peter goes to the party, Maria won't go. $p \rightarrow \neg q$

However, there is a consistent state that supports both (1) and (3), namely $|\neg p|$. So according to the above characterization, (3) does not reject (1). This example illustrates something quite fundamental: in general, reject-conditions cannot be derived from support-conditions. Thus, a semantics that aims to provide a comprehensive characterization of the proposals that speakers make when uttering sentences in conversation, needs to specify, at least, both support- and reject-conditions.[1]

Radical inquisitive semantics [4, 5] is a system in which support- and reject-conditions are stated, with the aim to deal with the type of phenomena discussed here. Suppositional inquisitive semantics is an extension of that system that intends to improve on it.

Suppositions

Besides full-fledged rejection, there is another type of non-supportive response to the conditionals in (1) and (2), exemplified in (4).

(4) Actually, Peter won't go to the party. $\neg p$

Suppose that A utters (1) and that B reacts with (4). One natural way to think about this response is as one that *dismisses a supposition* that A was making, namely that Peter may go to the party. If A utters the interrogative conditional in (2), she can also be taken to make this supposition; and if B reacts with (4), she can again be taken to dismiss this supposition. Clearly, the suppositions that a speaker makes in issuing a certain proposal, and responses that dismiss such suppositions, cannot be characterized purely in terms of the support-conditions for that sentence.

From radical to suppositional

At first sight it may seem that such suppositional phenomena can be captured if we formulate both support- and reject-conditions, and characterize states that dismiss a supposition of a sentence as states that both support and reject the sentence. This, roughly, is the strategy taken in radical inquisitive semantics.

This works fine for simple cases like $p \to q$, but does not give satisfactory results for slightly more complex cases like $(p \vee q) \to r$.

In radical inquisitive semantics, we get that $\neg p$ rejects but doesn't support $(p \vee q) \to r$, and hence it does not count as dismissing a supposition of it. Likewise, $\neg p$ supports but does not reject $\neg((p \vee q) \to r)$, which is equivalent with $(p \to \neg r) \vee (q \to \neg r)$, and hence it does not count as dismissing a supposition of it. The reason behind this is that support and rejection are both persistent in radical inquisitive semantics. Thus, given that the state $|p \to \neg r|$ rejects $(p \vee q) \to r$, so does $|\neg p|$, which is a substate of it. And for the same reason, given that the state $|p \to \neg r|$ supports $(p \to \neg r) \vee (q \to \neg r)$, so does $|\neg p|$.

This is clearly problematic. Intuitively, $\neg p$ should count as dismissing a supposition of $(p \vee q) \to r$, not as rejecting it, but only ruling out that it could still be supported. Likewise, $\neg p$ should count as dismissing a supposition of $(p \to \neg r) \vee (q \to \neg r)$, not as supporting it, but only ruling out that it could still be rejected. This is how things will work out in suppositional inquisitive semantics, where we not only state support- and reject-conditions, but also separate conditions which determine when a supposition of a sentence is dismissed.

Suppositional inquisitive semantics

We specify a suppositional inquisitive semantics, InqS, for the language of propositional logic, with $?\phi$ as an abbreviation of $\phi \vee \neg\phi$. The semantics is given by a simultaneous recursive definition of

[1] One can conceive of an even weaker notion of rejection, under which (1) would already be 'rejected' by responses like *Peter might go without Maria.* Note, however, that this response signals that the responder *refuses to accept* the given proposal, rather than signalling that the proposal is *unacceptable*, in the sense that updating with it would lead to an inconsistent state. The notion of rejection that we will model is the one where it really signals unacceptability of the given proposal.

three notions: $s \sup^+ \phi$, *s supports* ϕ, $s \sup^- \phi$, *s rejects* ϕ, and $s \sup^\circ \phi$, *s dismisses a supposition of* ϕ. We will first state the clauses and then go through them one by one.

Definition 18 (Suppositional inquisitive semantics)**.**

1. $s \models^+ p$ iff $s \neq \varnothing$ and $\forall w \in s\colon w(p) = 1$

   $s \models^- p$ iff $s \neq \varnothing$ and $\forall w \in s\colon w(p) = 0$

   $s \models^\circ p$ iff $s = \varnothing$

2. $s \models^+ \neg\phi$ iff $s \models^- \phi$

   $s \models^- \neg\phi$ iff $s \models^+ \phi$

   $s \models^\circ \neg\phi$ iff $s \models^\circ \phi$

3. $s \models^+ \varphi \wedge \psi$ iff $s \models^+ \varphi$ and $s \models^+ \psi$

   $s \models^- \varphi \wedge \psi$ iff $s \models^- \varphi$ or $s \models^- \psi$

   $s \models^\circ \varphi \wedge \psi$ iff $s \models^\circ \varphi$ or $s \models^\circ \psi$

4. $s \models^+ \varphi \vee \psi$ iff $s \models^+ \varphi$ or $s \models^+ \psi$

   $s \models^- \varphi \vee \psi$ iff $s \models^- \varphi$ and $s \models^- \psi$

   $s \models^\circ \varphi \vee \psi$ iff $s \models^\circ \varphi$ or $s \models^\circ \psi$

5. $s \models^+ \varphi \to \psi$ iff both of the following hold: (a) $\exists t \subseteq s\colon t \models^+ \phi$
   (b) $\forall t\colon$ if $t \models^+ \varphi$, then $\exists u \supseteq t\colon u \models^+ \phi$ and $u \cap s \models^+ \psi$

   $s \models^- \varphi \to \psi$ iff both of the following hold: (a) $\exists t \subseteq s\colon t \models^+ \phi$
   (b) $\exists t\colon t \models^+ \varphi$ and $\forall u \supseteq t\colon$ if $u \models^+ \phi$, then $u \cap s \models^- \psi$

   $s \models^\circ \varphi \to \psi$ iff at least one of the following holds:
   (a) $\forall t \subseteq s\colon t \not\models^+ \phi$
   (b) $s \models^\circ \phi$
   (c) $\exists t\colon t \models^+ \varphi$ and $\forall u \supseteq t\colon$ if $u \models^+ \phi$, then $u \cap s \models^\circ \psi$

Atoms

A state s supports an atomic sentence p just in case s is consistent and p is true in all worlds in s. Similarly, s rejects p just in case s is consistent and p is false in all worlds in s. Finally, s dismisses a supposition of p just in case s is inconsistent. The idea behind the latter clause is that in uttering p, a speaker makes the trivial supposition that p may or may not be the case—which can only be dismissed by an absurd, inconsistent response.

Negation

A state s supports $\neg\phi$ just in case it rejects ϕ. Vice versa, s rejects $\neg\phi$ just in case it supports ϕ. Finally, s dismisses a supposition of $\neg\phi$ just in case it dismisses a supposition of ϕ.

Conjunction

A state s supports $\phi \wedge \psi$ just in case it supports both ϕ and ψ, and it rejects $\phi \wedge \psi$ just in case it rejects either ϕ or ψ. Finally, s dismisses a supposition of $\phi \wedge \psi$ just in case it dismisses a supposition of ϕ or dismisses a supposition of ψ.

Disjunction

A state s supports $\phi \vee \psi$ just in case it supports either ϕ or ψ, and it rejects $\phi \vee \psi$ just in case it rejects both ϕ and ψ. Finally, s dismisses a supposition of $\phi \vee \psi$ just in case it dismisses a supposition of ϕ or dismisses a supposition of ψ.

Implication

The clause for implication is the most involved.

Support. Let us see what is necessary for s to support $\phi \to \psi$. First of all, the antecedent, ϕ, should be 'supportable' in s, i.e., some substate of s should support ϕ. Secondly, the idea is that s has to support the consequent, ψ, relative to any maximal state supporting the antecedent ϕ. So, among the states that support the antecedent ϕ, we have to look at all the maximal states, and whenever u is such a maximal state supporting ϕ, we have to check that $u \cap s$ supports the consequent ψ. There is one complication, however: it may be the case that there are no maximal supporting states for ϕ. This is why the clause is formulated as it is: we require that for every state t that supports ϕ there is a state $u \supseteq t$ that still supports ϕ and which is such that $u \cap s$ supports ψ (a technique adopted from [5]).

Reject. Now let us see what is necessary for s to reject $\phi \to \psi$. First of all, as in the case of support, the antecedent of the implication, ϕ, should be 'supportable' in s, i.e., some substate of s should support ϕ. Second, the idea is that s should reject ψ relative to at least one maximal state supporting ϕ. But again, it may be the case that there are no maximal states supporting ϕ. To circumvent this, we require that there be at least one state t supporting ϕ such that every state $u \supseteq t$ that still supports ϕ is such that $u \cap s$ rejects ψ.

Suppositional dismissal. Finally, let us see what is necessary for s to dismiss a supposition of $\phi \to \psi$. There are three cases in which this happens. The first is when the antecedent of the implication, ϕ, is not supportable in s, i.e., no substate of s supports ϕ. Notice that this is precisely the oppositive of the first requirement in the support clause and the rejection clause. The second case in which s dismisses a supposition of $\phi \to \psi$ is when s dismisses a supposition of the antecedent, $s \models^\circ \phi$. And finally, the third case is when s dismisses a supposition of the consequent, relative to some 'maximal' state supporting the antecedent. Again, it may be the case that there are no maximal states supporting the antecedent, so what we require is that there be at least one state t supporting ϕ such that every state $u \supseteq t$ that still supports ϕ is such that $u \cap s$ dismisses a supposition of ψ.

This semantics accounts for the motivating examples discussed above, and in the paper we show that it naturally accounts for many other (more complex) cases as well. We also present a treatment of epistemic modalities, and in particular their interaction with conditionals, in this suppositional framework.

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An arithmetical accessibility relation
Paula Henk

Preliminaries

In this note, we shall deal with modal logic and arithmetic. We denote by $\mathcal{L}_\Box$ the language of propositional modal logic, and by $\mathcal{L}_\mathsf{A}$ the language of arithmetic.

For the sake of simplicity, we formulate our results for the theory of Peano Arithmetic (PA), however we could replace it by any of its r.e. consistent Σ_1–sound extensions. We assume a gödelnumbering of the syntactical objects of PA, and identify syntactical objects with their codes. We use self–explanatory notation for $\mathcal{L}_\mathsf{A}$–formulas representing properties of syntactical objects and operations on them (such as substitution) in PA. The formula $\pi(x)$ denotes a fixed intensionally correct Σ_1–numeration of the axioms of PA in PA. Given $\pi(x)$, the provability predicate $\mathsf{Pr}_\pi(x)$ of PA is constructed in the usual way (see [Fef60]). We write $\overline{n}$ for the $\mathcal{L}_\mathsf{A}$–term that denotes the natural number n.

The modal system GL is K plus Löb's axiom $\Box(\Box A \to A) \to \Box A$. Let $\mathcal{K}$ be the class of frames that are transitive irreflexive finite trees. It is known that GL is sound and complete w.r.t. $\mathcal{K}$.

Definition 19. A *realisation* $*$ is a function from the propositional letters of $\mathcal{L}_\Box$ to sentences of $\mathcal{L}_\mathsf{A}$. The domain of a realisation is extended to all formulas of $\mathcal{L}_\Box$ by requiring that it commutes with the propositional connectives, and letting $(\Box A)^* := \mathsf{Pr}_\pi(A^*)$ for all $A \in \mathcal{L}_\Box$.

Theorem 1 (Solovay[Sol76])**.** *$\vdash_{\mathsf{GL}} A \Leftrightarrow$ for all realisations $*$, $\vdash_{\mathsf{PA}} A^*$.*

In order to define our arithmetical accessibility relation, we shall make use of the notion of a relative translation.

Definition 20. Let Σ and Θ be signatures[1]. A *relative translation* from Σ to Θ is a tuple $\langle \delta, \tau \rangle$, where δ is a Θ–formula with one free variable, and τ is a map from relation symbols R of Σ to formulas R^τ of Θ. We require the number of free variables in R^τ to be equal to the arity of R. We extend τ to a function from all formulas of Σ by requiring that it commutes with the propositional quantifiers, and furthermore

i. $(Rx_0 \dots x_n)^\tau = R^\tau(x_0 \dots x_n)$

ii. $(\forall x\, A)^\tau = \forall x\, (\delta(x) \to A^\tau)$

From a semantic perspective, a translation $(\delta, \tau) : \Sigma \to \Theta$ is a way of uniformly defining a model $\mathcal{M}^{(\delta,\tau)}$ of signature Σ inside a given model $\mathcal{M}$ of signature Θ, provided that $\mathcal{M} \models \exists x \delta(x)$. The domain of $\mathcal{M}^{(\delta,\tau)}$ consists of all $b \in \mathcal{M}$ with $\mathcal{M} \models \delta[b]$, and the interpretation of the relation symbols is given by τ. We say that $\mathcal{M}^{(\delta,\tau)}$ is an *internal model* of $\mathcal{M}$.

The t–internal Model Relation

The t–internal model relation is a strengthening of the internal model relation. Roughly speaking, $\mathcal{M}'$ is a t–internal model of $\mathcal{M}$ ($\mathcal{M} \rhd_\mathsf{t} \mathcal{M}'$) if $\mathcal{M}'$ is an internal model of $\mathcal{M}$, $\mathcal{M}$ has a definable truth predicate $\mathsf{tr}(x)$ for $\mathcal{M}'$, and it is internally true in $\mathcal{M}$ that $\mathcal{M}'$ is a model of PA.

[1] We only consider signatures of arithmetic, sometimes augmented with function symbols for primitive recursive functions. Replacing function symbols with relation symbols by a well–known algorithm, we can furthermore assume that the signatures are relational.

In order to express that $\mathsf{tr}(x)$ is well–behaved with respect to the atomic formulas and commutes with the quantifiers, the signature of $\mathcal{M}$ is augmented with a unary function symbol c. Intuitively, c names a function assigning domain constants to elements of the internal model. If φ is an $\mathcal{L}_{\mathsf{A}}$–formula whose free variables are among $k_0, \ldots, k_n$, we shall write $\varphi(c_{x_0} \ldots c_{x_n})$ for[2] :

$$\mathsf{sim\_sbst}(\overline{k_0}, \ldots \overline{k_n}, c(x_0), \ldots, c(x_n), \overline{\varphi(v_{k_0}, \ldots v_{k_n})}). \tag{8}$$

Note that if the function c is as above, then (8) is a sentence, given that for all $0 \leqslant i \leqslant n$, x_i is in the domain of the internal model.

Definition 21. Let $\mathcal{M} \models \mathsf{PA}$. Let Θ and Σ be the signatures of $\mathcal{M}$ and $\mathcal{M}'$ respectively. $\mathcal{M}'$ is a t*–internal model of* $\mathcal{M}$ (we write $\mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$) if there exists a quadruple $j = \langle \delta, \tau, \mathsf{tr}, c \rangle$ (we write $j : \mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$), where

i. $\langle \delta, \tau \rangle$ is a relative translation from Σ to Θ, and $\mathcal{M}' = \mathcal{M}^{\langle \delta, \tau \rangle}$.

ii. tr is a Θ–formula with one free variable.

iii. c is added to Θ as a new function symbol.

iv. Intuitively, the signature Σ' is obtained by extending[3] Σ with the domain constants as given by c. The following sentences are true in $\mathcal{M}$:

   i. $\forall x_0, \ldots, x_n\, (\delta(x_0) \wedge \ldots \wedge \delta(x_n) \rightarrow (R^{\tau} x_0 \ldots x_n \leftrightarrow \mathsf{tr}(R c_{x_0} \ldots c_{x_n})$, where R is an $n+1$-ary relation symbol of Σ.

   ii. $\forall \varphi \in \mathsf{sent}_{\Sigma'}, \forall \psi \in \mathsf{sent}_{\Sigma'}\, (\mathsf{tr}(\varphi \rightarrow \psi) \leftrightarrow (\mathsf{tr}(\varphi) \rightarrow \mathsf{tr}(\psi)))$

   iii. $\forall \varphi \in \mathsf{sent}_{\Sigma'}\, (\mathsf{tr}(\neg\varphi) \leftrightarrow \neg\mathsf{tr}(\varphi))$

   iv. $\forall \varphi \in \mathsf{sent}_{\Sigma'}, \forall u \in \mathsf{var}_{\Sigma'}\, (\mathsf{tr}(\forall u\, \varphi) \leftrightarrow \forall x\, (\delta(x) \rightarrow \mathsf{tr}(\varphi(c_x)))$

   v. $\forall \varphi \in \mathsf{sent}_{\mathcal{L}_{\mathsf{A}}}\, (\pi(\varphi) \rightarrow \mathsf{tr}(\varphi))$

Note that the above definition has the formula $\pi(x)$ as a parameter[4]. If $j : \mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$, we refer to the components of j by δ_j, τ_j, tr_j, and c_j.

Lemma 1. *If $j : \mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$, then for any sentence φ of the language of $\mathcal{M}'$,*

$$\mathcal{M} \models \varphi^{\tau_j} \leftrightarrow \mathsf{tr}_j(\varphi).$$

Note that as a consequence of Lemma 1 and clause 4.v of Definition 21, $\mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$ implies $\mathcal{M}' \models \mathsf{PA}$.

Lemma 2. *If $j : \mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$, then for any $\mathcal{L}_{\mathsf{A}}$–sentence φ,*

$$\mathcal{M} \models \mathsf{Pr}_{\pi}(\varphi) \rightarrow \mathsf{tr}_j(\varphi).$$

The following theorem gives us a way of obtaining strong interpretations.

[2] If n is an $\mathcal{L}_{\mathsf{A}}$–formula, then $\mathrm{Sbst}(k, m, n)$ is the formula that results when replacing the free variable v_k in n by $\overline{m}$. $\mathrm{Sbst}(k, m, n)$ is a primitive recursive function. $\mathsf{sbst}(x, y, z)$ is a function symbol representing $\mathrm{Sbst}(k, m, n)$ in PA in the sense that for all k, m, and for all $\mathcal{L}_{\mathsf{A}}$–formulas φ,

$$\vdash_{\mathsf{PA}} \mathsf{sbst}(\overline{k}, \overline{m}, \overline{\varphi}) = \overline{\mathrm{Sbst}(k, m, \varphi)}$$

The more general case of simultaneous substitution can be treated similarly.

[3] To be more precise, $\mathcal{M} \models \forall x\, (\mathsf{term}_{\Sigma'}(x) \leftrightarrow \mathsf{term}_{\Sigma}(x) \vee \exists y(\delta(y) \wedge x = c(y)))$.

[4] To generalise the result for a r.e. consistent Σ_1–sound extension T of PA, $\pi(x)$ is replaced by an intensionally correct Σ_1–numeration $\sigma(x)$ of T in T.

Theorem 2. *Let $\mathcal{M} \models \mathsf{PA}$. If $\mathcal{M} \models \neg\mathsf{Pr}_\pi(\varphi)$, then there is some $\mathcal{M}'$ such that $\mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$ and $\mathcal{M}' \models \neg\varphi$.*

Proof. By the formalised version of Gödel's Completeness Theorem, noting that a formula representing a Henkin set can be viewed as a truth predicate. □

Theorem 3. *Let $\mathcal{M} \models \mathsf{PA}$. Then for any $\mathcal{L}_{\mathsf{A}}$–sentence φ,*

$$\mathcal{M} \models \mathsf{Pr}_\pi(\varphi) \Leftrightarrow \textit{ for all } \mathcal{M}' \textit{ with } \mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}', \mathcal{M}' \models \varphi$$

Proof. For the direction from left to right, let $\mathcal{M} \models \mathsf{Pr}_\pi(\varphi)$, and let $j, \mathcal{M}'$ be such that $j : \mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$. By Lemma 2, $\mathcal{M} \models \mathsf{tr}_j(\varphi)$, whence by Lemma 1, $\mathcal{M} \models \varphi^{\tau_j}$. Since $\mathcal{M}' = \mathcal{M}^{(\delta_j, \tau_j)}$, we have $\mathcal{M}' \models \varphi$ by the internal model construction. For the other direction, suppose that $\mathcal{M} \models \neg\mathsf{Pr}_\pi(\varphi)$. By Theorem 2, there is some $\mathcal{M}'$ with $\mathcal{M} \rhd_{\mathsf{t}} \mathcal{M}'$ and $\mathcal{M}' \models \neg\varphi$ as required. □

An Arithmetical Kripke Model

Let $M = \langle W, R, V\rangle$, with $W = \{1, \ldots, n\}$ be a GL–model. We construct an arithmetical Kripke model $\mathfrak{M}_{\mathrm{big}}$ that is bisimilar to M.

Let $S_0, \cdots, S_n$ be the Solovay sentences corresponding to M, and $*$ the Solovay realisation, i.e. $p^* = \bigvee_{i:M,i \Vdash p} S_i$ (see [Sol76]). We define the Kripke model $\mathfrak{M}_{\mathsf{big}}$ as follows:

i. $W_{\mathsf{big}} = \{\mathcal{M} \mid \mathcal{M} \models \mathsf{PA}\}$

ii. R_{big} is $\rhd_{\mathsf{t}}$

iii. V_{big} is defined as: $\langle W_{\mathsf{big}}, \rhd_{\mathsf{s}}\rangle, \mathcal{M} \Vdash p :\Leftrightarrow \mathcal{M} \models p^*$

As an immediate consequence of Theorem 3 and the definition of $\mathfrak{M}_{\mathsf{big}}$, we have for all $\varphi \in \mathcal{L}_{\Box}$,

$$\langle W_{\mathsf{big}}, \rhd_{\mathsf{s}}\rangle, \mathcal{M} \Vdash \varphi \Leftrightarrow \mathcal{M} \models \varphi^* \tag{*}$$

Hence the forcing of modal formulas is independent (modulo $*$) of whether we consider $\mathcal{M}$ as a node in the Kripke model $\mathfrak{M}_{\mathsf{big}}$, or as a model of PA. Finally, we will show that M and $\mathfrak{M}_{\mathsf{big}}$ are bisimilar. Define the relation $Z : W \times W_{\mathsf{big}}$ as follows: $(i, \mathcal{M}) \in Z :\Leftrightarrow \mathcal{M} \models S_i$.

Theorem 4. *Z is a total bisimulation between M and $\mathfrak{M}_{\mathsf{big}}$.*

Proof. By the properties of the Solovay sentences. □

As a corollary, we get the arithmetical completeness of GL. If $\mathsf{GL} \nvdash A$, then by modal completeness of GL there is a GL-model M with $M = \langle\{1, \ldots, n\}, R, V\rangle$, and $M, 1 \Vdash \neg A$. Let $\mathfrak{M}_{\mathsf{big}}$ be the arithmetical Kripke model as above, and let Z be the total bisimulation between M and $\mathfrak{M}_{\mathsf{big}}$. Consider $\mathcal{M} \in W_{\mathsf{big}}$ such that $(1, \mathcal{M}) \in Z$, i.e. $\mathcal{M} \models S_1$. Since Z is a bisimulation, $\mathfrak{M}_{\mathsf{big}}, \mathcal{M} \Vdash \neg A$. By (*), this implies $\mathcal{M} \models \neg A^*$. Since $\mathcal{M} \models \mathsf{PA}$, we have that $\nvdash_{\mathsf{PA}} A^*$ as required.

Several directions suggest themselves for future research. For example, we intend to examine whether the result holds when one replaces PA by a weaker theory such as $\mathrm{I}\Sigma_1$. Furthermore, we want to investigate the modal logic of the arithmetical Kripke model $\mathfrak{M}_{\mathrm{big}}$, with the accessibility relation replaced by some other relation between models of PA. Some possibilities are: the internal model relation where we demand a truth predicate for the internal model, the internal model relation, and the end-extension relation. A difference from the t–internal model relation is that these relations need not be definable by an arithmetical formula.

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Inline contraction decomposition

Maarten Janssen

Introduction

Assigning POS tags to words in a text is typically done of three steps: tokenization, morphological analysis, and disambiguation. The last two of those steps are frequently done in a statistically-driven, language-independent fashion: although rule-based disambiguation tools contain language specific rules, most statistical disambiguation tools are in fact language independent. And also morphological analysis is done in a language-independent manner in many off-the-shelf POS taggers, where letter sequences are used to determine the possible POS tags for unknown words.

That means that the smallest of the three steps in POS tagging, tokenization, is the only part that is truly language dependent in current taggers. For the construction of a truly language-independent POS tagging system, tokenization is therefore a significant obstacle. Tokenization involves several problems, including sentences splitting, the decomposition of contractions, and the treatment of multiword expressions, including the recognition of named entities and dates. In this article I will show how the language-dependent contraction splitting rules can be avoided by making tokenization part of the tagging process itself. This method was developed for CorpusWiki, an online language-independent tagging environment, which is presented in the next section.

CorpusWiki

CorpusWiki (`http://www.corpuswiki.org`) is an online tool that aims to allow (non-computational) linguists to build POS tagged corpora for their language of choice. The system is meant to allow for the creation of tagged corpora, most relevantly for those languages for which no corpus data exist, and for which it would be very difficult to create tagged data by traditional means (although it has been used for large languages like Spanish and English as well). For large but less-resourced languages there are often corpus projects under way, in the case of Geogian there is for instance the corpus project by Paul Meurer [3], as well as corpora without POS tags, such as the dialectal corpus by Beridze & Nadaraia [1]. But for smaller languages such as for instance Ossetian, Urum, or Laz corpus projects of any size are much less likely. Corpora for these languages without POS tags often exist, for these specific languages in the TITUS project (Gippert), but annotating such corpora involves a computational staff that is typically not available for such languages.

CorpusWiki attempts to make it possible to create an annotated corpora without the need of involvement of computational linguistic staff, by having a user-friendly, language-independent interface in which the user only has to make linguistic judgements, and the computational machinery is taken care of automatically behind the screens. The system is designed for the construction of gold-standard style corpora of around 1 millions tokens that are manually verified, and all corpora are build in a collaborative fashion, in which all users can help in enlarging the corpus of a given language. For all languages, both the corpus and the tagger with parameter files are available for download.

In the CorpusWiki set-up, each text in the corpus is individually treated in three steps. First, the text is added to the system. Then the text is automatically assigned POS tags using an internal POS tagger, which is trained on all tagged texts already in the system. And finally, the errors made by the automatic tagger have to be corrected manually. Once the verification of the tags is complete, the tagger is retrained automatically. In this fashion, with each new text, the accuracy of the tagger improves and the amount of tagging errors that have to be corrected goes down. The only text that is treated differently in this set-up is the very first text, since for the first text, there are no prior tagged data. The system uses a canonical fable as the first text for each language to make the initial manual tagging of the first text go as smoothly as possible.

To make the tagging process more user friendly, tags are not presented as position-based tags, but rather as separated features, with a pull-down presenting the possible values for each feature, such as “singular/plural” for number on nouns (in languages with a binary number system for nouns). Which are the possible values in a given language is defined by the user, who indicates which classes, features, and values are used in that language, by selecting them from a long list of possible grammatical features and values. By having pre-defined pull-downs, the user does not have to study the tagset beforehand, but merely has to select from a list of explicitly named options.

Contractions

A contraction in the computational sense is a single orthographic word that grammatically functions as two separate words. A good example is the French words *au*, which is the contraction of a preposition (*à* = at, in) and the masculine definite determiner. There are two common ways of treating such contractions: by assigning a single, complex tag to the word as in (1), or splitting the word into two separate words as in (2), potentially while keeping the information about the contracted words the individual parts, as in (3).

$$\text{au PREP+DET} \tag{9}$$

$$\begin{array}{ll} \text{à} & \text{PREP} \\ \text{le} & \text{DET} \end{array} \tag{10}$$

$$\begin{array}{l} \text{à\_au PREP} \\ \text{le\_au DET} \end{array} \tag{11}$$

In many phonologically transcribed corpora, option (1) is preferred, since the contraction forms a single phonological word, which makes it hard to represent phonological transcription in (2) and (3). On the other hand, in (1) it is difficult to represent the lemmatized form of the two separate word, which is one of the reasons why (2) is more common in written corpora.

In the solution in (2), the contraction is simply treated as if it were two separated words. The separation of contractions is done in the tokenization process, so already at the level of morphological analysis, there are no longer any contractions in the text. As said before, in order to establish this, it is necessary to write language-specific rules that tell which strings in the language should be separated, which is not a feasible option for a system like CorpusWiki.

Instead, CorpusWiki pushes the separation of contractions further down the pipe, and only deals with word separation as part of the morphological analysis, hence getting rid of tokenization as a distinct step altogether. That is to say, the initial process only separates words simply on spaces (and punctuation marks), and only in a later stage do some of the space-delimited tokens that are in fact more than one word get subdivided into several tokens.

Not all contractions are alike: which words are written together differs greatly from language to language. In order to correctly deal with tokenization, CorpusWiki distinguishes between four different types of contractions:

1. *Lexical contrations*. Lexical contractions are individual combinations of words written together, such as for instance the case of *au* in French. They are not always transparent (there is no recognizable orthographic trace of the determiner *le* in *au*), and they are not always separable (*à* in Portuguese is a contraction of the preposition *a* and the determiner *a*, but there are no separate parts that can be assigned the role of PREP and DET). They do typically have a lexicalized form that is listed in the dictionary (*aux* is the plural form of *au*, which has a lexical entry in most French dictionaries).

2. *Orthographic contractions*. Orthographic contractions are productive and open-ended. For instance, whenever the French determiner *le* is followed by a word starting with a vowel, it is written together with that word: *le* + *eau* = *l'eau* (the water). The parts of an orthographic contraction are clearly recognizable, and in the majority of cases, there is one non-modified base word (*eau*), and another (trunctated) word adjoined to it (*l'*). Orthographic contraction do not have a citation form, and are never truly seen as one word.

3. *Clitics*. Clitics are in most respects like orthographic contractions, except that linguistically, they are viewed as something between an affix and a separate word. Most contractions do not have a lexicalized form, but there are cases in which the clitized word is said to have a lexical form. For instance, reflexive verbs with a clitic in Romance languages are typically represented with a cliticized citation form: in the Catalan sentence *vols rentar-te?* (do you want to wash yourself), the verb *rentar-te* is said to be a form of the pronominal verb *rentar-se*.

4. *Agglutinates*. Agglutinative morphemes are part of the inflectional morphology, and as such, should not be seen as contractions. However, some agglutinating affixes should be seen as contractions nevertheless, mostly class-changing affixes. Take the Turkish word *odamdayım* (I am in my room). Taken as a whole, this is a verb form in the 1st person singular indicative. But it does not have a lemmatized form as a verb, only a (lemmatized) root form: *oda* (room), which itself is not a verb but a noun. The verbal part of the word (*-yım*) behaves in several respects as separated from the nominal base. For instance, when an adjective, such as *kücük* (small) is placed in front of the word, it will modify only the noun, and not the word as a whole. Therefore, the part *-yım* is treated as a contracted part as well, in concordance with the treatment by for instance Bisazza & Federico [2].

Although there are differences in treatment between all these types of "contractions" in CorpusWiki, for the purpose of separation only the contrast between (1) and (2)-(4) is of importance. Lexical contractions are simply stored in the corpus as contractions, and when a lexical contraction is found in a new text, it will only be separated if it already was used as a lexical contraction in the training corpus. The productive types of contractions (2-4), on the other hand, are treated in the morphological analysis step.

Productive contractions

During the morphological analysis, productive contractions get separated in the following way: any word that can potentially be a contracted word is assigned a contracted structure as (one of) its morphological analyses, together with the lexical likelihood of that analysis. The potential contractions, like the rest of the morphological analysis in CorpusWiki, are established on the basis of the training corpus.

To establish this, contractions in the corpus are subdivided into sub-tokens, while keeping the space-delimited token as well. An example of the structure for the French contraction *l'eau* (the water) can be found in figure 1, presented in the internal XML-based format of the system. In this example, the space-delimited token is marked as a (orthographic) contraction, and the two individual tokens *l'* and *eau* are marked as a determiner and a common noun respectively, and provided with a lemma as well as a part of the contracted form. Furthermore, the first part (l') is marked as a contracted/truncated part, and the second as the unmodified base. Suppressed in figure 1 are the additional morphosyntactic feature/value pairs, indicating for instance that *l'* is the masculine singular form of the definite article.

```
<tok pos="CONTR" & type="orthographic">
     l'eau
     <dtok pos=DET lemma="le" dtype="contracted" form="l'"/>
     <dtok pos="SUB" lemma="eau" dtype="base" form="eau"/>
</tok>
```

Figure 5: Figure 1. CorpusWiki representation for *l'eau*

The subdivision in the training corpus is done by the annotator in two steps: first, the annotator has to indicate that the word is a contraction, and which of the four types of contractions it is, and then each part of the contraction is treated like any other word with pull-downs for feature-value pairs. And as with normal words, the percentage of cases in which the tagger will already correctly analyze the contraction correctly will grow as the corpus gets bigger.

When building the parameter files from the training corpus, the system will collect all contracted forms, and store them in a file (contraction lexicon), with an indication about their frequency, and whether they appear before or after the base form. So from figure 1, the system will add a record for a prefixing contracted form *l'* to the contraction lexicon.

When parsing a new text, this contraction lexicon is then used to parse (potential) contractions in the following way. When encountering the word *l'eau*, the system will mark it the same as it was marked in the previous text: as an orthographic contraction of the determiner *l'* and the noun *eau*. But when dealing with a word that has not appeared in the training corpus, say *l'administrateur* (the administrator), the system will assign it a potential analysis as a contraction of *l'* and *administrateur*, since it starts with a string that is in the contraction lexicon.

The probability of the analysis of *l'administrateur* as a contraction is calculated on the basis of all the words in the training corpus that start with *l'*, and the likelihood of *l'administrateur* being a contraction depends on the percentage of those words that is in fact a contraction. Since all words in the training corpus starting with *l'* will be contractions (since the apostrophe in French is an explicit truncation marker), the tagger will mark all new occurrences starting with *l'* as contractions as well. Notice that this is not based on the presence of the apostrophe itself, since there are many other languages in which the apostrophe does not mark contractions.

In cases (languages) where there is no explicit marking of the truncation, the system will decide in the disambiguation process whether the word is more likely to be a contraction or a simple word. So for instance, the Spanish word *hazte* (do/make yourself) is a form of the verb *hacer* (to do) with the clitic *the* (you) attached to it, without an explicit boundary between the two. Since there are many other words in Spanish ending on *-te* that are not cliticized words, the system will keep both options, with their respective lexical likelihood score, for disambiguation in context. Hence, as OOV items, both *parate* (stop yourself) and *karate* (karate) will be fed to the disambiguation step as either cliticized verbs or substantives.

After splitting off the contracted part (clitic, truncated form, or agglutinate), the system will continue with the morphological analysis of the remainder of the word. So when parsing the word *l'eau*, the system will first split the word in two parts, end then determine the potential POS tags for both parts the same way it does for any word. In this particular case, it will find that *l'* can be a masculine or feminine definite article, and *eau* can be only a feminine singular common noun, and determine the most likely of these in the disambiguation process.

Since this process is recursive, it can deal with words with multiple contracted parts without much problem. So when dealing with a word with two clitics, like the form *haztelo* (*haz* (do) + *the* (you) + *lo* (it)) in Spanish, it will first split off the indefinite pronoun *lo* and after that the personal pronoun *te*. And since contraction parts can appear on either side, there is no problem with words that have contracted parts on both sides either, such as the Catalan *d'anar-hi* (to go there), where the base form *anar* (to go) has an orthographic contraction to the left, and a clitic to the right.

Because the separation of contracted forms in this method is driven completely by examples provided in the training corpus, the process is dealt with in a statistically-driven, language-independent fashion. The overall result is in (almost) all cases the same in the end as a method in which language-specific tokenization rules are written, but works without the need to write explicit rules. This means that it is a tokenization method that is ideally suited for a language-independent system like CorpusWiki.

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Complexity of unification and admissibility with parameters in transitive modal logics

Emil Jeřábek

Introduction

Admissibility of inference rules is among the fundamental properties of nonclassical propositional logic: a rule is admissible if the set of tautologies of the logic is closed under the rule, or equivalently, if adjunction of the rule to the logic does not lead to derivation of new tautologies. Admissible rules of basic transitive modal logics (**K4**, **S4**, **GL**, **Grz**, **S4.3**, ...) are fairly well understood. Rybakov proved that admissibility in a large class of modal logics is decidable and provided semantic description of their admissible rules, see [15] for a detailed treatment. Ghilardi [5] gave a characterization of projective formulas in terms of extension properties of their models, and proved the existence of finite projective approximations. This led to an alternative proof of some of Rybakov's results, and it was utilized by Jeřábek [9, 11] to construct explicit bases of admissible rules. A sequent calculus for admissible rules was developed by Iemhoff and Metcalfe [8]. Methods used for transitive modal logics were paralleled by a similar treatment of intuitionistic and intermediate logics, see e.g. [15, 4, 6, 7].

Admissibility is closely related to unification [2, 1]: for equational theories corresponding to algebraizable propositional logics, E-unification can be stated purely in terms of logic, namely a unifier of a formula is a substitution which makes it a tautology. Thus, a rule is admissible iff every unifier of the premises of the rule also unifies its conclusion, and conversely the unifiability of a formula can be expressed as nonadmissibility of a rule with inconsistent conclusion. In fact, the primary purpose of Ghilardi [5] was to prove that unification in the modal logics in question is finitary.

In unification theory, it is customary to work in a more general setting that allows for extension of the basic equational theory by free constants. In logical terms, formulas may include atoms (variously called parameters, constants, coefficients, or metavariables) that behave like ordinary propositional variables for most purposes, but are required to be left fixed by substitutions. Some of the above-mentioned results on admissibility in transitive modal logics also apply to admissibility and unification with parameters, in particular Rybakov [15] proved the decidability of admissibility with parameters in basic transitive logics, and he has recently extended his method to show that unification with parameters is finitary in these logics [16, 17].

Various parts of the theory of admissible rules in transitive modal logic were generalized to the setting with parameters in [13]: this includes a characterization of projective formulas generalizing Ghilardi [5], the existence of projective approximations for logics satisfying suitable frame extension properties (called cluster-extensible, or clx, logics) leading to an alternative proof that admissiblity is decidable and unification is finitary in clx logics, a construction of bases of admissible rules (including independent bases, and finite bases where they exist) for clx logics, semantic descriptions of admissible rules, and various structural properties of clx logics.

The topic of this talk is the computational complexity of admissibility and unification with parameters in transitive modal logics, and clx logics in particular. The parameter-free case was studied in [10], where it was shown that admissibility is coNEXP-hard for transitive logics L such that, roughly speaking, all finite trees of depth 3 can be realized as skeletons of L-frames with prescribed final clusters, and on the other hand, this lower bound is matched by a coNEXP-completeness result for admissibility in logics satisfying suitable extension properties (while it is not stated that way in [10], the coNEXP-completeness result holds in particular for all clx logics of branching at least 2). Also, admissibility is coNP-complete for linear (i.e., branching at most 1) clx logics. For other results on the complexity of admissibility in nonclassical logics, see e.g. [3, 12, 18].

Based on the characterizations from [13], we will determine the complexity of admissibility with parameters in clx logics, as well as some related logics (intuitionistic logic, and variants of clx logics with a single top cluster). As we will see, the situation is more complicated than for parameter-free admissibility: without parameters, there are only two possibilities (coNP-complete or coNEXP-complete depending on linearity of the logic), whereas in the presence of parameters, the complexity of admissibility will reflect other semantic properties of the logic, leading to a richer landscape. In particular, admissibility with parameters has the same complexity as parameter-free admissibility for some logics, and larger complexity for other logics. Similarly to the parameter-free case, our lower bounds apply under very mild conditions, they are not limited to clx logics. Interestingly, the increased flexibility provided by parameters allows us to eliminate conclusions of rules, so that all the lower bounds for admissibility also apply to nonunifiability, and in particular admissibility has the same complexity as nonunifiability for every clx logic. (This can fail in the parameter-free case.)

The results reported in this talk will appear in [14].

Overview of the results

We work with transitive modal logics, i.e., normal modal logics $L \supseteq \mathbf{K4}$. We consider a propositional modal language with two kinds of atoms: variables and parameters. Substitutions are required to leave parameters intact. An *L-unifier* of a set of formulas $\varphi_1, \dots, \varphi_n$ is a substitution σ such that $\vdash_L \sigma(\varphi_i)$ for every i. A multiple-conclusion rule

$$\frac{\varphi_1, \dots, \varphi_n}{\psi_1, \dots, \psi_m}$$

is *L-admissible*, written as $\varphi_1, \dots, \varphi_n \mathrel{|\!\!\sim}_L \psi_1, \dots, \psi_m$, if every L-unifier of $\varphi_1, \dots, \varphi_n$ also unifies some ψ_j.

A particular class of logics where admissibility and unification with parameters is well-behaved was isolated in [13]. A logic $L \supseteq \mathbf{K4}$ with the finite model property is *cluster-extensible* (*clx*), if the following holds for every finite cluster type C and $n \in \omega$:

> Assume that L has at least one finite rooted frame with a root cluster of type C, and n immediate successor clusters. If F is any finite L-frame with n minimal clusters, then the frame obtained from F by attaching a new root cluster of type C is again an L-frame.

Here, a cluster type is an isomorphism type of a finite cluster qua Kripke frame: the possible cluster types are the irreflexive singleton type, and the n-element reflexive cluster types for each positive integer n. For extensions of $\mathbf{K4.2}$, one can relax the clx condition so that it only applies to frames F that have a unique top cluster. An analogous condition can be also considered for superintuitionistic logics: the logics $\mathbf{IPC}$, $\mathbf{KC}$, $\mathbf{T}_n$, and $\mathbf{KC} + \mathbf{T}_n$ (including $\mathbf{T}_0 = \mathbf{CPC}$, $\mathbf{T}_1 = \mathbf{LC}$) are clx. Clx logics have finite projective approximations, finitary unification type, and one can describe explicit bases of their admissible rules.

If L is a consistent linear (i.e., extending $\mathbf{K4.3}$) clx logic, then L itself, as well as L-admissibility without parameters, are coNP-complete. On the other hand, L-admissibility and L-nonunifiability with parameters are

- Σ^{P}_2-complete, if L has bounded cluster size, and branching 0 (i.e., depth 1). Examples: $\mathbf{S5} \oplus \mathbf{Alt}_n$, $\mathbf{K4} \oplus \Box\bot$, $\mathbf{CPC}$.
- PSPACE-complete, if L has bounded cluster size, and branching 1. Examples: $\mathbf{GL.3}$, $\mathbf{K4.3Grz}$, $\mathbf{S4.3Grz}$, $\mathbf{LC}$.
- NEXP-complete, if L has unbounded cluster size. Examples: $\mathbf{S5}$, $\mathbf{K4B}$, $\mathbf{S4.3}$, $\mathbf{K4.3}$.

If L is a nonlinear clx logic, then L is PSPACE-complete, and parameter-free L-admissibility is coNEXP-complete. Admissibility and nonunifiability with parameters are

- coNEXP-complete, if L has bounded cluster size. Example: $\mathbf{GL}$, $\mathbf{K4Grz}$, $\mathbf{S4Grz}$, $\mathbf{S4.2Grz}$, $\mathbf{IPC}$, $\mathbf{KC}$, $\mathbf{T}_n$ ($n \geqslant 2$).
- Θ^{EXP}_2-complete (see below for the definition of Θ^{EXP}_2), if $L \supseteq \mathbf{K4.2}$ has bounded inner cluster size, and unbounded top cluster size. Example: $\mathbf{S4.2} \oplus \mathbf{S4.1.4}$.
- Π^{EXP}_2-complete, otherwise. Examples: $\mathbf{K4}$, $\mathbf{S4}$, $\mathbf{S4.1}$, $\mathbf{S4.2}$.

Recall that NP is the class of languages recognizable in nondeterministic polynomial time, and coNP consists of their complements. Σ^{P}_k consists of languages that can be written as

$$x \in L \leftrightarrow \exists y_1 \, \forall y_2 \cdots Q y_k \, R(x, y_1, \dots, y_k), \tag{12}$$

where R is deterministically computable in time polynomial in the length of x (which also implicitly bounds the lengths of $y_1, \dots, y_k$). Π^{P}_k is defined dually. We have $\mathrm{NP} = \Sigma^{\mathrm{P}}_1$, $\mathrm{coNP} = \Pi^{\mathrm{P}}_1$. Equivalently, we can define the hierarchy inductively using oracle Turing machines: $\Sigma^{\mathrm{P}}_0 = \Pi^{\mathrm{P}}_0 = \mathrm{P}$, $\Sigma^{\mathrm{P}}_{k+1} = \mathrm{NP}^{\Sigma^{\mathrm{P}}_k}$, $\Pi^{\mathrm{P}}_{k+1} = \mathrm{coNP}^{\Sigma^{\mathrm{P}}_k}$. PSPACE consists of languages recognizable in polynomial space. NEXP are languages recognizable in nondeterministic exponential (i.e., 2^{n^c} for some constant c) time, and coNEXP are their complements. Σ_k^{EXP}-languages can be written as (12) with R computable in time exponential in the length of x, and Π_k^{EXP} is defined dually. Equivalently, $\Sigma_k^{\mathrm{EXP}} = \mathrm{NEXP}^{\Sigma_{k-1}^{\mathrm{P}}}$, $\Pi_k^{\mathrm{EXP}} = \mathrm{coNEXP}^{\Sigma_{k-1}^{\mathrm{P}}}$. Finally, we introduce Θ_2^{EXP} as the exponential analogue of the class sometimes denoted as Θ_2^{P}. Just like Θ_2^{P} itself, it has several equivalent definitions:

$$\Theta_2^{\mathrm{EXP}} := \mathrm{EXP}^{\mathrm{NP[poly]}} = EXP^{\|\mathrm{NP}} = \mathrm{P}^{\mathrm{NEXP}} = \mathrm{PSPACE}^{\mathrm{NEXP}}.$$

(Here, the [poly] means we only allow polynomially many oracle queries, $\|$ means that oracle queries are nonadaptive, and the space bound for $\mathrm{PSPACE}^{\mathrm{NEXP}}$ includes the oracle query tape.)

The classes mentioned above are ordered as

$$\mathrm{coNP} \subseteq \Sigma_2^{\mathrm{P}} \subseteq \mathrm{PSPACE} \subseteq \mathrm{(co)NEXP} \subseteq \Theta_2^{\mathrm{EXP}} \subseteq \Pi_2^{\mathrm{EXP}}.$$

While all the inclusions are generally assumed to be strict, and NEXP is assumed to be incomparable with coNEXP, it has only been proven that $\mathrm{coNP} \subsetneq \mathrm{(co)NEXP}$ and $\Sigma_2^{\mathrm{P}} \subsetneq \Pi_2^{\mathrm{EXP}}$.

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Positive formulas, minimal logic and uniform interpolation

Dick de Jongh and Zhiguang Zhao

Introduction

Minimal (propositional) logic MPC is obtained from the positive fragment, i.e. the $\neg, \bot$-free fragment of intuitionistic propositional logic IPC by adding a weaker negation: $\neg\varphi$ is defined as $\varphi \to f$, where the special propositional variable f is interpreted as the falsum. Therefore, the language of minimal logic is the $\neg, \bot$-free fragment of IPC plus f. Variable f has no specific properties, the Hilbert type system for MPC is as IPC's but without $f \to \varphi$. For the semantics, f is interpreted as an ordinary propositional variable, so we get the semantics of the $[\vee, \wedge, \to]$-fragment of IPC with an additional propositional variable f. In this article we give a semantic characterization of the positive formulas of IPC by the so-called top-model property. We the use this property to show that the positive fragment of IPC has a revised form of uniform interpolation and that this transfers to MPC.

The content of this article is the following:

In Section (4) we recall the syntax and semantics of IPC and MPC. In Section (4) we introduce the top-model property and show that it characterizes the positive formulas of IPC. In Section (4) we prove a revised version of uniform interpolation theorem and prove it for the positive fragment and for MPC.

Syntax and Semantics of MPC

In this section we recall the syntax of propositional language as well as the derivation systems of IPC and MPC, and their Kripke semantics. For more details, see [1].

Syntax

The propositional language $\mathcal{L}_I(P)$ of IPC consists of a countable or finite set P of propositional variables p_0, p_1, p_2, $\ldots$, propositional constants $\bot, \top$ and binary connectives $\wedge, \vee, \to$, $\neg\varphi$ is defined as $\varphi \to \bot$. The *positive fragment* of $\mathcal{L}_I(P)$ of IPC consists of the formulas of $\mathcal{L}_I(P)$ that do not contain $\neg$ or $\bot$. The propositional language $\mathcal{L}_M(P)$ of MPC consists of the formulas of the positive fragment to which the special propositional variable is f added. We take the axioms of IPC as in [1]. The axioms for MPC are the same except that $\bot\to\varphi$ is left out. So, derivations in MPC are the same as in IPC except that no $\bot$ or $\neg$ occurs, instead f may have occurrences.

For the proof of the uniform interpolation theorem of MPC in Section (4) we introduce the following notation: For any formula φ and any sequence $\vec{p} = (p_1, \dots, p_n)$ of propositional variables (here p_i can be f, but cannot be $\bot, \top$), $\varphi(\vec{p})$ is a formula with only propositional variables in $\vec{p}$.

Kripke Semantics

In this part we give the Kripke semantics of IPC and MPC.

A *Kripke frame* is a pair $\mathfrak{F} = (W, R)$ where W is a non-empty set and R is a partial order on it. A *Kripke model* is a triple $\mathfrak{M} = (W, R, V)$ where (W, R) is a Kripke frame and V is a *valuation* $V : P \cup \{f\} \to \mathscr{P}(W)$ (where $\mathscr{P}(W)$ is the powerset of W) such that for any $w, w' \in W$, $w \in V(p)$ and wRw' imply $w' \in V(p)$, where $p \in P \cup \{f\}$.

For formulas, the satisfaction relation is defined as usual with clauses for $p, f, \bot, \top, \vee, \wedge, \to$, where the semantics of f is the same as for the other propositional variables. If we define V on P and omit the clause for f, then we get the Kripke semantics of IPC; if we omit the clause for $\bot$, then we get the Kripke semantics of MPC. We use $\models_I$ and $\models_M$ to distinguish the satisfaction relation of IPC and MPC, and omit the index when it is not important or clear from the context. For IPC, we have the following completeness theorem (see e.g. [1]):

Theorem 1 (Strong Completeness of IPC)**.** *For any* IPC *formulas* Γ *and* φ*,* $\Gamma \vdash_{\mathsf{IPC}} \varphi$ *iff* $\Gamma \models_I \varphi$*.*

By a complete-via-canonicity proof, we have that MPC is strongly complete with respect to Kripke frames, i.e. for any Γ and φ, $\Gamma \vdash_{\mathsf{MPC}} \varphi$ iff $\Gamma \models_M \varphi$. The proof procedure is essentially the same as the complete-via-canonicity proof for IPC with respect to Kripke frames, just leave out $\bot$.

Theorem 2 (Strong Completeness of MPC)**.** *For any* MPC *formulas* Γ *and* φ*,* $\Gamma \vdash_{\mathsf{MPC}} \varphi$ *iff* $\Gamma \models_M \varphi$*.*

By a completeness-via-canonicity proof using adequate sets, we have the finite model property for IPC (again see [1]) and thereby for MPC:

Theorem 3 (Finite Model Property of MPC)**.** *For any* MPC *formula* φ*, if* $\nvdash_{\mathsf{MPC}} \varphi$*, then there is a rooted finite Kripke model* $\mathfrak{M}$ *falsifying* φ*.*

By the completeness theorem for MPC and IPC, since the semantic behavior of MPC in the language $\mathcal{L}_M(P)$ is exactly the same as that of IPC in the language $\mathcal{L}_I(P \cup \{f\})$ without $\bot$ (i.e. the positive $[\vee, \wedge, \to, \top]$-fragment $\mathcal{L}_I^+(P \cup \{f\})$ of $\mathcal{L}_I(P \cup \{f\})$), we can regard MPC as the positive fragment of IPC, and we have the following lemma:

Lemma 1. *For any formulas* Γ *and* φ *in* $\mathcal{L}_M(P) = \mathcal{L}_I^+(P \cup \{f\})$*,* $\Gamma \vdash_{\mathsf{MPC}} \varphi$ *iff* $\Gamma \vdash_{\mathsf{IPC}} \varphi$*.*

The Top-Model Property

We give a characterization of the $\neg, \bot$-free formulas of IPC by means of the following property:

Definition 22 (Top-Model Property)**.** We say that a formula φ has the *top-model property*, if for all Kripke models $\mathfrak{M} = (W, R, V)$, all $w \in W$, $\mathfrak{M}, w \models \varphi$ iff $\mathfrak{M}^+, w \models \varphi$, where $\mathfrak{M}^+ = (W^+, R^+, V^+)$ is obtained by adding a top point t (which is a successor of all points) such that all propositional variables are true in t.

For the top-model property we have the following proposition. For a proof, see [4] and [6].

Proposition 4.

1. *Every formula in $\mathcal{L}_I^+(P)$ and $\mathcal{L}_M(P)$ has the top-model property, and so has $\bot$.*

2. *For any formula φ in $\mathcal{L}_I(P)$, there exists a formula φ^+ in $\mathcal{L}_I^+(P)$ or $\varphi^+ = \bot$ such that for any top model $\mathfrak{M}$ and any point w in $\mathfrak{M}$, we have $\mathfrak{M}, w \models \varphi \leftrightarrow \varphi^+$.*

And this proposition leads to the following characterization.

Theorem 4. *A formula φ of* IPC *has the top-model property iff φ is equivalent to a $\neg, \bot$-free formula (in fact to φ^+) or to $\bot$.*

Proof. The direction from right to left is Proposition 4(1), so let us prove the other direction and assume that φ has the top model property, but is not equivalent to φ^+. Then there is a model $\mathfrak{M}$ with a world w so that φ and φ^+ have different truth values in $\mathfrak{M}, w$. Then, because both have the top-model property, φ and φ^+ have different truth values in $\mathfrak{M}^+, w$ as well. But that contradicts the fact given by Proposition 4 that φ and φ^+ behave identically on top models. □

Uniform Interpolation

In this section we prove a revised version of the uniform interpolation theorem for the positive fragment of IPC and for MPC by using the uniform interpolation theorem of IPC.

First of all we state the uniform interpolation theorem of IPC:

Theorem 5 (Uniform Interpolation Theorem of IPC)**.**

1. *For any formula $\varphi(\vec{p}, \vec{q})$ where $\vec{p}$, $\vec{q}$ are disjoint, there is a formula $\chi(\vec{p})$ (the* uniform post-interpolant *for $\varphi(\vec{p}, \vec{q})$) such that $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \chi(\vec{p})$, and for any $\psi(\vec{p}, \vec{r})$ where $\vec{r}$ and $\vec{p}$, $\vec{q}$ are disjoint, if $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \psi(\vec{p}, \vec{r})$, then $\vdash_{\mathsf{IPC}} \chi(\vec{p}) \to \psi(\vec{p}, \vec{r})$. We write $\exists \vec{q} \varphi$ for $\chi(\vec{p})$.*

2. *For any formula $\psi(\vec{p}, \vec{r})$ where $\vec{p}$, $\vec{r}$ are disjoint, there is a formula $\chi(\vec{p})$ (the* uniform pre-interpolant *for $\psi(\vec{p}, \vec{r})$) such that $\vdash_{\mathsf{IPC}} \chi(\vec{p}) \to \psi(\vec{p}, \vec{r})$, and for any $\varphi(\vec{p}, \vec{q})$ where $\vec{q}$ and $\vec{p}$, $\vec{r}$ are disjoint, if $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \psi(\vec{p}, \vec{r})$, then $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \chi(\vec{p})$. We write $\forall \vec{r} \psi$ for $\chi(\vec{p})$.*

This theorem is proved in [3] by a proof-theoretical method and in [2, 5] by the bisimulation quantifier method. In accordance with the latter we write $\exists \vec{q} \varphi(\vec{p}, \vec{q})$ for the post-interpolant and $\forall \vec{r} \varphi(\vec{p}, \vec{r})$ for the pre-interpolant.

In the form stated above uniform interpolation fails for the positive fragment, at least with regard to the pre-interpolant. It is namely so that $\forall p.p$ is $\bot$ and that is (up to equivalence) the only formula without p to imply p, and therefore no pre-interpolant exists in the positive fragment. We thank Albert Visser for this remark. However, this is in a way the only failure of the theorem; as long as we just consider positive formulas that are implied by at least one positive one containing only the relevant variables, the theorem holds. Using Theorem 5, we can prove the following form of uniform interpolation theorem for the positive fragment of IPC:

Theorem 6 (Uniform Interpolation Theorem for the positive fragment of IPC)**.**

1. *For any positive formula $\varphi(\vec{p}, \vec{q})$ where $\vec{p}$, $\vec{q}$ are disjoint, there is a positive formula $\theta(\vec{p})$ (the uniform post-interpolant for $\varphi(\vec{p}, \vec{q})$) such that $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \theta(\vec{p})$, and for any positive $\psi(\vec{p}, \vec{r})$ where $\vec{r}$ and $\vec{p}$, $\vec{q}$ are disjoint, if $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \psi(\vec{p}, \vec{r})$, then $\vdash_{\mathsf{IPC}} \theta(\vec{p}) \to \psi(\vec{p}, \vec{r})$. Morover, $\theta(\vec{p})$ is $(\exists \vec{q} \varphi)^+$.*

2. *For any positive formula $\psi(\vec{p}, \vec{r})$ where $\vec{p}$, $\vec{r}$ are disjoint, one of the following two cases holds:*

   (a) *There is a formula $\theta(\vec{p})$ (the uniform pre-interpolant for $\psi(\vec{p}, \vec{r})$) such that $\vdash_{\mathsf{IPC}} \theta(\vec{p}) \to \psi(\vec{p}, \vec{r})$, and for any $\varphi(\vec{p}, \vec{q})$ where $\vec{q}$ and $\vec{p}$, $\vec{r}$ are disjoint, if $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \psi(\vec{p}, \vec{r})$, then $\vdash_{\mathsf{IPC}} \varphi(\vec{p}, \vec{q}) \to \theta(\vec{p})$. Morover, $\theta(\vec{p})$ is $(\forall \vec{r} \psi)^+$.*

(b) *For any positive $\varphi(\vec{p},\vec{q})$ where $\vec{q}$ and $\vec{p}$, $\vec{r}$ are disjoint, $\not\vdash_{\mathsf{IPC}} \varphi(\vec{p},\vec{q}) \to \psi(\vec{p},\vec{r})$.*

Proof.

1. For the uniform post-interpolant, consider $\chi(\vec{p}) = \exists\vec{q}\varphi$ obtained from Theorem 5. Then $\theta(\vec{p}) = (\chi(\vec{p}))^+$ is the required formula:

   If $\not\models_I \varphi(\vec{p},\vec{q}) \to \theta(\vec{p})$, then $\mathfrak{M}, w \models_I \varphi(\vec{p},\vec{q})$ and $\mathfrak{M}, w \not\models_I \theta(\vec{p})$ for some $\mathfrak{M}, w$. By Proposition 4, $\mathfrak{M}^+, w \models_I \varphi(\vec{p},\vec{q})$ and $\mathfrak{M}^+, w \not\models_I \theta(\vec{p})$ (and thus also $\mathfrak{M}^+, w \not\models_I \chi(\vec{p})$), so $\not\models_I \varphi(\vec{p},\vec{q}) \to \chi(\vec{p})$, contradicting Theorem 5.

   Let $\models_I \varphi(\vec{p},\vec{q}) \to \psi(\vec{p},\vec{r})$. By Theorem 5, $\models_I \chi(\vec{p}) \to \psi(\vec{p},\vec{r})$. Suppose $\not\models_I \theta(\vec{p}) \to \psi(\vec{p},\vec{r})$, then $\mathfrak{M}, w \models_I \theta(\vec{p})$ and $\mathfrak{M}, w \not\models_I \psi(\vec{p},\vec{r})$ for some $\mathfrak{M}, w$. Then by Proposition 4, $\mathfrak{M}^+, w \models_I \theta(\vec{p})$ (and thus also $\mathfrak{M}^+, w \models_I \chi(\vec{p})$) and $\mathfrak{M}^+, w \not\models_I \psi(\vec{p},\vec{r})$. Hence $\not\models_I \chi(\vec{p}) \to \psi(\vec{p},\vec{r})$, a contradiction.

   We can see that $\theta(\vec{p}) \neq \bot$: Since $\models_I \varphi(\vec{p},\vec{q}) \to \theta(\vec{p})$ and $\varphi(\vec{p},\vec{q})$ is positive (thus satisfiable), $\theta(\vec{p})$ is also satisfiable. Therefore $\theta(\vec{p}) \neq \bot$.

2. The proof is similar to the one for the uniform post-interpolant. Note that in this case $(\forall q\theta)^+$ may be $\bot$, corresponding to the case (b).

□

Corollary 1 (Uniform Interpolation Theorem for MPC). *1. For any formula $\varphi(\vec{p},\vec{q})$ of* MPC *where $\vec{p}$, $\vec{q}$ are disjoint and may contain f, $\vdash_{\mathsf{MPC}} \varphi(\vec{p},\vec{q}) \to (\exists\vec{q}\varphi(\vec{p},\vec{q}))^+$, and for any positive $\psi(\vec{p},\vec{r})$ where $\vec{r}$ and $\vec{p}$, $\vec{q}$ are disjoint, if $\vdash_{\mathsf{MPC}} \varphi(\vec{p},\vec{q}) \to \psi(\vec{p},\vec{r})$, then $\vdash_{\mathsf{MPC}} (\exists\vec{q}\varphi(\vec{p},\vec{q}))^+ \to \psi(\vec{p},\vec{r})$.*

2. For MPC*-formula $\psi(\vec{p},\vec{r})$ where $\vec{p}$, $\vec{r}$ are disjoint, one of the following two cases holds:*

(a) *$(\forall\vec{r}\varphi(\vec{p},\vec{r}))^+$ is an* MPC*-formula, $\vdash_{\mathsf{MPC}} (\forall\vec{\varphi}(\vec{p},\vec{r}))^+ \to \psi(\vec{p},\vec{r})$, and for any $\varphi(\vec{p},\vec{q})$ where $\vec{q}$ and $\vec{p}$, $\vec{r}$ are disjoint, if $\vdash_{\mathsf{MPC}} \varphi(\vec{p},\vec{q}) \to \psi(\vec{p},\vec{r})$, then $\vdash_{\mathsf{MPC}} \varphi(\vec{p},\vec{q}) \to (\forall\vec{r}\psi(\vec{p},\vec{r}))^+$.*

(b) *For any* MPC*-formula $\varphi(\vec{p},\vec{q})$ where $\vec{q}$ and $\vec{p}$, $\vec{r}$ are disjoint, $\not\vdash_{\mathsf{MPC}} \varphi(\vec{p},\vec{q}) \to \psi(\vec{p},\vec{r})$.*

This means that in MPC the uniform post-interpolant exists for any formula, and the uniform pre-interpolant exists for any formula that is implied by at least one formula with the right variables. The result stands if instead of the formulation of the syntax with the additional variable f one chooses to formulate MPC with $\neg$ instead of f.

Example 1. The post-interpolant of $(p \to q) \to p$ w.r.t. p in the positive fragment and in MPC is not the formula $\exists q((p \to q) \to p)$, which is its post-interpolant in full IPC.

Proof. $\exists q((p \to q) \to p) = \neg\neg p$ and $(\neg\neg p)^+ = \top$. □

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Levels of knowledge and belief

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Introduction

Given a set of n agents and a proposition φ, a **level of knowledge** of φ is a (complete and consistent) description of who knows φ and what is known about each others' knowledge about φ. At one extreme, none of the agents knows anything about φ. The other extreme is when there is common knowledge of φ. There are many levels of knowledge between these two extremes that naturally arise when studying bounded reasoning in games [7] and analyzing social situations [10] (cf. also Parikh's discussion of *social software* in [9]).

Thinking abstractly about the levels of knowledge that can arise for a fixed proposition φ raises a number of interesting technical and conceptual questions. For instance, which levels of knowledge can arise under various circumstances of communication? This question has been extensively studied by researchers interested in logical models of distributed computing [5, 11, 4, 2],[1] and, more recently, by dynamic epistemic logicians (see, for example, [12]).

The question we focus on in this paper is: How many consistent levels of knowledge (or belief) are there? We adress these questions within the framework of epistemic logic, see [3] for details. Our main contribution is to unify and extend two previous results about the cardinality of levels of knowledge. The first result by Parikh and Krasucki (Theorem 3 of [11]) shows that there are only countably many *levels* of knowledge. A related result by Hart *et al.* (Theorem 2.2 of [6]) proves that there are, in fact, uncountably many *states* of knowledge. Of course, the results are not contradictory as the two papers are, in fact, counting different sets. The crucial difference is that Parikh and Krasucki only allow positive knowledge to be included in a description of a level of knowledge whereas Hart *et al.* focus on "know whether" statements in their description. We build on these and identify fragments of the standard epistemic language where there are countably many (consistent) descriptions of a level and those where there are uncountably many.

Results

To state our results formally, we give a formal definition of a level. The motivation behind the following definition is: For various reasoning contexts we are not interested in *all* formulae expressible by epistemic logic, but only in a certain fragment $\tilde{\mathcal{L}}$ of the language. The expressive power changes drastically with the choice of $\tilde{\mathcal{L}}$. The definition of a level is:

Definition 23. Let $\mathcal{L}$ be a modal language
i) The *level language* $\hat{\mathcal{L}}$ associated to $\mathcal{L}$ has the same logic symbols and the same modal operators as $\mathcal{L}$, but only one variable x and no constant symbols.
ii). An informational language $\tilde{\mathcal{L}}$ for $\mathcal{L}$ is any fragment of $\hat{\mathcal{L}}$.

[1] A classic result here is by Halpern and Moses [5] who show that common knowledge cannot arise in asynchronous situations

iii) A subset L of a $\tilde{\mathcal{L}}$ is a *level of knowledge* iff there is some maximally consistent subset $R \subset \hat{\mathcal{L}}$ such that $L = R \cap \tilde{\mathcal{L}}$

Equivalently, we could define levels of knowledge as follows: $L \subseteq \tilde{\mathcal{L}}$ is a level of knowledge iff there is a pointed Kripke model $(\mathcal{M}, w)$ and some proposition φ such that $L = \{\psi \in \tilde{\mathcal{L}} | (\mathcal{M}, w) \vDash \psi\frac{\varphi}{x}\}$, hence the connection to knowledge of a proposition φ. Our main classification results are:

Theorem 1. *For the following sublanguages $\tilde{\mathcal{L}}$ of the epistemic language there are countably many levels of information (where* At *denotes the set of agents):*

- $\tilde{\mathcal{L}}_K$ *the complete sublanguage generated by* $\{K_i : i \in \mathsf{At}, x\}$ *(the levels of knowledge as studied by Parikh)*
- $\tilde{\mathcal{L}}_L$ *the complete sublanguage generated by* $\{L_i : i \in \mathsf{At}, x\}$
- $\tilde{\mathcal{L}}_\wedge$ *the complete sublanguage generated by* $\{K_i : i \in \mathsf{At}, \wedge, x\}$
- $\tilde{\mathcal{L}}_{I\vee}$ *the language generated by* $\{D_I | I \subseteq \mathsf{At}, , x\}$*, where the operator* $D_I x$ *is defined via*

$$D_I x := \bigvee_{i \in I} K_i x$$

- $\tilde{\mathcal{L}}_{2\vee}$ *the complete sublanguage generated by* $\{K_1, K_2, \vee, x\}$.

For the other end of the spectrum we have the following characterisation results:

Theorem 2. *For the following sublanguages $\tilde{\mathcal{L}}$ of the epistemic language there are uncountably many levels knowledge if there is more than one agent*

- $\tilde{\mathcal{L}}_\neg$ *the complete sublanguage generated by* $\{K_i : i \in \mathsf{At}, \neg, x\}$
- $\tilde{\mathcal{L}}_{L,K}$ *the complete sublanguage generated by* $\{K_i, L_i : i \in \mathsf{At}, x\}$
- *the languages* $\tilde{\mathcal{L}}_B$ *generated by* $\{B_i : i \in \mathsf{At}, x\}$ *(and obviously any superset of* $\tilde{\mathcal{L}}_B$*)*
- *the language generated by* $\{X_i : i \in \mathsf{At}, x\}$*, where the operator* $X_i x$ *is defined as* $X_i x := K_i x \vee K_i \neg x$ *("knowing whether" as studied by Hart)*

In the paper we identify the general pattern behind the above result. We show that the general distinguishing factor between countably and uncountably many levels of knowledge is precisely the T-axiom of modal logic (and the fact that there are at least two agents). This allows us to carry over our analysis to many other settings, for instance to multi-agent deontic logics.

Apart from the cardinality questions discussed above there are other interesting questions related to levels. Much of the recent work in epistemic logic has focused on *dynamic* extensions modeling acts of communication and observation (cf. [12] for a discussion of this literature). A natural question here is how levels of knowledge change as the result of different dynamic operations. Since many dynamic operations such as public announcement or radical belief upgrade can be expressed as product updates, we constrain ourselfs to studying product updates only. See [1] for definitions and a general introduction. We say that a level of knowledge L is realized in some Kripke Model $(\mathcal{M}, w)$ for some φ iff $L = \{\psi \in \tilde{\mathcal{L}} | (\mathcal{M}, w) \vDash \psi\frac{\varphi}{x}\}$

Our main dynamic result is:

Theorem 3. *Let $L_1 \neq \emptyset$ and L_2 be levels of knowledge. Let $\mathcal{M}, s$ be a Kripke model realizing L_1. Then there is an event model $\mathcal{E}$ and a Kripke model $\mathcal{L}, t$ realizing L_2 with $\mathcal{M}, s \oplus \mathcal{E} = \mathcal{L}, t$ if and only if $L_1 \subseteq L_2$.*

A second result we are intersted in is then how to realize consistent levels of knowledge. Given a Kripke model $(\mathcal{M}, w)$, it is easy to read off the level of knowledge of w (of some φ). A natural question to ask is what can be said about the reverse direction: Given a consistent level of knowledge L , how can we construct a Kripke model realizing L. And what about a model of minimal cardinality?

Lemma 1. *Let $\tilde{\mathcal{L}}$ only contain positive formulae (i.e. formulae consisting of $K_i, \vee, \wedge$). Then there is a constructive map φ from the set of levels of knowledge (of some proposition p) to the set of Kripke models of $\mathcal{L}$.*

Combining this with a (constructive) map from Kripke models to Harsanyi type spaces (see [8]) gives a construcive way to generate minimal Kripke models realizing L.

For various philosophical as well as practical reasons we take *finite* Kripke models to be of special interest. Since there are only countably many finite Kripke models, we cannot in generally hope for every level of knowledge to be realizable in a finite Kripke model. However, for those levels mentioned in theorem 1 we do have the following result

Lemma 2. *Let $\tilde{\mathcal{L}}$ be one of the sublanguages of the epistemic language mentioned in* 1 *and let L be a level of knowledge of $\tilde{\mathcal{L}}$. Then there is a finite Kripke model $\mathcal{M}, s$ that realizes L (for a certain atomic proposition p)*

Conclusion and Outlook

Our results can be summarized as follows: Restricting the language to only positive knowledge (respectively epistemic possibility) and combinations thereof (conjunctive/disjunctive) yields a relatively low expressive power, in that only countably many different levels of knowledge are consistent. On the other hand, adding the ability to describe *ignorance* (i.e. negative knowledge) blows up the number of consistent descriptions of levels of knowledge. We furthermore identify the connection between levels of knowledge and dynamics as presented by product updates.

One starting point for further research is the following question: Until now, we have characterized all possible levels of knowledge of a particular proposition φ. How about the interdependencies between the levels of knowledge of different propositions. That is given levels of knowledge of φ and ψ, what can be said about the levels of $\varphi \vee \psi, \varphi \wedge \psi, \varphi \rightarrow \psi$ etc. More generally, we can define a map Ψ assigning each formula of some language $\mathcal{T}$ (for simplicity, assume $\mathcal{T}$ is a propositional language) a level of knowledge (in some sublanguage of $\mathcal{L}$). How can we characterize such maps?

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Dialect dictionaries with the functions of representativeness and morphological annotation in Georgian Dialect Corpus

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Introduction

The Georgian Dialect Corpus is being developed within the framework of the larger project *Linguistic Portrait of Georgia*. Presently, the GDC is available for researchers on the web at `http://mygeorgia.ge/gdc/`.

> The GDC consists of: 1 453 261 tokens; 301 203 word forms; 199861 contexts; 3017 texts.

The material included in the GDC has been recorded from 2703 informants at 812 villages in Georgia, Turkey, Iran, and Azerbaijan. The earliest data date back to the early 20^{th} century, while the latest ones have been recorded in 2012. The working team of the GDC processes the collection of dialect texts. It carries out the entire technological procedure, beginning from field work to the inclusion of a text into the corpus; hence, the requirements of representativeness have been taken into account at the very moment of text recording. The corpus incorporates samples of 17 sub-varieties of Georgian and of the Laz dialect of Zan. Currently, the corpus can be queried for entire words and for word subsequences (beginning, inner part, ending). Query hits are shown as KWIC concordances. It is also possible to view a text as a whole, which turns the corpus into a library of dialect texts.

New texts are continuously being added to the corpus, and at the same time, the morphological annotation of the material is being worked on; therefore, so far, the corpus can only be queried according to the following meta-textual (non-linguistic) features:

- language and dialect
- place of recording
- the informant's identity
- thematic and chronological features of a text

- text type (narrative, poetry, conversation . . .)

We plan to provide other query options (according to non-linguistic features) like text title, scientific author of a text, demographic data about an informant, etc.

Naturally enough, after the morphological annotation is in effect, querying for parts of speech and grammatical categories will be added.

To facilitate the morphological annotation of the corpus, we allocated a significant place to equipping dialect dictionaries with grammatical information and to applying them in the process of lemmatization and linguistic annotation. We decided to use the data of Georgian dialect lexicography to increase the lexical base (textual base) of the corpus as well.

The Problem of Representativeness in the GDC and Dialect Dictionaries

To achieve representativeness has been one of the primary tasks in corpus-building. Although the main principles of representativeness have long been established, a specific corpus project nevertheless faces the necessity of stating, 'defending' and implementing its own concept of representativeness. For instance, we expect any more or less important corpus initiative to be familiar with and to consider Douglas Biber's seminal paper [3]; however, truly enough, designers of individual corpora do not always follow Biber's canonical rules. Here is what G. Leech has to say about this: "A seminal article by Biber has frequently been cited, but no attempt (to my knowledge) has been made to implement Biber's plan for building a representative corpus" [6].

Clearly, whenever universal rules of representativeness are concerned, it should be borne in mind that the rules differ for general and special corpora [9]. It is, for instance, a necessary condition for a national corpus to be balanced according to genres and registers in order to function as a micro-model of a given language that is as accurate as possible, whereas for any other special corpus, dimensions of representativeness may be rather distinct.

Balance is conceived as a near synonym of representativeness in contemporary corpus linguistics; however, it should be admitted that corpus designers are more and more frequently facing conditions that oblige them 'to revise' this synonymic pair.

A dialect corpus is somehow a special corpus; as is known, the concept of representativeness is different in such a corpus [6, 7].

We chose an approach to corpus documentation of dialect data which implies that a dialect corpus should become a scientific source of a new type, one which would facilitate the representation and study not only of a language, but also of a linguistic-communicative model [4].

The representativeness concept of the Georgian Dialect Corpus was established with the national historico-cultural reality in mind, by considering the role and the place of this resource within the Georgian national scientific and cultural paradigm. The principal requirements according to this concept are as follows:

- completeness of lexical data
- completeness of linguistic annotation
- complete representation of dialect and intra-dialect strata
- completeness of age, gender, social variation and other metadata
- representation of sectoral, economic, folklore variation
- representationof mosaic variation caused by migration
- representation of features connected to chronological factors
- representation of speech features of smaller or marginal groups (for instance, speech of Georgian Jews ...)

We have already dealt with the composition of the textual base of the corpus in our other papers as well [1, 2]. We have also noted that, in order to increase the degree of representativeness of the corpus, we include non-linguistic components, that is, dictionaries.

The illustration block of a dictionary will be incorporated in the common concordance of the corpus in which a head word acts as a key word.

The decision was prompted by the following considerations: first, in accordance with the tradition established in Georgian dialectology, dialect data were described mainly for the sake of illustrating of scientific research and not for the creation of scientific publications of texts; hence, the data are fragments of valuable linguistic information, collected by qualified dialectologists, and they should be necessarily included in the common corpus 'context.'

Most Georgian dialect dictionaries are compiled by scholars who were native speakers of those dialects and had a thorough knowledge of both a dialect and the cultural, social, and economic space within which that dialect was spoken. Now, when this space is at the brink of extinction and referents of many words, as cultural and historical phenomena, no longer exist, these dictionaries preserve very valuable information.

Dictionaries are said to be a world arranged alphabetically. We want to create the most accurate dialect 'reflection' of this world by means of a corpus. So why shouldn't we be able to apply a dictionary to achieve this? Why shouldn't we use a head word as a key word and an illustration as a context? As they say, a concordance is 'a cut-off text.' A dictionary, like a concordance, cannot be used to reconstruct a primary text; however, it is a valuable material to represent a so-called 'cultural text'; moreover, the tissue of a dialectal 'cultural text' is destroyed on a daily basis, and a corpus, amalgamating the existing lexical repository as dialect texts, dialect dictionaries (and as other non-textual components), in the only way to preserve this cultural text.

Dictionaries and the Problem of Morphological Annotation in the Corpus

This paper describes the process of morphological annotation of the Upper Imeretian collection of the GDC based on the *Upper Imeretian Dictionary* [5].The annotation is based on **GeoTrans** (see [8]), an automated morphological dictionary of Standard Georgian. At the present experimental stage of the morphological annotation of the GDC, the following has been achieved:

- Formatting of the dictionary: development of the digital version of the dictionary and creation of a list of lemmas (totally 5671 lemmas)
- Automated selection and part-of-speech tagging of the forms from the list of lemmas of the dialect dictionary, coinciding with those of the standard. Totally 784 such lemmas were detected; a list of homonyms was identified, totaling 27 items. After the operation, 4860 'unidentified' elements were spotted in the list of lemmas, which were manually ascribed part-of-speech tags.
- By means of the received marked lists, the knowledge base of the automated morphological dictionary of Standard Georgian was enriched.This implies that a subsystem for morphological modeling of a given dialect variety was added to **GeoTrans**. In this system, each dialect form will be tagged in accordance with a respective part of speech and, frequently, marked in accordance with an inflectional pattern by means of which word forms are lemmatized.
- At the next stage, the **GeoTrans** standard language-analyzer enabled us **to select the lemmas** from the textual data of the corpus, **coinciding with those of the standard language**, amounting to3331 lemmas.

- The GeoTrans dialect analyzer specific dialect lemmas were selected and tagged, 472 **lemmas** in total.
- By means of the standard language analyzer, all the **word forms** underwent complete morphological analyses, coinciding with those of the standard, which amounted to 9285 forms. Here too, homonymous (528) and non-homonymous (8757) forms will be similarly distinguished.
- Following that, by means of ***lemmas and standard inflectional patterns***, lemmatization was performed and dialect (specific) word forms were morphologically tagged.

Conclusion: Equipping dialect dictionaries with morphological information and in such a way enriching the morphological knowledge base by means of the automated standard analyzer is an optimistic perspective for the automation of dialect corpus analysis. The concept of morphological annotation of the GDC envisages a differentiated approach to text data: to present dialect (specific) vocabulary, vocabulary common with the standard language, inflectional and derivational patterns common with the standard language, dialect-specific inflectional and derivational patterns as separate 'sectors' and then, to undertake the annotation strategy accordingly.

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Strategic Games over Łukasiewicz Logic

Ondrej Majer and Tomáš Kroupa

Łukasiewicz fuzzy logic

Fuzzy logics form an intensively studied and well understood family of many-valued logics, designed especially for handling gradable (i.e., capable of being more or less true) propositions. In this paper we shall only deal with Łukasiewicz logic. Detailed information on fuzzy logics can be found in the classical monograph [7].

The *standard semantics* of infinite-valued Łukasiewicz logic evaluates formulae in the real unit interval $[0,1]$. The truth value 1 is usually understood as the full truth of the proposition; the truth value 0 as the full falsity; and the intermediate truth values as the *degrees* of (partial) truth of gradable propositions. Łukasiewicz logic can be understood as a contraction-free substructural logic and hence it posses two disjunctions and two conjunctions.The propositional connectives of are interpreted truth-functionally, with the following truth functions on $[0,1]$:

$$\begin{aligned}
x \wedge y &= \min(x,y) \\
x \vee y &= \max(x,y) \\
\neg x &= 1-x \\
x \odot y &= \max(0, x+y-1) \\
x \oplus y &= \min(1, x+y) \\
x \to y &= \min(1, 1-x+y)
\end{aligned}$$

According to McNaughton theorem the class of functions $[0,1]^2 \to [0,1]$ providing an interpretation of a Łukasiewicz propositional formula in n variables coincides with the class of functions which are continuous, piecewise linear and with integer coefficients.

Strategic games with McNaughton functions

Imagine the following two-person game [8], which is a continuous variant of the well-known *Matching Pennies* [11]. Each of the two players (Xena and Yves) chooses secretly a real number from the unit interval $[0,1]$. Yves tries to guess a number closed to Xena's choice (his payoff in e.g. EUR is $10 \cdot (1 - \|x-y\|)$), while Xena wants to pick up a number maximally remote from Yves (her payoff is $10 \cdot \|x-y\|$) What is the maximum price p Xena is willing to pay for the participation in this game? The situation can be described as a two-player constant sum game, where the payoff functions are

$$f_1(x,y) = \|x-y\| \quad \text{and} \quad g_1(x,y) = 1 - \|x-y\|.$$

Interestingly, both f_1 and g_1 correspond to the Łukasiewicz formulas

$$\phi = (X \odot \neg Y) \oplus (\neg X \odot Y) \quad \text{and} \quad \neg\phi = (X \oplus \neg Y) \odot (\neg X \oplus Y),$$

respectively, where the former is known as the *Chang distance* [4] and the latter is its negation. Another example leading to payoffs described by McNaughton functions is the version of a game called *love and hate* [2] in which the payoff functions are

$$f_2(x,y) = \min(\|x-y\|, 1 - \|x-y\|) \quad \text{and} \quad g_2(x,y) = 1 - f_2(x,y)$$

.

If we use ϕ for Chang distance, the payoffs correspond to the formulas $\phi \wedge \neg\phi$ and $\neg\phi \vee \phi$ respectively. (Let us note that neither the first formula is a contradiction nor the second one is a tautology in Łukasiewicz logic.) This motivates the investigation of a general two-person (Xena and Yves) constant sum game such that:

- The strategy space of each player is the interval $[0, 1]$, each point from this interval is a *pure strategy.*
- The payoff function of Xena is an arbitrary McNaughton function of two variables $f(x, y)$, the payoff function of Yves is given by $g = 1 - f$.

Nash equilibrium is one of the basic solution concepts in game theory. It is a pair (in two person games) of strategies that is stable in the sense that no player can improve her payoff by unilaterally deviating from her equilibrium strategy. Let's denote $s_f(x, y)$ the payoff function for Xena. We say that a pair of (pure) strategies (x^*, y^*) is a (pure) *Nash equilibrium* of the game $\mathcal{G}_f$ iff

$$s_f(x, y^*) \leqslant s_f(x^*, y^*) \leqslant s_f(x^*, y),$$

for every pair of pure strategies (x, y). (Note that the payoffs sum up to one, so an increase for Xena means a decrease for Yves).

Neither the Chang distance game, nor Matching Pennies have a Nash equilibrium in pure strategies. The standard extension of strategic space of the players is to allow them randomizing among the pure strategies, the randomizing strategies are called *mixed.* If the space of pure strategies is finite, randomizing consists of assigning probability to each pure strategy such that they sum up to one. The payoff is then calculated as a weighted average and the mixed Nash equilibrium is defined in the very same way as the pure one, just the pure strategies are replaced by mixed ones. The game of Matching Pennies has an equilibrium in the extended strategy space each player assigns equal probability 1/2 to each of the pure strategies. According to the famous Nash theorem every strategic game the payoffs of which are expressed by a finite matrix has an equilibrium.

If the strategy spaces are infinite, the situation gets more complicated. The strategy space of our Chang distance game is a continuum, so we shall allow not only probability distributions among a finite number of points, but an arbitrary probability distribution.

- a *mixed strategy* is a probability measure defined on Borel subsets of $[0, 1]$.
- every pure strategy $x \in [0, 1]$ can be identified with the Dirac measure δ_x and conversely.

When Xena and Yves play a pair of mixed strategies (μ, ν), the expected payoff of Xena is

$$s_f(\mu, \nu) = \int_{[0,1]^2} f \quad \mathrm{d}(\mu \times \nu).$$

We say that a pair of mixed strategies (μ^*, ν^*) is a *Nash equilibrium* of the game $\mathcal{G}_f$ if

$$s_f(\mu, \nu^*) \leqslant s_f(\mu^*, \nu^*) \leqslant s_f(\mu^*, \nu),$$

for every pair of mixed strategies (μ, ν).

There is no generalization of Nash theorem to the whole class of games with infinite space of strategies (there are some games in this class for which the equilibrium does not exist). Most famous result for infinite strategic games is the Glicksberg's theorem [6].

Theorem 1 (Glicksberg)**.** *Let strategy sets be compact Hausdorff topological spaces and payoff functions be real and continuous. Then there exists a Nash equilibrium* (μ^*, ν^*)*.*

The proof of this theorem is based on a generalization of Kakutani's fixed point theorem and hence does not provide any method for calculating or estimating the equilibria. Therefore, the quest for methods and algorithms to recover Nash equilibria is one of the main problem for various classes of continuous games [12]. In particular, sufficient conditions for the existence of finitely-supported mixed equilibria are sought. Our game satisfies the conditions of Glicksberg's theorem and moreover the Nash equilibrium is very simple.

Example 1. One Nash equilibrium pair in game $\mathcal{G}_{f_1}$ is (μ^*, μ^*), where $\mu^* = 1/2(\delta_0 + \delta_1)$. Since

$$s_{f_1}(\mu^*, \mu^*) = 1/2,$$

Xena won't pay no more than $p = 1/2 \cdot 10 = 5$ euros for the game ticket. There is another pair of Nash equilibria $(\mu^*, \delta_{1/2})$, which means that the optimal response of Yves to the Xena's random choice of 0 and 1 is the constant selection of number 1/2.

Although the strategy space of our game is uncountably infinite, its solution is finite – the probability distribution concentrates in finitely many points. Are all the games with McNaughton payoffs like that? Such a hypothesis seems to be indirectly supported by the following theorem.

Proposition 5. *Let f be a McNaughton function of two variables. Then there is a triangulation $\mathcal{T}_f$ of the unit square $[0,1]^2$ such that f is linear over each triangle of $\mathcal{T}_f$.*

The theorem shows, that in some sense every McNaughton function can be encoded by finitely many points (vertexes of the triangulation).

Conjecture: *every game $\mathcal{G}_f$, where f is a 2-variable McNaughton function, possesses a Nash equilibrium pair whose mixed strategies are finitely-supported Borel probability measures.*

If this conjecture is true remains an open question. There are some general results about some classes of games for which finitely supported Nash equilibria exist, but neither of them covers the case of the games with McNaughton payoffs. We have achieved only partial results in this direction so far. They are based on the triangulation of of the space of strategies (unit square) mentioned above.

Let $V(\mathcal{T}_f)$ be the set of all vertexes of the triangulation $\mathcal{T}_f$ of a McNaughton function f. Our goal is to approximate $\mathcal{G}_f$ by a matrix game whose equilibria will also be the equilibria of the original game $\mathcal{G}_f$. The matrix game will be given by the following requirements:

1. The strategy sets are nonempty finite subsets $M, N \subset [0,1]$.
2. The payoff function of Xena is the restriction of f to $M \times N$, the payoff function of Yves is the restriction of $\neg f$ to $M \times N$.

Let $M = \{x_1, \ldots, x_m\}$ and $N = \{y_1, \ldots, y_n\}$. An *$(M \times N)$-grid $\mathcal{A}$* is given by the vertex set $M \times N$ and the line segments having the endpoints of the form

$$(x_i, y_j), (x_i, y_{j+1}) \quad \text{or} \quad (x_i, y_j), (x_{i+1}, y_j).$$

The matrix game $\mathcal{G}_f^{\mathcal{A}}$ corresponding to $\mathcal{G}_f$ and an $(M \times N)$-grid $\mathcal{A}$ is called a *grid game*.

Theorem 2. *Let f be a McNaughton function of two variables and $\mathcal{T}_f$ be an associated triangulation. If there exists an $(M \times N)$-grid $\mathcal{A}$ such that*

- *$V(\mathcal{T}_f) \subseteq M \times N$ and*
- *each line segment of $\mathcal{A}$ belongs to some triangle of $\mathcal{T}_f$,*

then every Nash equilibrium of the matrix game $\mathcal{G}_f^{\mathcal{A}}$ is a Nash equilibrium of the game $\mathcal{G}_f$. Consequently, the game $\mathcal{G}_f$ has a pair of finitely-supported Nash equilibrium strategies.

For instance, both games $\mathcal{G}_{f_1}$ and $\mathcal{G}_{f_2}$ satisfy the hypothesis of the previous theorem. There are McNaughton functions such that the associated game does not fulfill the sufficient condition of Theorem 2, but they still have the required equilibria. The standing conjecture is that the for every game with McNaughton payoff function there is a finitely-supported Nash equilibrium.

Games with McNaughton payoffs are definitely interesting from the game-theoretical point of view, but an obvious question is a connection to logic. The well known systems dealing with imperfect information is Independence Friendly (IF) logic of Hintikka-Sandu [9]. IF logic introduces a new notation which allows e.g. to make the existential quantifier in the formula $\forall x \exists y(x = y)$ independent of the general one ($\forall x \exists_{/x} y(x = y)$ in the IF notation.) Without going into the details of IF logic it seems clear, that we can represent the formula by a strategic game analogous to Matching Pennies: the first player chooses the value x and the second player chooses independently the value of y if they match, the second player wins, if not the first does. There is a semantics for IF logic identifying the value of an IF formula with the (mixed) equilibrium payoff in the corresponding semantics game (limited to the finite domains in which the existence of an equilibrium is guaranteed). This semantics makes IF logic a many-valued logic its values being rationals in the interval $[0, 1]$. It was proposed to introduce an IF version of Łukasiewicz logic (see [5]), but the resulting system has not been investigated yet. From a point of view of motivation this logic might be interesting because it combines two kinds of uncertainty: vagueness captured by the fuzzy part and probability captured by the IF part (or its equilibrium semantics). The class of games with McNaughton payoffs would correspond to a special fragment of IF Łukasiewicz logic. An investigation of deeper logical interpretation of the results about the games with McNaughton payoffs will be a subject of future research.

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Morphosyntactic annotation of the Georgian National Corpus – The diacronic dimension

Paul Meurer

By the end of last year, we launched a five-year project that aims at building a Georgian National Corpus. In its initial stage, the untertaking is financed by the Volkswagen Foundation in Germany, with additional funding by the Rustaveli Foundation in Georgia.

The main goal of this project is to build a balanced, diacronic corpus of the Georgian language, representing all important stages and varieties of the written language, from the earliest manuscripts to present-day fiction and non-fiction, including dialect texts.

All texts will have extensive metadata, describing their provenance, creation date, authorship, level of annotation, language variety, and more. Historical texts will be included on the basis of text-critical editions, with facsimile, diplomatic, and normalized levels of annotation, building on work done in Frankfurt in the Titus/Armazi project [1]. As annotation format we have chosen TEI XML.

An important project objective is to equip all texts with fine-grained morphosyntactic annotations. This is being carried out on the basis of a morphosyntactic analyzer for Modern Georgian which I developed earlier [2], and which is being extended, in cooperation with Georgian and German linguists, to cover dialectal variation, and, most importantly, the earlier stages of the Georgian language, including Middle and Old Georgian.

In this talk, I will present the technical aspects of the GNC project, focusing mainly on the challenges posed by the extension of the morphosyntactic analyzer to Middle and Old Georgian, and on the disambiguation techniques chosen.

The core of the morphological analyzer is a finite state transducer that operates solely on the concatenative level, yielding an intermediate morphophonemic representation on the lower side, and a lemma form plus morphosyntactic features on the upper side, based on a lexicon derived from major dictionaries of Old and Modern Georgian. This concatenative transducer is common to all variants of the language covered. In a second step, the core transducer is composed both on the upper and on the lower side with transducers that are specific for each particular language variety; they treat both phonotactic and orthographic (on the lower side) and morphosyntactic (on the upper side) pecularities of those varieties.

In this way, it is, for instance, possible to capture the different surface representation of the 2nd person subject and 3rd person object markers in the Xanmeti and Haemeti texts, versus their virtual nonexistence in Modern Georgian.

In the upper-side transducers, the case, person and number agreement marking of the verbs is computed. Here, the subtle interplay of the plural object marker with the person markers and the object case (and its nonexistence in Modern Georgian) can be expressed in an elegant way.

The different mapping of morphological form to tense in Old and Middle/Modern Georgian, which is a consequence of the fundamentally different aspect systems, is also coded in the upper-side transducer.

At the first stage, disambiguation of the morphosyntactic annotation is done in two ways: Ambiguity that can be resolved reliably from the sentence context is removed automatically using hand-written Constraint-Grammar rules. Those rules are, again, specific to a given language variety. The remaining ambiguity is resolved manually. This manual work is carried out by trained linguists and students in the Georgian project group, using a specially devised Web interface that conveniently allows both viewing, disambiguating and error correcting the texts.

At a later stage, we are planning to refine the rule-based system, and to train a stochastic disambiguator on the manually disambiguated texts, which will be used to disambiguate a larger part of the modern non-fictional texts.

When annotating such a large body of highly diverse texts, we inevitably encounter many out-of-vocabulary words. Among them are words or inflected forms of the standard language(s) that for some reason are missing in the lexicon, dialect forms, incorrect, deviant and emphatic spellings, transcriber errors, and more. The bulk of the unknown words are names: anthroponyms and toponyms in the older texts, diverse named entities in the modern non-fictional texts. At the present early stage of the project, the coverage of the analyzer is approximately 96% (token coverage).

These out-of-vocabulary words are extracted, collected and categorized, and then added to the lexicon and the morphological analyzer. Similarly, when missing words or readings are detected in the manual disambiguation process, they are collected and added to the analyzer. Since the amount of named entities in the modern texts will be too large for manual treatment, we are planning to develop a Named Entity Recognizer that will help to annotate them correctly.

The corpus will be searchable through a powerful corpus query interface [3] and will be hosted at mirrored sites in Georgia, Germany and Norway.

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A Formalization of Frame Theory in Dependence Logic using (explicit) Strategies

Ralf Naumann and Wiebke Petersen

We present a formalization of the Barsalou-Löbner frame theory (BLFT) in Dependence Logic with explicit strategies. In BLFT frames are conceived of as fundamental representations of knowledge in cognition. In its present formalization, Petersen (2007), frames are defined as a particular kind of typed feature structures (Carpenter 1992). On this approach, the semantic value of a lexical item is reduced to its contribution to the truth conditions of sentences in which it occurs. However, empirical data both from neuroscience (N400; e.g. Baggio and Hagoort 2011) and cognitive linguistics (quantifiers, Geurts and Nouwen 2007) provide evidence that the meaning of lexical items cannot be reduced in this way. In our formalization of the BLFT the above data are analyzed as dependence relations. Two different types of such relations are distinguished: (i) semantic relations between lexical items are modeled by dependence formulas from Dependence Logic (Väänänen 2007). In game-theoretic terms they are atomic games which require uniform winning strategies. The second type is used to distinguish expressions whose semantic values in terms of truth conditions are the same but which differ at the cognitive level, as, for example, it is the case for 'at least n' and 'more than $n-1$'. Such expressions are interpreted w.r.t. the same game (or game tree) but different admissible strategies. This requires that strategies be made explicit (Pacuit and Simon 2011). On our frames-as-strategies formalization the cognitive significance of frames is to be seen in the fact that they model meaning components that are not related to truth conditions.

Barsalou (1992) argues that the concept of a frame provides the fundamental representation of knowledge in human cognition. The basic building blocks frames are sets of attribute-value pairs. Attributes are taken as concepts that describe an aspect of at least some category member whereas values are subordinate concepts of an attribute. These sets of attributes are restricted by a variety of relations which function as constraints, in particular as constraints on the distribution of values of different attributes. Barsalous hypothesis was strengthened in Löbner (2012) to the following two theses: (H1) The human cognitive system operates with one general format of representations and (H2) If the human cognitive system operates with one general format of representations, this format is essentially a Barsalou frame. These hypotheses will be called the "Barsalou-Löbner Hypothesis" (BLH). The BLH requires a frame model that (i) is sufficiently expressive to capture the diversity of representations and (ii) that is sufficiently precise and restrictive in order to be testable. Given these two constraints, it follows that the gap between cognitive linguistics, brain science and formal semantics has to be filled (Petersen 2013). A formalization of the BLH was presented in Petersen (2007). Petersen defines frames as generalizations of typed feature structures in the sense of Carpenter (1992). According to this definition, a frame is a connected, directed graph such that (i) there is a central node (but not necessarily a root node), (ii) each node is assigned a type and (iii) there are no nodes having two outgoing arcs that are labeled with the same attribute.

Although we basically agree with the BLH, we will argue that any attempt at defining frames as typed feature structures is doomed to fail because it cannot satisfy the two constraints H1 and H2 above. Furthermore, and even more importantly, (i) if frames can be reduced to a particular type of feature structures, what is *specific* about a theory of frames, or, to put it differently, is there really a *genuine* theory of frames?, and (ii) in what exactly does the cognitive significance of frames lie? After presenting empirical evidence in support of this criticism, we will develop an alternative formalization of the BLH that is based on a game-theoretical interpretation of Dependence Logic extended by the notion of a strategy (or a protocol).

The problem of (in-)dependence between the values of different attributes. Consider the well-known example from FOL: "All humans are mortal" and its generalization "All A's are B's". If human beings are modeled as sets of attribute-value pairs, the above formula expresses the constraint that the value of the attribute 'mortal' depends on the value of the attribute 'human'. Assuming that the values of both attributes are true, one gets (1)

(1) human(true) $\rightarrow$ mortal(true)

In database theory (see Abiteboul et al. 1995) such constraints are called functional dependences because the values of some attributes of a tuple uniquely or functionally determine the values of other attributes of that tuple. They are required to hold for all possible instances of a database scheme. Applied to a frame theory, this means that (1) has to be valid for all frames that are of a particular type, say of the type human being and all of its subtypes. The next example shows that a functional dependence need only hold for "particular" individuals.

(2) This melon sounds ripe.

(2) can be interpreted as expressing a functional relation between the attributes 'sound' and

‘ripeness’ of a melon: a melon is ripe if it sounds muffled (if it emits a muffled sound). However, in this case the dependence only holds for “normal” melons so that the relation (or the inference) is defeasible. The third example of a relation between the values of different attributes is related to the N400, an event-related potential (ERP) component (see e.g. Baggio and Hagoort 2011).

(3) a. The girl put a sweet in her mouth/pocket after the lesson.
    b. The girl was writing a letter when her friend spilled coffee on the paper/tablecloth.

An N400 effect is observed for ‘pocket’ as opposed to ‘mouth’ in (3a) and for ‘tablecloth’ compared to ‘paper’ in (3b). In neurophysiology this effect is usually explained as a predictability or priming effect that results from a relation of semantic closeness. For example, in the context of a writing event, paper as a medium on which is written is semantically more expected than a tablecloth. In a frame theory, such relations can also be interpreted as dependencies between the values of different attributes. For example, if the value of the attribute “activity” is “writing”, the value of the attribute “medium” is “‘paper’ or ...” As indicated by the dots, the dependence relation is multi-valued and in addition based on a relation of semantic closeness.

The above examples show that the semantic value of a lexical item cannot be reduced to the contribution it makes to the truth conditions of sentences in which it occurs. Besides that part of its meaning which contributes to truth conditions, there is an additional second meaning component that expresses relations to other elements in the lexicon. Thus, meaning cannot be defined ‘in isolation’ but must already be seen in the context of a (semantic) network of interconnected relations. From a cognitive perspective both the existence and the advantages of such relations can be explained as follows. Items in the Memory repository (see Hagoort 2005) of the brain are learnt in particular situations which already exhibit those relations. Writing events often involve paper but rarely tablecloth as material and human beings are (often sadly) known to eventually die. The advantage of dependence relations is that they admit an agent to improve or even build up strategies (plans) for action or anticipation (projection) of continuations in sentences or discourses. Dependence relations in (1)-(3) will be called *type I dependences*.

Another set of data shows (i) that modifier expressions like ‘already’ need not change the truth conditions of sentences but that they nevertheless enrich its meaning in a cognitively significant way and (ii) that quantifiers which express the same set-theoretic relation can differ in other meaning components. Consider the following data, discussed in Löbner (1987, 1989).

(4) a. It is late.
    b. It is already late.

According to Löbner and others, both expressions have the same truth conditions: it has to be late at the parameter point. However, they differ with respect to the condition imposed on the temporal ‘environment’ of the parameter point. Whereas ‘already late’ requires that there be a negative phase satisfying ‘not late’ preceding the positive phase in which the parameter point falls, this is not the case for (unmodified) ‘late’. Thus, ‘late’ requires information about the way in which the truth conditions came about. According to Generalized Quantifier Theory (GQT), the meaning of sentences involving a quantifier expression Q (“$Q \quad A$ are B”) is always defined in terms of a set-theoretic relation between the sets denoted by A and B. On this account, scalar quantifiers like the superlative “at least n” and the comparative “more than $n-1$” are interdefinable because they define the same set-theoretic relation, viz., $\text{card}(A) \cap \text{card}(B) \geqslant n$. However, this view runs into several problems. For example, on this view the following two sentences should be equally acceptable, which they are not (Geurts and Nouwen 2007).

(5) a. I will invite at least two people, namely Jack and Jill.
    b. ?I will invite more than one person, namely Jack and Jill.

As noted by Geurts and Nouwen, while (5a) is perfectly acceptable, and allows for the possibility that more than two people will be invited, (5b) is less felicitous and rules out that possibility. Dependence relations like those in (4) and (5) will be called *type II dependencies.*

Outline of the formal theory

We will develop a Game-Theoretical Semantics (GTS) of the Barsalou-Löbner Frame Theory in Dependence Logic (DL) (Väänänen 2007, Hodges and Väänänen 2010) extended with generalized quantifiers (Engström 2011) and explicit strategies (Pacuit and Simon 2012). Recall that the meaning of a lexical item comprises at least two components: (i) semantic value contributing to the truth conditions of sentences and (ii) dependence relations of either type I or type II. The basic building blocks are extensive games of perfect information. They are used to model the first meaning component related to truth conditions. In order to account for dependence relations these games have to be lifted to extensive games of imperfect information (see Hodges and Väänänen 2010 for a similar proposal in the domain of mathematics, which is based on the well-known type-shifting rules from formal semantics).

At least two different types of lift must be distinguished: (i) lifts that make it possible to express type I dependences and (ii) lifts that make it possible to express type II dependences. Let L be the logic for basic extensive games of perfect information. Common to both types of lift is a shift from single assignments to sets of assignments. The effect of lifts of the first kind consists in making it possible to add dependence formulas '$= (x_1, \ldots, x_{n-1}, x_n)'$ from DL to L. The second type of lift is used for expressions like 'already'. In this case an atomic game (of perfect information) is sequentially composed with a non-atomic extensive game of imperfect information on which an indistinguishability relation is defined on the set W of game positions (see Gosh, Ramanujam and Simon 2010 for details). This relation induces an equivalence relation on the set of maximal paths in the underlying game tree. Each equivalence class is associated with a particular strategy. The effect of this kind of lift is twofold. First, the winning positions of the resulting game for a player are those of the non-lifted atomic extensive game. Second, the difference to the underlying atomic game is that the resulting game also encodes information about how such winning positions can be attained, following one of the different possible strategies. The similarities and differences between type I and type II dependence relations can be characterized as follows. Semantically, both types require a shift from single assignments to sets of assignments in order to express the condition that winning strategies must be uniform. They differ w.r.t. the way strategies are encoded in the underlying logic L. For type I depence, L is extended to a logic L' by adding dependence formulas '$= (x_1, \ldots, x_{n-1}, x_n)$'. This is done in a way similar to Väänänen (2007) and Hodges and Väänänen (2010). By contrast, type II dependence relations require strategies to be made explicit. Here we follow Pacuit and Simon (2011). They propose to use action expressions of Propositional Dynamic Logic to explicitly describe different strategies. On this approach, a formula $[\pi]\phi$ has the interpretation 'ϕ is guaranteed to be true by following the protocol π'. Semantically, strategies are defined as finite labeled trees. A strategy is enabled in a game G at state if the tree corresponding to the strategy can be embedded in (the unwinding of) G at w. In our theory, formulas are of the form $[\pi](\phi L' \wedge \psi_L)$ where ψ_L is a boolean combination of formulas of L (i.e. no occurrences of dependence atoms) and ϕ_L is a boolean combination of dependence atoms. With respect to explicit strategies, we will use the mu-calculus as an alternative to PDL (see van Benthem 2012 and below for arguments).

As a first example, consider again (5) above. Geurts and Nouwen (2007) argue that this difference in the admissibilty of the 'namely'-construction is due to a difference in definiteness. Whereas 'at least' refers to a definite object to which one can refer using 'namely', this is not the case for 'more'. In our frame theory this difference is explained as follows in terms of strategies. Both determiners are defined with respect to the same extensive game for which the winning positions are defined in terms of the set-theoretic relation $\mathrm{card}(A) \cap \mathrm{card}(B) \geqslant n$. This captures the equivalence with respect to truth-conditional content. The difference lies in the admissible strategies. If strategies are defined using Propositional Dynamic Logic (van Benthem 2011, 2012, Pacuit and Simon 2011), the strategy corresponding to 'at least' is formulated using a deterministic while-program. By contrast, for 'more' the strategy is expressed using the choice program operator, which leads to a non-deterministic strategy. Thus, definiteness is related to deterministic programs yielding deterministic strategies. Computationally, the difference consists in the fact that deterministic PDL is less complex than non-deterministic PDL (see Harel, Kozen and Tiuryn 2000). As a second example, consider 'late'. Abstracting away from possible type I dependences, 'late' can be represented as a test program using PDL as L: late?. After lifting, one gets the formula $[\pi]$ (late?). Here, π can be taken to be a combination of a liveness and a safety property: first there is a phase of not late (i.e. the test late? fails) until late holds for the first time (liveness) and from then on late continuously holds (i.e. the test late? succeeds) (saftety property).

Suppose lexical items in the Memory repository of the brain can be modeled by networks, say neural networks (see e.g. d'Avila Garcez, Lamb and Gabbay 2009 for a formal approach) or the inhibition networks of Leitgeb (2004). In our frame theory such networks can be taken to represent all information about the lexical item that has been learnt, therefore also including world knowledge (though some information may even be innate). Applied to our theory, this means that such a network stores all information, that is, information related to truth conditions as well as type I and type II dependence relations. When a linguistic expression is parsed, it is represented as a formula $[\pi](\phi_{L'} \wedge \psi_L)$. In the brain this triggers a cascading parallel activation of the following form: the ψ_L is responsible for the activation (of part of) the network representing the item (say 'already' as opposed to 'still' or 'write' as opposed to 'type'; however for 'at least' and 'more' the same network is activitated due to sameness of truth conditional meaning components). The $\phi_{L'}$-component triggers type 1 dependences and therefore preactivates other items which are semantically related to the given one ('write' preactivates 'paper' but not 'tablecloth' as medium on which it is written). Finally, the π-component activates a particular strategy that has to be used and therefore only a part of the network. For example, 'already late' requires activation of the network leading to a representation in which the late-phase is preceded by a not-late-phase. Likewise, 'at least' leads to a representation admitting reference to a definite (plural-)object, whereas this is not the case for 'more'. Thus on our frames-as-strategy view, their cognitive significance can be described as follows: (i) they lead to an activation of (part of) the network representing the item modeled by the frame in the Memory repository, (ii) they lead to a preactivation of other networks and (iii) they (possibly) activate only that part of the network leading to a representation in accordance with the particular strategy encoded by the frame. The cognitive significance of frames-as-strategies, as opposed to other formalisms aiming at capturing cognitive meaning aspect (e.g. Van Lambalgen and Hamm 2004), thus lies in the fact that they make it possible to formalize non truth-conditional meaning components in a compositional and modular way.

Uniqueness and possession: Typological evidence for type shifts in nominal determination

Albert Ortmann

Introduction

The Concept Type and Determination approach (Löbner 2005, 2011) assumes a classification of nouns and their uses into the four types: sortal, relational, individual, and functional concept (SC, RC, IC, and FC, respectively). These four types arise from a cross-classification of the properties 'relational' und 'unique (= unambiguous) reference': Sortal nouns (henceforth SNs) classify the objects of the universe, while RNs do so in relation to an argument which is normally realised as a possessor. Accordingly, the logical type of the former is <e,t>, while that of the latter is <e,<e,t>. An IN unambiguously singles out a particular individual, depending on a given time/world coordinate (formally to be specified in terms of a situational argument, which will however be ignored here). An FN, too, singles out a particular individual, however in relation to a possessor argument. Their logical types are therefore e and <e,e>, respectively.

Definiteness and Uniqueness

A crucial distinction within Löbner's (1985, 2011) Concept Type and Determination theory is that between semantic and pragmatic uniqueness. Semantic uniqueness entails that the reference of a noun is unambiguous because of its lexical semantics, independent of the context or situation. Pragmatic uniqueness, by contrast, refers to those uses of nouns whose unambiguous reference only comes about by the context of utterance, as is the case with deictic and anaphoric use: the dog involves unique reference that comes about by anaphoric or deictic use, hence pragmatic uniqueness. We are thus dealing with a type shift from sortal to individual (SC → IC; formally: <<et>,e>)). The indefinite uses of ICs and FCs (a sun, a mother) involve a shift in the opposite direction, that is, IC/FC → SC (<e, <et>> and <<e,e>,<et>>, respectively).

The semantic vs. pragmatic uniqueness distinction motivates the various asymmetries with regard to the (non-)occurrence of definite articles in diverse languages, which in (cf. Ortman in print) are characterised as belonging to one of the following two types:

Split I: Pragmatic uniqueness is marked by the definite article, whereas semantic uniqueness is unmarked (e.g., in West Slavic and Old Georgian, see Boeder 2010).

Split II: Pragmatic and semantic uniqueness is morphosyntactically distinguished by different article forms, typically in terms of an opposition of phonologically strong vs. weak articles as in Germanic (e.g., in Fering Frisian (see Ebert 1971), and numerous dialects of German).

Relationality and Possession

Fully along the lines of the opposition of semantic and pragmatic uniqueness, in this paper I advocate the view that the contrast of inalienable and alienable possession should be re-interpreted as semantic and pragmatic possession. Semantic possession is called so because some relation of affiliation between the noun's referential argument (the 'possessum') and the possessor is inherent to the lexical semantics of a an RN. Pragmatic possession is called so because the POSS relation is established by the context rather than by the lexical meaning of the possessum, often depending on the utterance situation. Consequently, the paper argues for the following analogy: RNs are semantically possessed, hence undergo inalienable possession, in exactly the same way as semantically unique concepts (ICs and FCs) do not take the definite article. The shift from sortal noun to relational concept (SN → RC) is displayed by what is traditionally called alienable possession, in exactly the same way as the shift from sortal noun to an individual concept (SN

→ IC) (as well as that from relational noun to functional concept, RN → FC) is displayed by a strong definite article in case of pragmatic uniqueness.

Obviously, one and the same concept need not be treated the same in all languages, and there is some cross-linguistic variation as to the class of nouns that may enter inalienable possession. E.g., in Ewe (Niger-Congo) body part but not kinship terms are treated as alienable, while Acholi (Nilotic) displays the reverse picture (see the contributions in Chappell & McGregor 1995, as well as Nichols 1988: 572 on North American languages). For Georgian the conceptual range of inalienability is narrower than in most other languages, in that only kinship terms of the ascending generations qualify as inalienable. This asymmetry (which is also known from Australian languages) is conceptually motivated by the fact that the existence of older generations is a prerequisite for the younger to exist. A theoretical implication is that the propensity of RCs to be treated as inalienable is a default that may be overwritten by idiosyncratic specification.

In order to relate this semantic contrast to the morphological and syntactic facts (on which see, among others, Seiler 1983, Nichols 1988, as well as the introduction of Chappell & McGregor 1995 and references there), I start with a brief overview of some major morphological and syntactic modes of expressing an (in)alienability distinction in possession:

- The noun is straightaway possessible vs. only possessible via a connective morpheme: This strategy is particularly common in languages of the Americas. In Yucatec Maya, semantically possessed nouns such as *la'ak* 'friend' directly combine with a possessor as in (1-a). By contrast, the noun *nah* 'house' must be morphologically extended by the suffix *-il* as in (1-c) in order to be possessed.

(1) Yucatec (Mayan, Mexico; Lehmann 1998: 52; 56)

a. *in* *la'ak*
1SG.P'OR friend
'my friend'

b. *le* *nah-o'*
DEF house-DISTAL
'the house'

c. *in* *nah-il*
1SG.P'OR house.POSS
'my house'

- In another very common strategy, possessor agreement is directly attached to the noun vs. attached to possessive classifier:

(2) Paamese (Oceanic < Austronesian, Vanuatu; Crowley 1996: 384ff)

a. *yati-n* *ēhon*
head-3SG child
'the child's head'

b. *ani* *emo-n* *ēhon*
coconut POSSCL.POTABLE-3SG child
'child's drinking coconut'

In the pragmatically possessed case in (2-b) the classifier mediates between the possessum and the specification of the possessor (irrespective of whether the latter is also lexically realised).

- The P'OR is realised as a suffix vs. as a free possessive pronoun: In Georgian, in case of inalienable possession the pronominal marker is affixed to the noun. By contrast, the pronominal marker for alienable possession is attached to a relator it is realised as a free possessive pronoun which shows case concord with the head noun.

(3) Georgian:

a. *deda-čem-i*
mother-P'OR1SG-NOM
'my mother'

b. *čem-i* *čign-i*
P'OR1SG-NOM book-NOM
'my book'

All of these contrasts are attained by straight affixation to the possessum or juxtaposition of the possessor on the inalienable/semantically possessed side, and 'mediation' by a classifier, a connective, a free (possessive) pronoun, or a case marker on the possessor on the alienable/pragmatically possessed side.

Note that the widespread phenomenon of a 'deleted' possessor of an RN/FN may be seen as an extreme case of morphosyntactically unmarked possession. As pointed out by Seiler (1983: 18), it is frequently found where the conceptual closeness of possessor and possessum is highest, especially when body-part terms are combined with verbs whose agent or experiencer argument is co-referent with the possessor as in French *Il a levé le bras*, lit. He has raised the arm, thus giving rise to an NP-external realisation of the possessor.

The analytic idea that I pursue in this paper is that morphological means of 'alienability' such as connectives and classifiers are interpreted as establishing a non-inherent, contextual, hence pragmatic POSS relation. This programme is shared by L"obner (2011), but it will be pursued more radically here, by presenting evidence for the following claim:

(4) Claim: The morphological means of pragmatic possession should be analysed as instantiating a shift from SN to RC.

This way, the semantic vs. pragmatic distinction accounts re-interprets the alienability contrast: Sortal nouns occur morphologically unmarked in their non-possessed use, while in many languages they are overtly marked by con?nectives and classifiers when combined with a possessor. These markers overtly exhibit a type shift SC → RC for pragmatic possession; formally: <<et>,<e,<et>>>.

This type shift and its effect when applied to the sortal possessum noun *nah* from Yucatec, illustrated in (1b, c), is sketched in (5):

(5)

a.	sortal noun:	*nah:*	$\lambda x.\text{HOUSE}(x)$
b.	POSS type shift SC → RC:	*-il:*	$\lambda N.\lambda y.\lambda x.[N(x) \wedge \text{POSS}(y,x)]$
c.	result of POSS type shift:	*nah-il:*	$\lambda y.\lambda x.[\text{HOUSE}(x) \wedge \text{POSS}(y,x)]$
d.	saturation of the p'or argument:	*in=nah-il:*	$\lambda x.[\text{HOUSE}(x) \wedge \text{POSS}(\text{SPEAKER},x)]$

What I propose here, then, is a radical lexicalist solution: semantic operation is paired with morphological material. (Note that a template that is equivalent to the POSS type shift is also proposed by Barker 1995 on purely compositional semantic grounds for English; see also Vikner & Jensen 2002 and Partee & Borschev 2003 for discussion.) I furthermore argue that possessive classifiers are best analysed as encompassing the function of a relator plus some additional, more specific information concerning either the sortal properties of the possessum (for example, edibles, domestic animal), or characterising its utility for the possessor.

For the semantic status of the possessor, this solution implies that all possessors are logically treated as individuals, including the possessor agreement markers. Thus, whatever is assumed as the semantics of personal pronouns will characterize the markers at issue. This is a consequence of the POSS type shift, and it has at least two further advantages:

- It correctly predicts that for RNs such as 'friend' the possessor affixes can occur without prior application of the POSS shift, due to the relational semantics of the noun; see (1-a);
- It accounts for the fact that the same 'set A' of pronominal markers occurs with transitive verbs, where they also have pronominal status (the Mayan language generally exhibiting pro-drop).

These two facts would not be accounted for if one were to assume a distinctive semantics for these markers that would make reference to possession: This distinct POSS semantics would have to be turned off for inalienable possession as well as for subject marking, both of which involve the very same markers. It is obvious that this would result in undesirable polysemy.

As the converse to the previous shift, relative nouns occur canonically and unmarked in their possessed use, whereas the omission of a possessor in some languages calls for an overt morphological marker. I treat this as a type shift RC → SC; formally: <<e,<et>>,<et>>.

Evidence comes from quite a few genetically unrelated languages of the Americas and of Melanesia. While some linguists speak of this morphological operation as *absolutivisation*, Seiler 1983 proposes the term *de-relationisation*, hence my gloss 'DEREL' for the marker at issue:

(6) Mam (Mayan, Guatemala; England 1983: 69)

a. *n-yaa'=ya*
1SG.ERG-grandmother=NON3RD
'my grandmother'

b. *yaa-b'aj*
grandmother-DEREL
'grandmother'

(Likewise, Yucatec employs the suffix *-tsil* for licensing the non-possessed use of a relational noun; see Lehmann 1998: 70ff.) In terms of concept types, de-relativising suffixes can be conceived of as denoting a shift from relational concept to sortal concept, thus, RC → SC, We are dealing with a morphologically overt operation that reduces the argument structure of the noun, similarly, much in the same way as passive and antipassive morphology: The variant with reduced argument structure is morphologically marked, which corresponds to the fact that it is derived from the variant with the full argument structure; formally $\lambda R.\lambda x.\exists y.R(x,y)$.

Results of the paper

The distinction of semantic vs. pragmatic is successful in explaining morphosyntactic asymmetries regarding the two dimensions of nominal determination:

Semantic uniqueness implies that the reference of a noun is unambiguous because of its lexical semantics. Pragmatic uniqueness refers to those uses of nouns whose unambiguous reference only comes about by the context of utterance. Weak articles are semantically redundant, they merely signal the presence of an IC/FC, while strong articles denote a <<et>,e> type shift.

Semantic possession implies that the relation between the noun's referential argument and the possessor is inherent to the noun's lexical semantics. Pragmatic possession implies that the POSS relation is contextually established, and often depends on the utterance situation.

The opposition of semantic vs. pragmatic possession is reflected by, and accounts for, what is known as the alienability distinction:

– 'Inalienable' morphology is found with relational nouns and merely signals the inherence of a relation of affiliation, and is therefore morphologically unmarked.

– ‘Alienable’ morphology denotes a <<et>,<e<et>>> shift from SC to RC.

With regard to the distinction of inherent to the lexical semantics vs. pragmatically established, inalienable constructions corresponds to either weak or absent definite articles in case of semantic uniqueness as mentioned in section 2 on article asymmetries. Alienable constructions correspond to the phonologically strong articles of those languages that exhibit an article split.

The two dimensions of nominal determination, definiteness and possession, are thus largely parallel in the following regards: (i) the distinction of semantic vs. pragmatic, (ii) the type shifts from underlying concept type to actual use, and (iii) the close correlation of conceptual and morpho-syntactic markedness that is displayed by split systems.

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Intensionalisation of logical operators

Vít Punčochář

Entailment can be viewed as a relation which preserves some specific value. According to the standard picture the value in question is truth. This paper investigates an alternative: The relation of entailment will be defined as assertibility preservation. We will explore some possible formulations of propositional and first order logic based on this idea.

While truth is relative to possible worlds, assertibility is relative to contexts. We will work with a very simple concept of context which was introduced by Robert Stalnaker: Contexts are sets of possible worlds (see e.g. [4]). For some technical reasons, we will exclude the empty context from the space of all contexts. On the level of propositional logic, possible worlds can be identified with classical valuations, i.e., functions from atomic formulas to truth values $\{0, 1\}$.

Our first semantics is based on two relations between contexts and formulas of classical propositional language. Besides assertibility relation ($\Vdash^+$), we need also deniability relation ($\Vdash^-$), since assertibility of $\neg\varphi$ will be defined as deniability of φ. An atomic formula is assertible (deniable) in a given context iff it is true (false) in every world of the context. Symbolically:

A1 $C \Vdash^+ p$ iff for all $v \in C$, $v(p) = 1$.

D1 $C \Vdash^- p$ iff for all $v \in C$, $v(p) = 0$.

These conditions are motivated in the following way: A context is just a set of possible worlds and we can suppose that the actual world is one of the worlds contained in the context but it is not determined which one it is. So there is enough evidence in the context that p is true iff p is true in all the worlds of the context. Similarly, there is enough evidence that p is false iff p is false in all the worlds. Assertibility and deniability conditions for complex formulas are:

A2 $C \Vdash^+ \neg\varphi$ iff $C \Vdash^- \varphi$.

D2 $C \Vdash^- \neg\varphi$ iff $C \Vdash^+ \varphi$.

A3 $C \Vdash^+ \varphi \vee \psi$ iff $C \Vdash^+ \varphi$ or $C \Vdash^+ \psi$.

D3 $C \Vdash^- \varphi \vee \psi$ iff $C \Vdash^- \varphi$ and $C \Vdash^- \psi$.

A4 $C \Vdash^+ \varphi \wedge \psi$ iff $C \Vdash^+ \varphi$ and $C \Vdash^+ \psi$.

D4 $C \Vdash^- \varphi \wedge \psi$ iff $C \Vdash^- \varphi$ or $C \Vdash^- \psi$.

A5 $C \Vdash^+ \varphi \rightarrow \psi$ iff for all $D \subseteq C$, such that $D \Vdash^+ \varphi$, $D \Vdash^+ \psi$.

D5 $C \Vdash^- \varphi \rightarrow \psi$ iff for all $D \subseteq C$, such that $D \Vdash^+ \varphi$, $D \Vdash^- \psi$.

These assertibility and deniability conditions will be motivated in the full paper. The constant $\perp$ can be introduced as a formula that is by definition assertible in no context and deniable in all contexts.

In this framework, entailment is defined as assertibility preservation: $\Gamma \models \varphi$ iff φ is assertible in every context in which everything from Γ is assertible. As a result we receive an elegant logic. Let us call it L_1. In the full paper a sound and complete natural deduction system for this logic will be presented.

L_1 has some common features with various logics known from the literature: We can mention, e.g., Veltman's data semantics (see [6]), Gauker's logic for conditionals (see [3]), Nelson's constructive logic (see e.g. [5]), Wansing's constructive connexive logic (see [8]). L_1 is very similar to inquisitive semantics (see [2]) even though the informal interpretation of the framework is different. From the technical point of view, L_1 and inquisitive semantics differ only in the way negation is treated. In inquisitive semantics negation of φ is defined as $\varphi \to \bot$ which is not equivalent with the negation defined in L_1.

We will consider one more alternative definition of negation in this framework. Consider the semantics which is based on the above formulated assertibility conditions for atomic formulas (A1), conjunction (A3), disjunction (A4), and implication (A5) plus the following assertibility condition for negation which replaces the condition A2:

A2* $C \Vdash^+ \neg\varphi$ iff $C \nVdash^+ \varphi$.

This weak negation can be interpreted as denial of assertibility and accordingly $\neg\varphi$ can be read as "φ is not assertible". Let us call the resulting logic L_2. From the technical point of view, this logic can be understood as a conservative extension of inquisitive semantics with greater expressive power. The reason is that while inquisitive negation ($\sim$) can be expressed in L_2 (e.g. $\sim\varphi =_{Df} \varphi \to \bot$), the L_2-negation cannot be expressed in inquisitive semantics. For there is no formula φ containing only connectives from $\{\to, \wedge, \vee\}$ which would be logically equivalent with, e.g., $\neg p$. This can be seen from the fact that the set of contexts in which $\neg p$ is assertible is not closed under subcontexts (i.e. subsets) and there is no "inquisitive formula" with this feature.

If weak negation of the logic L_2 is available, two kinds of modal operators can be introduced very naturally as defined symbols:

$$\Box\varphi =_{Df} \sim\sim \varphi \qquad \Diamond\varphi =_{Df} \neg\Box\neg\varphi.$$
$$\blacksquare\varphi =_{Df} \sim \neg\varphi \qquad \blacklozenge\varphi =_{Df} \neg\blacksquare\neg\varphi.$$

The meaning of these operators is the following: It holds that

$\Box\varphi$ is assertible in C iff φ is (classically) true in all worlds of C.

$\blacksquare\varphi$ is assertible in C iff φ is assertible in all subcontexts of C.

The assertibility conditions for the possibility operators are dual: "all" is just replaced by "some".

In the full paper, also L_2 will be characterized by a sound and complete natural deduction system. Here we mention only two specific features of the system. First, the standard conditional proof is not its valid rule of inference. For instance, it holds in L_2 that $\Diamond p, \sim p \models \bot$ but $\Diamond p \not\models \sim p \to \bot$ (for in the contexts which contain both, a world assigning 1 to p and a world assigning 0 to p, the formula $\Diamond p$ is assertible but the formula $\sim p \to \bot$, i.e. $\Box p$, is not).[1]

However, a restricted version of conditional proof can be used in the natural deduction system: In a subordinate proof only $\neg$-free formulas and formulas of the form $\varphi \to \psi$ from its outer proof can be used.

Second, the following defined disjunction-like connective plays a crucial role in the formulation of the system:

$$\varphi_1 \oplus \ldots \oplus \varphi_n =_{Df} \Box(\varphi_1 \vee \ldots \vee \varphi_n) \wedge (\Diamond\varphi_1 \wedge \ldots \wedge \Diamond\varphi_n).$$

$\oplus$ represents a disjunction-like connective which reflects some pragmatic aspects of the ordinary word "or": $\oplus$ is flexible arity operator and by assertion of a sentence of the form $\varphi_1 \oplus \ldots \oplus \varphi_n$ it is roughly said that every disjunct is true in at least one possible world and, at the same time, in every possible world at least one disjunct is true. So the disjuncts are all open possibilities covering together the whole context.[2]

The dual of $\oplus$ is $\otimes$ defined as

$$\varphi_1 \otimes \ldots \otimes \varphi_n =_{Df} \Diamond(\varphi_1 \wedge \ldots \wedge \varphi_n) \vee (\Box\varphi_1 \vee \ldots \vee \Box\varphi_n).$$

[1]This phenomenon reminds the framework of update semantics (see, e.g., [7]). However, our approach is not based on the concept of an update.

[2]A connective that corresponds to our $\oplus$ is used in the field of coalgebraic logic where it is called nabla operator. See e.g. [1].

For these operators the following relationships will play an important role ($\equiv$ stands for logical equivalence):

$\Diamond\varphi_1 \wedge \ldots \wedge \Diamond\varphi_n \equiv \blacklozenge(\varphi_1 \oplus \ldots \oplus \varphi_n)$,

$\Box\varphi_1 \vee \ldots \vee \Box\varphi_n \equiv \blacksquare(\varphi_1 \otimes \ldots \otimes \varphi_n)$.

If time allows, in the presentation will be shortly discussed also an extension of the semantics to the level of predicate logic. Here contexts are defined as pairs $\langle D, W\rangle$, where D is a nonempty set of objects and W is a nonempty set of possible worlds understood as structures of first order predicate logic whose domain is D. The relation of assertibility between contexts and formulas of first-order predicate logic can be introduced in a very straightforward way. Naturally such relation is also relative to an evaluation of variables e. Here are the conditions for quantifiers:

$C \Vdash_e^+ \forall x\varphi$ iff for all $a \in D$, $C \Vdash_{e_a^x}^+ \varphi$.

$C \Vdash_e^+ \exists x\varphi$ iff for some $a \in D$, $C \Vdash_{e_a^x}^+ \varphi$.

As on the level of propositional logic, there are three natural but non-equivalent ways how to introduce negation: we can define strong negation using the deniability conditions, the intuitionistic-like negation, and the weak negation. When we have weak negation, we can define the modalities as before and moreover we can introduce a new quantifier (∇) which corresponds to the operator $\oplus$, and its dual (Δ) which corresponds to $\otimes$:

$\nabla x\varphi =_{Df} \forall x\Diamond\varphi \wedge \Box\exists x\varphi$,

$\Delta x\varphi =_{Df} \exists x\Box\varphi \vee \Diamond\forall x\varphi$.

For these quantifiers the following relationships, which combine different kinds of modalities, are analogous to those concerning $\oplus$ and $\otimes$ which were mentioned above.

$\blacklozenge\nabla x\varphi \equiv \forall x\Diamond\varphi$,

$\blacksquare\Delta x\varphi \equiv \exists x\Box\varphi$.

Besides presenting some new technical results, it will be argued in the lecture that logic based on the assertibility conditions models some linguistic phenomena better than logic based on truth conditions.

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Altenative Semantics for Visser's Basic Propositional Logic

Katsuhiko Sano and Minghui Ma

Introduction and Motivation

Visser [8] introduced the basic propositional logic (**BPL**) and used the extension **FPL** of **BPL** with the Löb's rule to interpret formal provability in Peano Arithmetic. Gödel-Mckinsey-Tarski translation [4, 5] faithfully embeds intuitionistic logic (**Int**) into modal logic **S4**. Visser [8, p.179] considered two variants G_0 and G_1 (in our terminology) of Gödel-Mckinsey-Tarski translation and showed that both of them faithfully embed the logic **FPL** into provability (Gödel-Löb) logic by Solovay's completeness theorem. The only difference between G_0 and G_1 is on the atomic clause. While G_0 sends a variable p to $\Box p$, G_1 sends p to $p \wedge \Box p$. It is well-known that the translation G_0 embeds the logic **BPL** into modal logic **K4** ([7] studies Gödel-Mckinsey-Tarski translation in an expanded syntax of **BPL** with a new implication symbol). As far as the authors know, it has not been investigated which modal logics we can obtain via the translation G_1. In this paper, we give an answer to this question: G_1 embeds **BPL** into **wK4**, where **wK4** was shown by Esakia [2] to be the modal logic of *weakly transitive* Kripke frames. (A Kripke frame (W, R) is weakly transitive if wRv and vRu and $w \neq u$ imply wRu.) A consequence of this result is that we can provide a *topological* semantics for **BPL**, since **wK4** is the logic of all topological spaces if the diamond $\Diamond$ is interpreted as the *derivative operator* or the *limit operator* [3, 1]. With the help of the translation result, we semantically consider two kinds of semantics (ordinary semantics and proper-successor semantics) over both Kripke frames and topological spaces and show several new completeness results of **BPL** (see Theorems 2 and Theorem 3). A key idea of our proper-successor semantics over Kripke model consists in focusing on the proper-future points in the semantic clause for the implication, i.e., disregarding the current evaluation point. By extending this idea to the topological setting, we can naturally obtain our topological semantics for **BPL** by the notion of the co-derivative operator, i.e., the dual of the derivative operator.[1]

Full and Faithful Embedding of BPL into Modal Logic

The syntax $\mathcal{ML}$ of modal logic consists of the set Prop of propositional variables, $\wedge$, $\vee$, $\bot$, $\to$ and a modal operator $\Box$. Then, the set $\mathsf{Form}_{\mathcal{ML}}$ of all $\mathcal{ML}$-formulas is generated by the following grammar:

$$\mathsf{Form}_{\mathcal{ML}} \ni \alpha ::= \bot \mid p \mid \alpha \wedge \beta \mid \alpha \vee \beta \mid \alpha \to \beta \mid \Box\alpha \qquad (p \in \mathsf{Prop}).$$

Definition 24. A *Kripke frame* is a pair $\mathfrak{F} = (W, R)$ of a non-empty set W and $R \subseteq W \times W$. A *Kripke model* $\mathfrak{M}$ is a pair of a basic Kripke frame $\mathfrak{F} = (W, R)$ and a valuation mapping $V : \mathsf{Prop} \to \mathcal{P}(W)$. We say that a Kripke frame $\mathfrak{F} = (W, R)$ (or a model $\mathfrak{M}$) is *transitive* if wRv and vRu imply wRu for all $w, v, u \in W$. We also say that $\mathfrak{F} = (W, R)$ (or $\mathfrak{M}$) is *weakly-transitive* if wRv and vRu and $w \neq u$ imply wRu for all $w, v, u \in W$.

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It is easy to see that transitivity implies weak-transitivity. Given any Kripke model, the notion of truth or satisfaction $\mathfrak{M}, w \Vdash \alpha$ is defined as usual. Given any class $\mathbb{F}$ of frames, we say that α is *valid* in $\mathbb{F}$ (notation: $\mathbb{F} \Vdash \alpha$) if $(\mathfrak{F}, V), w \Vdash \alpha$ for all $\mathfrak{F} \in \mathbb{F}$, all valuations V on $\mathfrak{F}$ and all $w \in W$. Define $\mathrm{MLog}(\mathbb{F}) = \{ \alpha \in \mathsf{Form}_{\mathcal{ML}} \mid \mathbb{F} \Vdash \alpha \}$. We denote by $\mathbb{WT}$ and $\mathbb{TR}$ the class of weakly-transitive Kripke frames and the class of transitive Kripke frames, respectively.

We say that a set $\Lambda \subseteq \mathsf{Form}_{\mathcal{ML}}$ of formulas is a *normal modal logic*, if it contains all propositional tautologies, $\Box(p \to q) \to (\Box p \to \Box q)$, and it is closed under modus ponens, uniform substitution, $\Box$-necessitation (from φ infer $\Box\varphi$). Henceforth, we always consider normal modal logics that contain $(w4)$ $p \wedge \Box p \to \Box\Box p$, which defines weak-transitivity of R. We denote the minimal normal modal logic containing $(w4)$ by **wK4**. By **K4**, we mean the minimal modal logic containing $\Box p \to \Box\Box p$. It is well-known that $\mathbf{K4} = \mathrm{MLog}(\mathbb{TR})$.

Fact 1 (Esakia 2001)**.** $\mathbf{wK4} = \mathrm{MLog}(\mathbb{WT})$.

Let us move to Visser's basic propositional logic. The syntax $\mathcal{L}$ for **BPL** is the result of deleting the modal operator $\Box$ from $\mathcal{ML}$. $\mathsf{Form}_{\mathcal{L}}$ of all $\mathcal{L}$-formulas is defined by:

$$\mathsf{Form}_{\mathcal{L}} \ni \varphi ::= \bot \mid p \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \to \psi \qquad (p \in \mathsf{Prop}).$$

Now we present Suzuki and Ono [6]'s Hilbert-style axiomatization of **BPL**.

Definition 25. **BPL** is defined by the following axioms and inference rules:

(A1)	$p \to p$	(A7)	$p \to (q \to p \wedge q)$
(A2)	$p \to (q \to p)$	(A8)	$p \to p \vee q$
(A3)	$(p \to q) \wedge (q \to r) \to (p \to r)$	(A9)	$q \to p \vee q$
(A4)	$p \wedge q \to p$	(A10)	$(p \to r) \wedge (q \to r) \to (p \vee q \to r)$
(A5)	$p \wedge q \to q$	(A11)	$p \wedge (q \vee r) \to (p \wedge q) \vee (p \wedge r)$
(A6)	$(p \to q) \wedge (p \to r) \to (p \to q \wedge r)$	(A12)	$\bot \to p$
(MP)	From α and $\alpha \to \beta$ infer β		
(Sub)	From α infer α^σ, where σ is a uniform substitution		

Let us introduce Visser's Kripke semantics for **BPL**.

Definition 26. Given a Kripke frame $\mathfrak{F} = (W, R)$, we say that $V : \mathsf{Prop} \to \mathcal{P}(W)$ is *persistent* if $w \in V(p)$ and wRu imply $u \in V(p)$ for all $w, u \in W$ and all $p \in \mathsf{Prop}$. We say that a Kripke model $\mathfrak{F} = (W, R, V)$ is *persistent* if a valuation V is persistent.

Given any persistent Kripke model $\mathfrak{M} = (W, R, V)$, the satisfaction relation $\mathfrak{M}, w \models \varphi$ (note that we use '$\models$' here, which is different from '$\Vdash$' for our modal syntax) is defined as follows:

$\mathfrak{M}, w \models p$	iff	$w \in V(p)$
$\mathfrak{M}, w \models \bot$		Never
$\mathfrak{M}, w \models \varphi \wedge \psi$	iff	$\mathfrak{M}, w \models \varphi$ and $\mathfrak{M}, w \models \psi$
$\mathfrak{M}, w \models \varphi \vee \psi$	iff	$\mathfrak{M}, w \models \varphi$ or $\mathfrak{M}, w \models \psi$
$\mathfrak{M}, w \models \varphi \to \psi$	iff	(wRu and $\mathfrak{M}, u \models u \models \varphi$) imply $\mathfrak{M}, u \models \psi$, for all $u \in W$

Remark that we do not require any condition on R here. It is known that transitivity of R and the persistency condition for all atoms imply the persistency condition for all formulas. We can also weaken the assumption of R as follows.

Proposition 6. *Let $\mathfrak{M}$ be persistent and weakly-transitive. If $\mathfrak{M}, w \models \varphi$ and wRu then $\mathfrak{M}, u \models \varphi$, for all $w, u \in W$ and all $\varphi \in \mathsf{Form}_{\mathcal{L}}$.*

Given any class $\mathbb{F}$ of frames, we say that $\varphi \in \mathsf{Form}_{\mathcal{L}}$ is *valid* in $\mathbb{F}$ (notation: $\mathbb{F} \models \varphi$) if $(\mathfrak{F}, V), w \models \varphi$ for all $\mathfrak{F} \in \mathbb{F}$, all persistent valuations V on $\mathfrak{F}$ and $w \in W$. Define $\mathrm{Log}(\mathbb{F}) = \{ \varphi \in \mathsf{Form}_{\mathcal{L}} \mid \mathbb{F} \models \varphi \}$.

Remark 1. While (MP) does *not* preserve validity on a fixed persistent Kripke model, it preserves validity on the class of persistent Kripke models.

Fact 2 (Visser 1981)**.** $\mathbf{BPL} = \mathrm{Log}(\mathbb{TR})$.

Note that $\mathbb{F} \subseteq \mathbb{G}$ implies $\mathrm{Log}(\mathbb{G}) \supseteq \mathrm{Log}(\mathbb{F})$. Since $\mathbb{TR} \subseteq \mathbb{WT}$ clearly holds, $\mathrm{Log}(\mathbb{WT}) \supseteq \mathrm{Log}(\mathbb{TR})$. Therefore, Fact 2 enables us to derive the following.

Proposition 7. $\mathbf{BPL} \subseteq \mathrm{Log}(\mathbb{WT})$.

Definition 27. We define the syntactic translation G_1 from $\mathsf{Form}_{\mathcal{L}}$ to $\mathsf{Form}_{\mathcal{ML}}$ by:

$$\begin{array}{rlrl} G_1(p) & = p \wedge \Box p & G_1(\varphi \vee \psi) & = G_1(\varphi) \vee G_1(\psi) \\ G_1(\bot) & = \bot & G_1(\varphi \wedge \psi) & = G_1(\varphi) \wedge G_1(\psi) \\ G_1(\varphi \to \psi) & = \Box(G_1(\varphi) \to G_1(\psi)). & & \end{array}$$

Proposition 8. $\vdash_{\Lambda} G_1(\varphi) \to \Box G_1(\varphi)$ *for any normal modal logic* Λ *with* $(w4) \in \Lambda$.

Definition 28. $(\mathrm{MP}_{\Box})$ is the rule: from p and $\Box(p \to q)$ infer q.

We say that $(\mathrm{MP}_{\Box})$ is *admissible* in a normal modal logic Λ if, for any uniform substitution σ in $\mathsf{Form}_{\mathcal{ML}}$, $p^{\sigma} \in \Lambda$ and $\Box(p^{\sigma} \to q^{\sigma}) \in \Lambda$ imply $q^{\sigma} \in \Lambda$. Note that the addition of the admissible rule to Λ does not change the set of all theorems in Λ. We need this admissibility because of (MP) in $\mathbf{BPL}$ (recall Remark 1).

Theorem 1. *Let* $(w4) \in \Lambda$*. Suppose that* $(\mathrm{MP}_{\Box})$ *is admissible in* Λ*, and that* $\Lambda \subseteq \mathrm{MLog}(\mathbb{F})$ *and* $\mathrm{Log}(\mathbb{F}) \subseteq \mathbf{BPL}$*. Then* $\vdash_{\mathbf{BPL}} \varphi$ *iff* $\vdash_{\Lambda} G_1(\varphi)$*.*

By $\mathbf{K4} = \mathrm{MLog}(\mathbb{TR})$ and Fact 2, we can obtain that $\vdash_{\mathbf{BPL}} \varphi$ iff $\vdash_{\mathbf{K4}} G_1(\varphi)$ for all formulas $\varphi \in \mathsf{Form}_{\mathcal{L}}$. By Theorem 1, Fact 2 and Proposition 7, we finally obtain the following.

Corollary 2. $\vdash_{\mathbf{BPL}} \varphi$ iff $\vdash_{\mathbf{wK4}} G_1(\varphi)$ *for all* $\varphi \in \mathsf{Form}_{\mathcal{ML}}$*.*

Alternative Kripke Semantics for Visser's BPL

Given a persistent Kripke model $\mathfrak{M} = (W, R, V)$ and $\varphi \in \mathsf{Form}_{\mathcal{ML}}$, we define the alternative satisfaction relation $\mathfrak{M}, w \models^{\bullet} \varphi$ by replacing the truth clause of $\varphi \to \psi$ with the following clause:

$$\mathfrak{M}, w \models^{\bullet} \varphi \to \psi \quad \text{iff} \quad (wRu \text{ and } w \neq u \text{ and } \mathfrak{M}, u \models^{\bullet} \varphi) \text{ imply } \mathfrak{M}, u \models^{\bullet} \psi, \text{ for all } u \in W.$$

An underlying idea of this new clause is to disregard the current evaluation point w by adding the condition of '$w \neq u$'. In other words, we restrict our attention to the *proper* future or successor points. In this sense, let us call the new semantics the *proper-successor semantics*.

Proposition 9. *Let* $\mathfrak{M}$ *be persistent and weakly-transitive. If* $\mathfrak{M}, w \models^{\bullet} \varphi$ *and* wRu *then* $\mathfrak{M}, u \models^{\bullet} \varphi$*, for all* $w, u \in W$ *and* $\varphi \in \mathsf{Form}_{\mathcal{L}}$*.*

We use $\mathbb{F} \models^{\bullet} \varphi$ to mean that $(\mathfrak{F}, V), w \models^{\bullet} \varphi$ holds for all frames $\mathfrak{F} \in \mathbb{F}$, all persistent valuations V and all points w of $\mathfrak{F}$. Given any class $\mathbb{F}$ of frames, we define $\mathrm{Log}^{\bullet}(\mathbb{F}) = \{ \varphi \in \mathsf{Form}_{\mathcal{L}} \mid \mathbb{F} \models^{\bullet} \varphi \}$. We define that $\mathbb{PRE}$ is the class of all pre-ordered set (i.e., R is reflexive and transitive) and that $\mathbb{PO}$ is the class of all partially ordered set (i.e., R is reflexive, transitive and antisymmetric).

Theorem 2. $\mathbf{BPL} = \mathrm{Log}^{\bullet}(\mathbb{WT}) = \mathrm{Log}^{\bullet}(\mathbb{TR}) = \mathrm{Log}^{\bullet}(\mathbb{PRE}) = \mathrm{Log}^{\bullet}(\mathbb{PO}) = \mathrm{Log}(\mathbb{WT})$.

sem. \ frame class	weakly-transitive	transitive	pre-order	partial order
ordinary sem.	$\mathrm{Log}(\mathbb{WT}) = \mathbf{BPL}$	$\mathrm{Log}(\mathbb{TR}) = \mathbf{BPL}$ [8]	$\mathrm{Log}(\mathbb{PRE}) = \mathbf{Int}$	$\mathrm{Log}(\mathbb{PO}) = \mathbf{Int}$
proper-suc. sem.	$\mathrm{Log}^{\bullet}(\mathbb{WT}) = \mathbf{BPL}$	$\mathrm{Log}^{\bullet}(\mathbb{TR}) = \mathbf{BPL}$	$\mathrm{Log}^{\bullet}(\mathbb{PRE}) = \mathbf{BPL}$	$\mathrm{Log}^{\bullet}(\mathbb{PO}) = \mathbf{BPL}$

Topological Semantics for Visser's BPL

Since **wK4** is the logic of all topological spaces if the diamond $\Diamond$ is interpreted as the *derivative operator* [3], it is quite natural to ask if we can also provide a topological semantics with **BPL**. This section shows that this is the case. Our key of discovering a topological semantics for **BPL** consists in the following question: what is the corresponding notion in the topological setting to the proper-successor semantics over Kripke models? Our answer will be given in terms of the notion of co-derivative operator, i.e., the dual of the derivative operator.

We say that (W, τ) is a *topological space* if W is non-empty and $\tau : W \to \mathcal{P}\mathcal{P}(W)$ satisfies the following requirements:

- $\tau(w)$ is upward-closed and closed under finite intersections and $\tau(w) \neq \varnothing$ $(w \in W)$.
- if $X \in \tau(w)$ implies $w \in X$ for all $X \subseteq W$ and all $w \in W$.
- if $\Box_\tau(X) \subseteq \Box_\tau(\Box_\tau(X))$ for all $X \subseteq W$,

where $\Box_\tau(X) := \{\, w \in W \mid X \in \tau(w) \,\}$, i.e., the interior of X.

Given a topological space (W, τ), we say that a valuation $V : \mathsf{Prop} \to \mathcal{P}(W)$ is *τ-persistent* if $V(p) \subseteq \Box_\tau(V(p))$. We say that $\mathfrak{M} = (W, \tau, V)$ is *τ-persistent* if V is τ-persistent.

Let us introduce two kinds of semantics: ordinary semantics and proper-successor semantics. Let $\mathfrak{M} = (W, \tau, V)$ be a τ-persistent model. We define the ordinary semantics $\models$ of $\mathcal{L}$ by fixing the semantic clause similarly to Kripke semantics except:

$$\mathfrak{M}, w \models \varphi \to \psi \quad \text{iff} \quad X \cap [\![\varphi]\!] \subseteq [\![\psi]\!] \text{ for some } X \in \tau(w),$$

where $[\![\varphi]\!] := \{\, w \in W \mid \mathfrak{M}, w \models \varphi \,\}$. On the proper-successor semantics $\models^\bullet$ for $\mathcal{L}$, the semantic clause for implication is defined as:

$$\mathfrak{M}, w \models^\bullet \varphi \to \psi \quad \text{iff} \quad (X \backslash \{\, w \,\}) \cap [\![\varphi]\!]^\bullet \subseteq [\![\psi]\!]^\bullet \text{ for some } X \in \tau(w),$$

where $[\![\varphi]\!]^\bullet := \{\, w \in W \mid \mathfrak{M}, w \models^\bullet \varphi \,\}$.

Given a topological space (W, τ), we define the *derivative operator* $\mathbf{d}_\tau$ by

$$\mathbf{d}_\tau(X) := \{\, w \in W \mid X \backslash \{\, w \,\} \in \tau(w) \,\}.$$

The *co-derivative operator* $\mathbf{t}_\tau$ is defined by $\mathbf{t}_\tau(X) = W \backslash \mathbf{d}_\tau(W \backslash X)$. It is easy to show that $x \in \mathbf{t}_\tau(X)$ iff $X \cup \{\, x \,\} \in \tau(x)$. Then, the proper-successor semantics $\models^\bullet$ for the implication is reformulated as:

$$\mathfrak{M}, w \models^\bullet \varphi \to \psi \quad \text{iff} \quad w \in \mathbf{t}_\tau((W \backslash [\![\varphi]\!]^\bullet) \cup [\![\psi]\!]^\bullet).$$

Proposition 10. *For any τ-persistent topological model $\mathfrak{M} = (W, \tau, V)$, both $[\![\varphi]\!] \subseteq \Box_\tau([\![\varphi]\!])$ and $[\![\varphi]\!]^\bullet \subseteq \Box_\tau([\![\varphi]\!]^\bullet)$ hold for all $\varphi \in \mathsf{Form}_\mathcal{L}$.*

Given a class S of topological spaces, we define the notions of the logic of S with respect to the ordinary semantics (written: $\mathrm{Log}(\mathsf{S})$) and the logic of S with respect to the proper-successor semantics (written: $\mathrm{Log}^\bullet(\mathsf{S})$) similarly to the case of Kripke semantics.

Let (W, τ) be a topological space. (W, τ) is T_0 if $x \neq y$ implies $x \notin X$ or $y \notin Y$ for all $x, y \in W$, $X \in \tau(x)$ and $Y \in \tau(y)$. (W, τ) is T_d if $\mathbf{t}_\tau(X) \subseteq \mathbf{t}_\tau(\mathbf{t}_\tau(X))$ for all $X \subseteq W$. We denote the class of all topological spaces by Top. By $T_0\mathsf{Top}$ and $T_d\mathsf{Top}$, we mean the class of all T_0-topological spaces and the class of all T_d-topological spaces, respectively. By Theorem 2, we establish the following.

Theorem 3. $\mathrm{Log}^\bullet(\mathsf{Top}) = \mathrm{Log}^\bullet(T_0\mathsf{Top}) = \mathrm{Log}^\bullet(T_d\mathsf{Top}) = \mathbf{BPL}$.

sem. \ space class	topological spaces	T_0-spaces	T_d-spaces
ordinary sem.	$\mathrm{Log}(\mathsf{Top}) = \mathbf{Int}$	$\mathrm{Log}(T_0\mathsf{Top}) = \mathbf{Int}$	$\mathrm{Log}(T_d\mathsf{Top}) = \mathbf{Int}$
proper suc. sem.	$\mathrm{Log}^\bullet(\mathsf{Top}) = \mathbf{BPL}$	$\mathrm{Log}^\bullet(T_0\mathsf{Top}) = \mathbf{BPL}$	$\mathrm{Log}^\bullet(T_d\mathsf{Top}) = \mathbf{BPL}$

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On the licensing of argument conditionals

Kerstin Schwabe

The paper is about constructions where the propositional argument of a matrix predicate is realized by a conditional as in the German construction (1):

(1) *Frank bedauert es, wenn Maria krank ist*
'Frank regrets it if Maria is ill'.

It will discuss the syntactic and semantic status of argument *wenn*-clauses and propositional correlates associated with them as well as the semantic properties of the verb classes that license these argument adverbials.

An argument conditional is neither an object- nor a subject-clause introduced by a non-canonical complementizer as suggested by Böttcher & Sitta (1972) for German and Pullum (1987) for English. It is an adverbial which restricts the matrix clause and simultaneously provides the propositional argument of the matrix predicate – cf. Williams (1974), Rothstein (1995), and Hinterwimmer (2012) for English, Fabricius-Hansen 1980 for German, and Quer (2002) for Spanish and Catalan. The reasons for the adverbial status of argument conditionals are, among others:

i. An obligatory propositional argument has to be realized by a correlate if the *wenn*-clause is preposed – cf. (2a). Preposed complement clauses like *dass*-, *wh*-, and *ob*-clauses leave a trace in the complement position thus prohibiting a correlate there – cf. (2b).

(2) a. *Wenn Maria krank ist, bedauert *(es) Frank*
if Maria ill is regrets *(it) Frank
'If Maria is ill, Frank regrets it'.

b. *Dass Maria krank ist, bedauert (*es) Frank*
that Maria ill is regrets it'.

ii. Unlike *dass*-argument clauses, argument *wenn*-clauses cannot be adjoined to DPs or PPs, respectively – cf. (3a, b).

(3) a. *Das, dass/ *wenn Maria krank ist, ...*
'The fact that/*if Maria is ill ...'
b. *Darüber, dass/*wenn Maria kommt, ...*
'the perspective that/* if Maria will come.

iii. The paraphrase of a construction with an argument *wenn*-clause contains a *dass*-clause in the complement position of the matrix predicate – cf. (4).

(4) *Wenn Maria krank ist, bedauert Frank, dass Maria krank ist.*
if Maria ill is regrets Frank that Maria ill is ill

iv. Preference predicates admit *wh*-extraction out of *dass*-clauses. *Wh*-extraction out of *wenn*-clauses is not allowed – cf. (5).

(5) *Wohin zieht Frank vor, dass/*wenn Maria fährt?*
where prefers Frank that/*if Maria will go?'

Similarly to Pesetsky (1991), I consider extraposed argument *wenn*-clauses as base-generated VP- or vP-adjuncts which are indirectly licensed by a correlate or, if there isn't any, by a propositional *pro* – cf. (6a-c).

(6) a. Frank$_j$ bedauert$_i$ es$_k$ t$_j$ [VP [VP [V' t$_k$/pro t$_i$]] wenn Mia krank ist]
Frank regrets it/pro if Mia ill is
b. Frank$_j$ freut$_i$ sich t$_j$ [$_{\text{VP}}$ [VP [V' darüber/pro ti]] wenn Mia glücklich ist]
Frank is glad *da*-about if Mia happy is
c. Eigentlich ist es$_k$ [$_{\text{vP}}$ [vP t$_k$ sonderbar] wenn er schweigt]
actually is it odd if he is silent

Wenn-clauses in the left periphery as in (2) originate in the Spec-position of the functional category TP where they are licensed by T^0 – cf. (7).

(7) [Wenn Mia krank ist]$_i$ bedauert$_j$ es$_k$ [$_{\text{TP}}$ t$_i$ ... Frank [$_{\text{VP}}$ [$_{\text{V'}}$ t$_k$/*pro t$_j$]]

Argument conditionals that are out of VP are ordinary adverbials. If they are inside VP, however, they are rather complement-like. Correlates like *es/das* and *pro* as well as prepositional correlates like *darüber* 'pro-about' are theta-marked by the matrix predicate and thus licensed by it. A pro - positional *pro* must additionally m$_e$-command the argument conditional in terms of Pesetsky (1991).

Unlike Pesetsky (1991) and like Rothstein (1995), I regard the correlate to be a pronoun. Like its related clause, it refers to an abstract object, to a proposition. The latter is regarded as an abstract object in a particular context – cf. Asher (1993) and Schwabe (2007). As to (6a), the context-givenness of the proposition 'Maria is ill' is represented by a proposition index which is assigned to the related clause by dass 'that' or wenn 'if' – cf. the index σ at Mia is ill in (8a).

(8) a. [Frank bedauert es]$_\tau$ wenn$_{\langle\sigma,\tau\rangle}$ [Mia krank ist]$_\sigma$
Frank regrets it if Mia ill is

The proforms *es*, ProPP and *pro* are represented as propositional variables p. The context-givenness of the proposition they refer to is represented by an index variable i at the proposition variable p. As to (7) where the proform *es* refers to an anaphorically given proposition, the index variable absorbs the index of this proposition. If the proform m$_e$-commands the argument conditional, what is possible for the *es*, the ProPP and the *pro*, the argument *wenn*-clause provides the index for the proform variable – cf.

8b. [frank_regrets_p$_\sigma$] if [Mia is ill]$_\sigma$
$\sigma \Rightarrow \tau(\sigma)$

The conjunction *wenn* 'if' creates an implication where the argument conditional is the antecedent and the matrix clause is the consequent. The antecedent focuses on a truth condition of the matrix predicate. In (8), it is that σ is an element of v, i.e. the set of propositions that are valid, but not propositionally tautological – cf. (8c) and the potentially factive predicates of Schwabe & Fittler (2012), Williams (1974), and Fabricius-Hansen (1980) as well as Rothstein (1995), Quer (2002), Hinterwimmer (2011) and Thomson (2012) who follow Kratzer's (1986) slightly different analysis. Thus, (8) can be paraphrased by *If σ is valid* but not tautological, *Frank regrets it that σ*.

8c. $\sigma \in v \Rightarrow \tau(\sigma)$

A sentence like (8a) and a corresponding one with a *dass*-clause like *Frank bedauert es, dass Mia krank ist* 'Frank regrets it if Mia is ill' have the same truth values provided Mia is ill. But they are not equivalent because their truth values differ if Mia is not ill. As for matrix verbs allowing the embedding of question extensions like for instance *hören* 'hear' or *wissen* 'know', a sentence with an embedded polar interrogative is very similar to a sentence with an argument conditional – cf. (9) and (10).

(9) Max erfährt es, wenn jemand vor der Tür steht.
Max finds out it if anyone at the door is
Anyone is at the door $\Rightarrow$ Max finds out that there is someone at the door

(10) Max erfährt es, ob jemand vor der Tür steht.
Max finds out it if/whether anyone at the door is
(Max finds out that there is someone at the door) or (Max finds out that there isn't anyone at the door)

Here again, both variants have the same truth values if the antecedent is true. But again, they are not equivalent. If there isn't anyone at the door, the implication given with (9) is true whereas the disjunction given with (10) could be false.

The logical form of constructions with potentially factive predicates as in (8) is not appropriate for a construction with a preference predicate like (11) because preference predicates are not potentially factive – cf. Fabricius-Hansen (1980). Neither Fabricius-Hansen nor Kayati (2010) find an appropriate paraphrase for such constructions.

(11) [Mia zieht es$_\sigma$ vor]$_\tau$ wenn$_{\langle\sigma,\tau\rangle}$ [Lea Klavier spielt]$_\sigma$
Mia prefers it if Lea the piano plays
*[lea_plays_the piano]$_\sigma$ $\in$ v $\Rightarrow$ [mia_prefers_p$_\sigma$]
*$\sigma \in v \Rightarrow \tau(\sigma)$

I propose that the argument conditional focuses on the truth condition that σ is an element of the union $v \cup f$, i.e. of v, the set of valid but not propositionally tautological propositions, with f, the set of false but not contradictory propositions. In this way, the argument conditional is a faultless antecedent for the matrix clause which says 'Mia prefers σ (= Lea plays the piano) if $\sigma \in v \cup f$:

(12) [Mia zieht es$_\sigma$ vor]$_\tau$ wenn$_{\langle\sigma,\tau\rangle}$ [Lea Klavier spielt]$_\sigma$
lea_plays_the_piano] $\in v \cup f \Rightarrow$ [Mia_prefers_p$_\sigma$]
$\sigma \in v \cup f \Rightarrow$ Mia zieht es vor dass σ

Constructions like (12) are equivalent to the corresponding ones with a *dass*-clause like *Mia zieht es vor, dass Lea Klavier spielt* 'Mia prefers it that Lea plays the piano' as long as the embedded statement is contingent. It follows that there is only one conjunction wenn 'if'. It indicates an implication and introduces the antecedent of this implication. It depends on the matrix predicate whether the antecedent denotes a proposition that is either true but not tautological or contingent.

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Pragmatic effects in processing superlative and comparative quantifiers: the role of clausal implicatures

Maria Spychalska

The right semantic interpretation of so-called superlative quantifiers, such as *at most n* and *at least n*, (n represents a bare numeral) has been widely discussed in the linguistic and philosophical literature [4], [3], [9], [2]. According to Generalized Quantifier Theory, these quantifiers are equivalent to comparative quantifiers, i.e. *fewer than n+1* and *more than n-1* respectively [1]. Various empirical data, however, question that these equivalences hold in natural languages. For instance it has been shown that speakers generally do not accept the logically valid inference *at most n+1 As are B* from *at most n As are B* [5], [3]. There is also ample data concerning differences between processing of superlative and comparative quantifiers. One of the most important results is that the verification of sentences with superlative quantifiers requires supposedly more time than the verification of sentences with respective comparative quantifiers [6], [5]. The processing of quantifiers is also influenced by their monotonicity: Although the downward monotone quantifiers *at most n* and *fewer than n* take a longer time to be verified than the upward monotone quantifiers *at least n* and *more than n*, they are actually falsified faster [6].

To account for the empirical data, [4] and [5] propose that whereas comparative quantifiers have their conventional meaning, the semantics of superlative quantifiers has a modal component, and consequently both *at most n As are B* and *at least n As are B* logically imply that *it is possible that there are exactly n As that are B.* In contrast, Cummins and Katsos [3] propose that such a modal statement is a result of a pragmatic rather than a logical inference, namely a so-called *clausal implicature.* This implicature is generated by statements with superlative quantifiers, since the semantics of these quantifiers can be represented in a disjunctive form, i.e. the following equivalences hold: *at most n As are B* $\leftrightarrow$ *(exactly) n or fewer than n As are B* and *at least n As are B* $\leftrightarrow$ *(exactly) n or more than n As are B.*

Based on the two Gricean Maxims – *Quantity* and *Quality* – the correct use of a disjunctive sentence *p or q* assumes that the speaker is not yet decided about the truth-values of the disjuncts. It follows that the (clausal) implicatures: *possibly p*, *possibly not p*, *possibly q* and *possibly not q* are inferred [7].

In order to test the processing predictions of the pragmatic and the semantic theory regarding the modal component in the meaning of superlative quantifiers, I conducted a sentence-picture verification experiment with reaction time measurement. The experiment was designed to compare the dependence of the subjects' accuracy and response time on the linguistic form of the used quantifier and on the model in which the quantified sentence is evaluated (shown picture). The picture determined the truth, falsity or pragmatic infelicity of the evaluated sentence. The predictions were as follows: On the one hand, if superlative quantifiers have modal semantics, which means that the sentence ϕ: *At most n As are B* logically implies ψ : *It is possible that there are exactly n As that are B*, then ϕ should be rejected in models in which there are fewer than *n As* that are *B*. Since ψ, which is a logical consequence of ϕ, is false in such models, then ϕ cannot be true in those models either. On the other hand, if ψ is merely a pragmatic inference from ϕ, then it should be defeasible and ϕ should be evaluated as true in models that have fewer than *n As* that are *B*. In this case we expect that the evaluation of sentences with superlative quantifiers will result in a longer response-times and/or a higher mistakes ratio compared to the evaluation of sentences with comparative quantifiers, since the latter do not involve any pragmatic effects. The important hypothesis is that this effect should strongly depend on the model type and a delay in evaluating superlative is expected only in those models in which the use of these quantifiers can be considered as pragmatically infelicitous. The critical distinction is made between so-called "borderline models" i.e. models that have exactly N target objects (As that are B), and models that have fewer than N or more than N target objects, where N is the size of the maximal/ minimal witness set for the downward and upward monotone quantifiers respectively, and $N = n$ for *at most n/ at least n.*

I propose that the use of the downward monotone superlative quantifier *at most n* is more felicitous in "borderline" models, i.e. with *exactly n* target objects, than in models with fewer than n target objects. This assumption requires a more specific explanation. In fact, if the disjunctive equivalent of *at most n*, i.e. *exactly n or fewer than n* triggers clausal implicatures, then both $\psi = \diamond\exists^{=n}$ (*possibly exactly n*) and $\xi = \diamond\exists^{<n}$ (*possibly fewer than n*) are implicated. Whereas the presence of ψ determines the infelicity of sentences with *at most n* in models with fewer than n target objects, the presence of ξ makes the use of this quantifier infelicitous in models with exactly n target objects. It follows that the use of the quantifier *at most n* is infelicitous in any model, in which it is actually true. At first, this observation might seem paradoxical, however, it tells us something about the communicative role of superlative quantifiers in natural language. Their purpose is not to provide descriptions of situations but to express the available options in the case in which a situation is underspecified. The case of model-verification of sentences with *at most n* is from this point of view artificial. In a standard communication, if we have for instance three red dots, the correct description is that *three dots are red* or that *there are three red dots*, and not that *at most three dots are red.* Thus, it is true that from a pragmatic perspective the use of a superlative quantifier is not felicitous in any of those cases in which it is supposed to play a purely descriptive role. Nevertheless, I propose that the use of superlative quantifiers can be considered *more felicitous* in some models, due to a hierarchy of implicatures or typicality effects. Assuming that there is some hierarchy of implicatures that are associated with a given sentences (either a hierarchy of importance or of order in which they are actually generated), I hypothesize that the implicature *possibly exactly n* is the preferred one for the quantifier *at most n.* Then, in the case of verifying *at most n* in models with exactly n target objects, the implicature that is primarily generated, i.e. *possibly exactly n* is confirmed and the implicature *possibly fewer than n* is suppressed. However, if one tries to verify *at most n* in models with fewer than n target objects, the implicature *possibly exactly n* is contradicted. The process of cancelation this default implicature *possibly exactly n* and generating the second implicature: *possibly fewer than n* requires then an extra reasoning step and should be associated with longer processing time. In principle, the case of the quantifier *at least n* should be analogous to the case of *at most n*: there are two implicatures *possibly exactly n* and *possibly more than n.* The question is whether the hierarchy of these implicatures is the same. On the one hand the borderline models should be preferred (and thus also the implicature *possibly exactly n*), on the other hand the borderline models constitute in this case the lower bound of the truth-conditions, which might lead to a dispreference of those models. Therefore, it is not clear whether we can expect an isomorphism between the pragmatic effects for *at least n* and for the downward monotone superlative quantifier *at most n.*

Experiment

Fifty-six (twenty-nine women) right-handed German native speakers were tested (mean age: 24.25, SD: 4.26) in the experiment. They were asked to evaluate sentences with the upward monotone quantifier *at least 3* and the downward monotone quantifier *at most 3*, as well as with other, presumably equivalent, quantifiers; all of them in 5 different types of models. All sentences were of the form *Q As are B*, where Q is quantifier, A denotes a shape from the set L_1 = {star, circle, square, triangle, arrow, heart} and B denotes a color from the set L_2 = {red, blue, yellow, orange, violet, green}. The following quantifiers (*Qforms*) were considered: the superlative *at most three/ at least three* (*Sup*), the disjunctive *three or fewer than three/ three or more than three* (*Dis*), the comparative *fewer than four / more than two* (*Comp*), the negative comparative *not more than three/ not fewer than three* (*NegComp*) and the basic numeral *three* (*Num*) (Table 3).

Table 3: Quantifiers tested in the experiment

Quantifiers	*superlative*	*comparative*	*disjunctive*	*negative comparative*
Upward monotone	*at least three*	*more than two*	*three or more than three*	*not fewer than three*
Downward monotone	*at most three*	*fewer than four*	*three or fewer than three*	*not more than three*
Bare numeral			*three*	

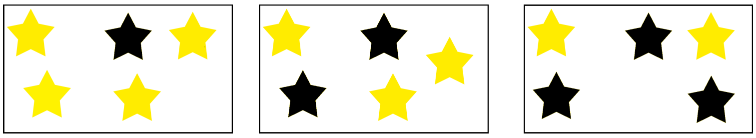
Models were pictures containing objects of various shapes and colors. They were varied with respect to the number of *target* objects: N_T. For the present experiment, where the borderline of truth-conditions N was always equal to 3, N_T was chosen to vary between $N+2$ and $N-2$. With this setup the number of target objects was always at least one and never larger than five, which is the limit of the so-called subitizing range, i.e. range in which immediate accurate number judgements are possible. This gave five *basic categories of models* (*Mforms*) with $N_T = \{1, 2, 3, 4, 5\}$. All upward monotone quantifiers were true in models with 3, 4 or 5 target objects and false in models with 1 or 2 target objects, whereas all downward monotone quantifiers were true in models with 1, 2, 3 target objects, and false in models with 4 or 5 target objects. (See Table 4 and Figure 6 for example pictures).

In each trial a presentation of a sentence was followed by a presentation of a picture. The time for presenting sentences was calculated based on their length to mirror the natural reading time. Since all the equivalent forms were evaluated in isomorphic (with respect to cardinalities) pictures it can be assumed that all the reaction time differences result from the processing differences that are linked to the specific *Qforms*. Pictures were presented until a response was given but not longer than for 10 seconds. Subjects were asked to respond with "yes" or "no" to whether a presented sentence was true about the subsequent picture. Half of the subjects had "yes" on their right-hand side, half on their left-hand side. They were instructed to respond accurately, but also as fast as possible. Each *Qform* was evaluated in a given model type three times. For each trial with a fixed quantifier and a model, A and B were selected randomly from the predefined lists L_1 and L_2 by a computer program. After the sentence content was determined the picture was generated to match the sentence and the model. The elements were randomly scattered on the white screen. The order of the questions in the experiment was also randomized. There were 216 experimental trials: 2 (monotonicity) × 4 (*Qform*) × 8 (*Mform*) × 3 (reapeats) + 3 (repeats) × 8 (*Mform*) [bare numeral form]. As fillers additional 30 sentences with other quantifiers were used: *two*, *four*, *some*, *all* and *no*. This amounts to 246 trials.

Table 4: Logical truth-value of quantifiers of a given monotonicity in a given model

Model (number of target objects)	1	2	3	4	5
Upward monotone *Qforms*	F	F	T	T	T
Downward monotone *Qforms*	T	T	T	F	F

Figure 6: At most three stars are yellow/ Fewer than four...



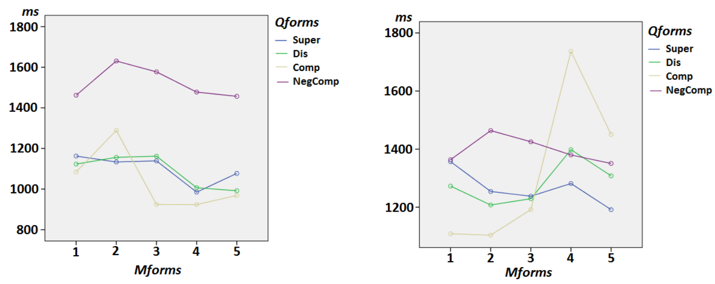
(a) False (b) True and Infelicitous (c) True

Results

I now summarize the most important findings from the study. The analysis of subjects' accuracy revealed that although subjects generally accepted the downward monotone superlative quantifier (*at most 3*) (mean accuracy in each model over 2.6, value range: $0-3$), they made significantly fewer mistakes in models with 3 target objects ("felicitous models") compared to models with 1 target object ($p = .001$), and compared to models with 2 target objects ($p = .02$). This effect supports the hypothesis that *at most three* is considered semantically consistent with models in which there are only 1 or 2 target objects, even though subjects tend to make more mistakes in such cases, which might be a result of the pragmatically infelicitous use of this quantifier.

With respect to the reaction time results, a repeated measure analysis of variance proved that both monotonicity (*Mon*) and quantifier form (*Qform*), had a significant effect on subjects' time taken to respond correctly ($p < .001$). All interactions between our three factors (monotonicity, quantifier, model) turned out to be significant ($p < .001$). Pairwise comparisons for the *Qform* showed, however, that only the negative comparative form was evaluated significantly slower than every other form ($p < .001$), but the comparisons between other forms were not significant.

Figure 7: Mean reaction time for each *Qform* and *Mform*



(a) *Upward monotone forms* (b) *Downward monotone forms*

The main research hypothesis was that the differences between processing of superlative and comparative quantifiers (and presumably also other equivalent forms) depend on the pragmatic felicity of the use of those quantifiers in the given models. In order to investigate this hypothesis, a repeated measure analysis of variance was conducted for each *Mform* and monotonicity separately. The results support our hypothesis and allow the conclusion that the differences in processing between various quantifiers can be explained by extra pragmatic processes that are triggered only in some models and only for some quantifier forms.

The results were especially convincing in the case of the downward monotone quantifiers. Whereas in the "infelicitous models" (1 or 2 target objects) the superlative quantifier (*at most three*) was evaluated significantly slower than the comparative one (*fewer than four*) ($p < .008$), in the "felicitous models" (3 target objects) there was no significant difference in the reaction time taken to verify these two forms.

It is also worth noting that the time of evaluating the disjunctive and the superlative quantifier forms (of both monotonicities) did not differ significantly in any of the models, which suggests that the processing costs of these two forms are comparable. This result supports the proposed theoretical model, in which the disjunctive and the superlative forms share the same pragmatic properties. For more details see Figures 7 and Tables 5 and 6.

Table 5: Repeated measure analysis results (with effect sizes) for upward monotone quantifiers, for each model. Pairwise comparisons after Bonferonni correction.

Model	1	2	3	4	5
Qform	F(2.194,120.680) =17.227 $\eta^2 = .239$ $p < .001$	F(2.194,121.661) =18.201 $\eta^2 = .256$ $p < .001$	F(2.03,109.598) =29.875 $\eta^2 = .356$ $p < .001$	F(1.754,96.462) =23.374 $\eta^2 = .298$ $p < .001$	F(2.081,114.438) =26.222 $\eta^2 = .323$ $p < .001$
sup vs. comp	–	$p = .037$	$p < .004$	–	–
sup vs. dis	–	–	–	–	–
sup vs. neg comp	$p < .001$	$p < .001$	$p < .001$	$p < .001$	$p < .001$
comp vs. neg comp	$p < .001$	$p = .005$	$p < .001$	$p < .001$	$p < .001$
comp vs. dis	–	–	$p = .001$	–	–
dis vs. neg comp	$p < .001$	$p < .001$	$p < .001$	$p < .001$	$p < .001$

Table 6: Repeated measure analysis results (with effect sizes) for downward monotone quantifiers, for each model. Pairwise comparisons after Bonferonni correction)

Model	1	2	3	4	5
Qform	F(3,159)= = 7.462 $\eta^2 = .123$ $p < .001$	F(2.195,120.713)= = 11.545 $\eta^2 = .173$ $p < .001$	F(1.54,83.134)= = 13.781 $\eta^2 = .203$ $p < .001$	F(2.445,134.49)= =16.247 $\eta^2 = .228$ $p < .001$	F(2.513,135.729)= =3.88 $\eta^2 = .067$ $p = .015$
sup vs. comp	$p < .001$	$p = .007$	–	$p < .001$	$p = .052$
sup vs. dis	–	–	–	–	–
sup vs. neg comp	–	—	$p = .002$	$p = .002$	–
comp vs. neg comp	$p = .001$	$p < .001$	$p = .001$	$p = .001$	–
comp vs. dis	–	–	—	$p < .001$	–
dis vs. neg comp	–	$p = .008$	$p = .001$	$p < .001$	–

The results for the upward monotone quantifiers diverge, however, from the expected pattern. In the models with 3 target objects, opposite to the results obtained for the downward monotone quantifiers, there was a significant difference between the comparative and the other quantifiers, including the most relevant superlative quantifier: the comparative quantifier was evaluated significantly faster ($p < .004$). In the models with 4 or 5 target objects, there was, however, no difference between the processing time of the superlative and the comparative quantifiers. This suggests that the processing mechanisms for the upward monotone quantifiers differ from the mechanisms for the downward monotone quantifiers. It is nevertheless remarkable that there was a significant difference between the processing time of the superlative and the comparative quantifiers only in the models with 3 target objects, but not in the other models in which these quantifiers can be evaluated as true. This result differs again from the data so far reported: the superlative quantifiers require indeed more processing time than the comparative ones but only in some models and not generally.

It is worth paying attention to the fact that both the upward and the downward monotone comparative quantifiers were evaluated significantly slower than the other quantifiers in models in which the number of target objects was equal n, i.e. the numeral mentioned in the quantifier. Thus, *fewer than four* was evaluated slower than the other downward monotone quantifiers in the models with 4 target objects, and *more than two* was evaluated slower than the other upward monotone quantifiers in the models with 2 target objects. This result can be related to the fact that, in the case of comparative quantifiers, the numeral n mentioned in the quantifier is not a part of the quantifier's truth conditions, thus in models in with n target objects, the sentence with the comparative quantifier is false and has to be rejected. In contrast, in the case of all the other considered quantifiers: superlative, disjunctive and negative comparative, the numeral n mentioned in the quantifier belongs to the truth-conditions. Hence, minimal model to reject sentences with those quantifiers must have $n + 1/n - 1$ target elements for downward or upward quantifiers respectively.

The results of this experiment provide evidence that the processing of natural-language quantifiers (comparative, superlative, disjunctive), depends not only on their linguistic form, but also highly on the model in which they are verified/ falsified. This goes against the results reported so far that the superlative quantifiers are generally processed slower than the comparative quantifiers. In contrast, I show that they are processed slower only in some specific types of models. The picture of the processing load that is linked to specific linguistic forms of quantifiers turns out to be more complex than the literature has suggested so far.

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Quantifier domain restrictions and cross-contextual assessments of truth value

Isidora Stojanovic

The initial puzzle

Imagine a dinner taking place during a logic conference, at which Tarek says to his neighbor:

(1) Everyone is a logician.

Suppose furthermore that there are no non-logicians at that dinner. Then (1) is intuitively true – or, to put it more weakly, (1) has a reading on which it is intuitively true. But now consider the following, uttered right after (1):

(2) Kylie Minogue is a logician.

By the rule of universal instantiation, (2) logically follows from (1). Yet (2) is false (on any reading). Thus we get something of a puzzle: on the assumption that (1) is true and that it logically entails (2), (2) must come out true, yet (2) comes out false.

A closely related puzzle involves a structural rule that allows adding idle premises to an already valid inference, and ensures, in particular, that $p, q \vdash p$ is a logically valid inference (since the idle premise q has been added to the Identity axiom $p \vdash p$). Consider:

(3) Everyone is a logician.

(4) Kylie Minogue isn't a logician.

(5) Therefore, everyone is a logician.

Suppose again that (3) is uttered in the context of a dinner party at which no non-logician is present. Then (3) will be heard as true, and (4) is also true, but (5), which consists of the same sentence as (3) but is said after (4), will be heard as false.

The mainstream solution

The dominant approach to contextual restrictions on quantifier domains posits some sort of implicit argument, or "hidden indexical", which accounts for the fact that a sentence such as (1) can take different truth values in different contexts (cf. e.g. Westerståhl 1985, Stanley and Williamson 1995, Stanley and Szabó 2000, Peters and Westerståhl 2006, to mention only a few). Let us restate our initial puzzle more explicitly as follows:

(i) "Everyone is a logician. Therefore, Kylie Minogue is a logician" is an instance of the rule of universal instantiation, $\forall x Fx \vdash F[x/b]$, where F may be any predicate, and b any individual constant; it is therefore a logically valid inference.

(ii) (1) is intuitively true. Hence, if S1 is the formal representation of the sentence in (1), and if $i_1 = (c_1, s_1)$ where c_1 stands for the context relevant to the interpretation of (1) (presumably, the logic dinner) and s1 stands for the circumstances (world, time, etc.) relevant to the determination of (1)'s truth value (presumably, the actual world, the time at which (1) is said, etc.), then we ought to have $[\![S_1]\!]^i_1 =$ True.

(iii) (2) is intuitively false. Hence, if S_2 is the formal representation of the sentence in (2), and if $i_2 = (c2, s2)$ where c_2 stands for the context relevant for the interpretation of (2) and s_2 stands for the circumstances relevant to determining the truth of (2), then we ought to have $[\![S_2]\!]^i_2 =$ False.

(iv) (1) and (2) are uttered in the same context, and the circumstances relevant to determining their truth values are the same, too. In other words, $i_1 = i_2$.

Claim 1. Taken together, i, ii, iii and iv lead to contradiction.

On the mainstream view, the contradiction is avoided by giving up the assumption (i). In a nutshell, the formal representation associated with the sentence in (1) will not be $\forall x \text{Philosopher}(x)$. To begin with, 'everyone' is treated as a restricted quantifier, where the restrictor consists at least of the sortal expressed by the suffix '-one', which, for simplicity, let us take to be the predicate of being human. Then, the formal representation that generalized quantifier theory provides for (1) becomes:

$$[\forall x : \text{Human}(x)]\ \text{Logician}(x).$$

The crucial further move is to posit a hidden argument in the restricting clause of the quantifier, consisting of a 2nd order variable π that takes predicates as its values:

$$[\forall x : \mathrm{Human}(x) \wedge \pi(x)]\ \mathrm{Logician}(x).$$

In the context of (1), π will presumably take as its value some complex predicate along the lines of '(to be) at this dinner', and since Kylie Minogue does not satisfy the predicate, one of the premises in the inference is false, so the inference may remain valid with its conclusion being false.

The problem of cross-contextual assessments

While the mainstream view handles the puzzle fine enough, it falls short of extending to slightly more complicated puzzles. Thus consider a variation on our working example. At a dinner attended by logicians only, Tarek says:

(6) Everyone is a logician.

When Kylie Minogue joins the party, Marula says, in reference to Tarek's utterance of (6):

(7) That's no longer true.

(7') What you said isn't true any more.

While people were inclined to judge the utterance in (6) as being true, at the time when it was made, people are also inclined to think that it is possible to later reassess it as being false, as is done in (7) or (7').[1] In other words, (7) and (7'), though presumably not true *simpliciter*, both have readings on which they are true, readings that can be enhanced either by lexical material (such as the introduction of 'no longer') or by pragmatic factors. The question then becomes how to account for those readings.

While people readily say things such as (7) or (7'), it is not obvious what such claims really amount and what the semantics of sentences such as "That's not true" is. Part of this indeterminacy might stem from an indeterminacy as to what the word 'that' refers to on such uses. In turn, this raises the theoretical question of what can be assigned a truth value, since 'that' will presumably refer to something that can be said to be true or false. This is a vexed question that we shall not try to settle here, but let me point to the most obvious ways of "disambiguating" the sentence in (7):[2]

i: the utterance itself of (6) used to be true, e.g. at the time of (6), but is no longer true at the time of (7).

ii: the proposition, or the semantic content, expressed by (6) used to be true, e.g. at the time of (6), but is no longer true at the time of (7).

iii: the sentence in (6) used to be true, e.g. when it was interpreted in the context of (6), but is no longer true when reinterpreted in the context of (7).

The upshot is that this will give us various ways of analyzing sentences such as (7) or (7'), in which a person reassesses a previously made claim as having a different truth value than it

[1] To be sure, the data are subtle, and in particular, a note about the presence of 'no longer' in (7) and of 'any more' in (7') is in order. Although the presence of such phrases is not crucial to the argument, they do enhance the reading on which (12) and (12') come out true.

[2] I am not claiming that i, ii and iii exhaust the ways in which (7) may be analyzed. If one takes 'that' to be a genuine demonstrative (in the spirit of Davidson 1968), then it should be possible for this demonstrative to stand for any utterance, proposition or sentence (depending on what one allows among the things that may be said to be true or false) that is sufficiently salient in the context of (7).

appeared to have at the time when it was made. What is more, I believe that these different analyses can all be made to work, albeit with some stretches and strains. The weight of arguments coming from considerations about reports such as (7) or (7') will not be overwhelming. Still, considerations of the sort are to be taken into account, hence in the remainder of this section, I shall shed doubt on the account that the mainstream view gives up. For the sake of concision, let me move straight to what I take to be the most plausible account within the mainstream view. Recall that the formal rendering of (6) would go as follows:

$$[\forall x : \text{Human}(x) \wedge \pi(x)] \text{ Logician}(x).$$

The context of (6) will then supply a value to π, and plausibly that will be a predicate such as 'belongs to this dinner party'. The proposition that (6), under this assignment of value to π, will express in its context, will be a temporal proposition; that is, one that can take different truth values at different times. The proposition at stake will be true when evaluated at the time of (6), since at that time everyone present at the dinner is (or was) a logician, and it will false when evaluated at the time of (7), after Kylie Minogue joined the dinner party, since at that time there are individuals who satisfy the predicates in the restriction clause who are not logicians.

Regardless of its overall plausibility, this account stumbles upon the following problem. The context of (6) will supply (at least) two candidate values for the predicate variable π: the temporally neutral predicate 'belongs to this dinner party' and the temporally specific predicate '*now* belongs to this dinner party', distinguished from the former in that it contains the temporal indexical 'now'. However, neither the speaker of (6) nor the addressee will have any principled reason to prefer either value assignment to the other, since the choice between the two will have no impact on the truth value of (6). But now, note that the true reading of (7), in the scenario envisaged, is only possible if the value assigned to π is the temporally neutral predicate. For, were it the temporally specific predicate, then the only way for (7) to be true is that someone actually belonging to the contextually restricted domain of (6) should cease being a logician by the time of (7). In turn, the ambiguity in (7) that we started with, and, in particular, the true reading, would then have to depend on the issue of which value the context of (6) *actually* supplied to π. Yet we have seen that there are two equivalent values, so to speak, the choice between which makes no impact on the truth value of (6) and need not be resolved in the context of (6).

The proposed solution

Our proposal, a version of which has been given in Stojanovic (2012), draws on the idea (from Kaplan (1989) and widely accepted nowadays) that context, *qua* a parameter in a formal semantic framework that truth value assignment is relative to, has two roles. One role is to supply semantic values for indexicals as well as implicit arguments (whether or not we allow for implicit arguments for quantifier domain restriction); the other role is to supply values for the parameters of circumstances of evaluation (at which sentences are evaluated for a truth value). This makes room for an account of the context-sensitivity of quantifier phrases that does not need to posit any phonetically unrealized arguments. In a nutshell, the proposal is that the sentence in (1) has the same formal counterpart, namely $\forall x : \text{Human}(x)]\ \text{Logician}(x)$, regardless of the context in which it is used. However, it can still take different truth values in different contexts, because its 'content' (to use Kaplan's term), or the domain-neutral proposition that it expresses, may be evaluated at different circumstances: evaluated at those whose domains contain only logicians, it is true, and false otherwise.

Applied to the initial puzzle, the proposal is, then, to reject the assumption (iv): (1) and (2), while interpreted with respect to the same context, are evaluated at different domains. That is to say, we have $i_1 \neq i_2$ because $s_1 \neq s_2$, although $c_1 = c_2$.

Let me briefly wrap up by showing how this proposal handles our modified puzzle, which involves a cross-contextual assessment of truth value. Recall that the gist of the proposal is that a sentence, even when used on a particular occasion, can still take different truth values depending on which domain(s) of quantification we evaluate it at. The same will go for (7): even when uttered on a particular occasion, e.g. just as Kylie Minogue has joined the logicians' dinner party, it will not have a fixed truth value, but will only receive one when evaluated at particular circumstances, specifying the world, the time and the domain. If the first conjunct, viz. that it was true in the past that everyone is a logician, is evaluated at the domain restricted to the dinner party as of the context of (6), and if the second conjunct, viz. that it is now not true that everyone is a logician, is evaluated at the larger domain of (7) including Kylie Minogue, then the outcome is value True. But if both conjuncts are evaluated at the domain of (7), or if both are evaluated at the domain of (6), then the outcome will be value False. Thus a clear advantage of the present proposal over the mainstream view is that it can straightforwardly predict the ambiguity in sentences like (7) and (7'). This being said, it remains true that the data regarding cross-contextual assessments are subtle and involve linguistic constructions whose syntax, semantics and pragmatics are not well understood. A more thorough defense of the proposal outlined here thus needs to be postponed for a future research.

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Censors for Boolean Description Logic

Thomas Studer and Johannes Werner

Controlled query evaluation is a recent approach to guarantee privacy preserving query answering in database and knowledge base systems. The general idea is that a database may not be queried directly but only via a so-called censor that protects sensitive information. In case the censor has to answer a critical query, that is a query that would leak sensitive information if answered truthfully, it can choose between two options to keep the secret. The censor can refuse to answer the query [7] or it can give an incorrect answer [6], that is it lies. The framework of controlled query evaluation has been applied for a variety of data models and control mechanisms, see for instance [2, 3, 4, 5]. However, it has not been studied for description logic knowledge bases.

In our work, we extend controlled query evaluation so that it also applies to Boolean description logics [1]. We present precise formal definition of all relevant notions and study both truthful and lying censors.

We start with recalling some basic notions of description logic and fixing the notation. The language $\mathcal{L}_{\mathcal{ALC}}$ consists of $\mathcal{ALC}$ subsumptions, which are of the form $C \sqsubseteq D$, meaning that the interpretation of the concept C is a subset of the interpretation of the concept D, and Boolean combinations of subsumptions. As usual, the symbol $\models$ denotes the semantic consequence relation.

A knowledge base $\mathcal{CK}$ is a set of $\mathcal{L}_{\mathcal{ALC}}$-formulae. A query q is given by an $\mathcal{L}_{\mathcal{ALC}}$-formula. The evaluation function

$$\mathsf{eval} : \wp(\mathcal{L}_{\mathcal{ALC}}) \times \mathcal{L}_{\mathcal{ALC}} \to \{t, f, u\}$$

evaluates a query q over a knowledge base $\mathcal{CK}$ as follows

$$\mathsf{eval}(\mathcal{CK}, q) := \begin{cases} t & \text{if } \mathcal{KB} \models \varphi \\ f & \text{if } \mathcal{KB} \models \neg\varphi \text{ and } \mathcal{KB} \text{ is satisfiable} \\ u & \text{otherwise} . \end{cases}$$

A privacy configuration is a triple that consists of the attacker's knowledge, the knowledge-base that can be queried, and the set of secrets that should not be revealed to the attacker.

Definition 29 (Privacy Configuration)**.** A *privacy configuration* is a tripel $\mathcal{PC} = (\mathcal{CK}, \mathcal{AK}, \mathcal{SK}) \in \wp(\mathcal{L}_{\mathcal{ALC}}) \times \wp(\mathcal{L}_{\mathcal{ALC}}) \times \wp(\mathcal{L}_{\mathcal{ALC}})$ s.t.

PCA-A) $\mathcal{CK} \models \mathcal{AK}$ (Truthful start).

PCA-B) $\mathcal{CK}$ is satisfiable (and hence so is $\mathcal{AK}$) (Consistency).

PCA-C) $\mathcal{AK} \not\models \sigma$ for each $\sigma \in \mathcal{SK}$ (Hidden secrets).

A censor provides an answering function for each privacy configuration. This answering function may distort the answer to a query if the correct answer would leak sensitive information.

Definition 30 (Censor)**.** A *censor* is a mapping assigning an answering function $\mathrm{Cens}_{(\mathcal{CK},\mathcal{AK},\mathcal{SK})} : \mathcal{L}_{\mathcal{ALC}}^{\mathbb{N}} \to \mathbb{A}^{\mathbb{N}}$ to each given privacy configuration. A sequence $\mathbf{q} \in \mathcal{L}_{\mathcal{ALC}}^{\mathbb{N}}$ is called a *query-sequence*. The set $\mathbb{A}$ contains the potential answers a censor might give. In this paper only $\{t, f, u, r\}$ and $\{t, f, u\}$ are possible choices for $\mathbb{A}$.

We need the epistemic content of an answer to a knowledge base query. To this end we introduce two prefixes, $\Box$ (necessity) and $\Diamond$ (possibility), which are used as markers. Marked formulae cannot be combined any further.

Definition 31 (Content)**.** Let $\psi \in \mathcal{L}_{\mathcal{ALC}}$ and $a \in \mathbb{A} \subseteq \{t, f, u, r\}$. The *content* of a as response to ψ is given by

$$\mathrm{Cont}(\psi, a) = \begin{cases} \{\Box\psi\} & \text{if } \; a = t \\ \{\Box\neg\psi\} & \text{if } \; a = f \\ \{\Diamond\psi, \Diamond\neg\psi\} & \text{if } \; a = u \\ \varnothing & \text{if } \; a = r \end{cases}$$

We gather marked formulae in sets, called *clouds*, which represent the knowledge of the querying agent in the view of the censor. A cloud is satisfied by a set of $\mathcal{ALC}$-interpretations if and only if all formulae prefixed with a $\Box$ are satisfied in all of the interpretations and for each formula prefixed with a $\Diamond$, there is at least one interpretations that satisfies it.

A censor is credible if its answers do not contradict each other, that is if they provide a consistent view to the attacker. A censor is effective if it keeps all secrets. Continuity is a technical notion saying that the answers a censor provides only depend on previous queries and answers (but not on future ones).

Definition 32. Let $\mathcal{PC} = (\mathcal{CK}, \mathcal{AK}, \mathcal{SK})$ be a privacy configuration.
We define the *state cloud* wrt. a query-sequence $\mathbf{q} \in \mathcal{L}_{\mathcal{ALC}}^{\mathbb{N}}$ at stage n by

$$\mathscr{SC}_{\mathcal{PC},\mathbf{q}}(n) := \bigcup_{\varphi \in \mathcal{AK}} \mathrm{Cont}(\varphi, t) \cup \bigcup_{i=1}^{n} \mathrm{Cont}(q_i, a_i),$$

where $\mathbf{a} := \mathrm{Cens}_{(\mathcal{CK},\mathcal{AK},\mathcal{SK})}(\mathbf{q})$. A censor Cens is called

- *credible* for $\mathcal{PC}$ iff for every sequence $\mathbf{q} \in \mathcal{L}_{\mathcal{ALC}}^{\mathbb{N}}$ and every $n \in \mathbb{N}$, it holds

$$\mathscr{SC}_{\mathcal{PC},\mathbf{q}}(n) \text{ is satisfiable} \qquad (C^n_{\mathcal{PC},\mathbf{q}})$$

- *effective* for $\mathcal{PC}$ iff for all sequences $\mathbf{q} \in \mathcal{L}_{\mathcal{ALC}}^{\mathbb{N}}$ and every $n \in \mathbb{N}$ it holds

$$\mathscr{SC}_{\mathcal{PC},\mathbf{q}}(n) \not\models \Box\sigma \text{ for every } \sigma \in \mathcal{SK} \qquad (E^n_{\mathcal{PC},\mathbf{q}})$$

  (i.e. no secret is semantically implied by a state cloud)

- *continuous* for $\mathcal{PC}$ iff for all sequences $\mathbf{q}, \mathbf{r} \in \mathcal{L}_{\mathcal{ALC}}^{\mathbb{N}}$ and all $n \in \mathbb{N}$, it is

$$\mathbf{q}|_n = \mathbf{r}|_n \to \mathrm{Cens}_{\mathcal{PC}}(\mathbf{q})|_n = \mathrm{Cens}_{\mathcal{PC}}(\mathbf{r})|_n \quad ,$$

  where $\mathbf{a}|_n$ denotes the initial segment of $\mathbf{a}$ of length n, i.e. $(a_1, \ldots, a_n)$.

A censor is called credible [effective, continuous], if it is credible [effective, continuous] for every privacy configuration.

A censor is truthful if it does never lie. But it may refuse to answer a query. A censor is lying if it may give incorrect answers.

Definition 33 (Truthful)**.** The censor Cens is called *truthful* iff for all privacy configurations, for all question sequences $\mathbf{q}$ and for all i:

$$a_i \in \{r, \mathsf{eval}(\mathcal{CK}, q_i)\} \ ,$$

where $\mathbf{a} := \mathrm{Cens}(\mathbf{q})$.
A censor that is not truthful is called *lying*.

A censor is minimally invasive if it distorts an answer only if otherwise a secret would be leaked.

Definition 34 (minimally invasive)**.** Let Cens be effective and credible. Cens is called *minimally invasive* iff whenever $a_i \neq \mathsf{eval}(\mathcal{CK}, q_i)$ replacing a_i by $\mathsf{eval}(\mathcal{CK}, q_i)$ would lead to a violation of either effectiveness or credibility.

A censor is repudiating if for any sequence of queries, there is an alternate knowledge-base in which all secrets are false and which, given as input to the censor, would produce the same answers as the ones computed from the actual knowledge-base. Hence a repudiating censor can plausibly deny all secrets even if the algorithm of the censor is known to the attacker.

Definition 35 (Repudiation)**.** A censor Cens is called *repudiating* if for each privacy configuration $(\mathcal{CK}, \mathcal{AK}, \mathcal{SK})$ and each query sequence $\mathbf{q}$ there are knowledge-bases $\mathcal{KB}_i$, s.t.

R-A) $\mathrm{Cens}_{\mathcal{CK},\mathcal{AK},\mathcal{SK}}(\mathbf{q})|_n = \mathrm{Cens}_{\mathcal{KB}_n,\mathcal{AK},\mathcal{SK}}(\mathbf{q})|_n$,

R-B) for all $i \in \mathbb{N}$ and all $\sigma \in \mathcal{SK} : \mathcal{KB}_i \not\models \sigma$,

R-C) for all i $(\mathcal{KB}_i, \mathcal{AK}, \mathcal{SK})$ is a privacy configuration.

We establish two main theorems. One about truthful censors and one about lying, but not refusing, censors.

Theorem 1 (Truthful censors)**.** *1. There exists a censor that is credible, effective, truthful, continuous, and repudiating but not minimally invasive;*

2. There exists a censor that is credible, effective, truthful, continuous, and minimally invasive but not repudiating.

3. The above censors are optimal. Namely, a continuous truthful censor cannot simultaneously be credible, minimally invasive and repudiating.

Theorem 2 (Lying censors)**.** *There is a censor that is credible, effective, lying (but not refusing), continuous, repudiating, and minimally invasive.*

Hence, in contrast to truthful censor, we can simultaneously have all desired properties for lying censors. Therefore, there is no need to consider censors that combine both lying and refusing.

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Breaking the curse of deontic logic

Vladimír Svoboda

G.H. von Wright, the founding father of deontic logic, noted in the early 1990s that the branch of logic that originated, in its modern form, in the early 1950s "has remained something of a problem child in the family of logical theories" (von Wright 1991, p. 256). Unfortunately, logical theories subsumed under the term "deontic logic" have kept their somewhat problematic reputation even in the first decades of the new millennium.

In my paper, I argue that the problem stems to a large extent from the fact that theories subsumed under the heading "deontic logic" commonly have rather unclearly defined goals and that their creators tend to conflate perspectives that are discordant. Thus, if we search for a remedy to this unfortunate situation we should pay special attention to the proper formulation of the questions that particular theories from the area of deontic logic should answer and, more generally, to the philosophical foundation of the research.

If we focus on the question of the delineation of deontic logic, we will quickly find that the task is rather complex. Deontic logic, often characterized as the field of logic that is concerned with obligation, permission and related concepts, namely with the contribution that notions like *permitted*, *forbidden*, *obligatory*, *ought*, *must not* or *may* make to what follows from what. Sometimes it is, however, characterized more broadly as the logic of normative reasoning or the logic of prescriptive discourse. In the narrow understanding, deontic logic is just a branch of modal logic that focuses on specific logical features of statements containing modal operators *obligatory*, *permitted*, *forbidden* (O, P, F in the standard notation). But already von Wright pointed out that "deontic sentences in ordinary usage exhibit a characteristic ambiguity. Sometimes they are used as norm-formulations. We shall call this their prescriptive use. Sometimes they are used for making what we called normative statements. We call this their descriptive use. When used descriptively, deontic sentences express what we called norm-propositions" (von Wright 1963, p. 132).

Von Wright decided to "retain the same ambiguity" that is typical for natural language in the formal language as well. He was aware that such a decision is controversial in many respects, but he underestimated, in my view, its drawbacks. Many of his followers who are less cautious and philosophically minded than he was suffer from what could be called "deontic schizophrenia". A typical symptom of their problem is that they admit that deontic sentences interpreted as prescriptions don't have any truth values, but they are ready to work with formulas like $p \to \mathrm{O}q$ or $\mathrm{O}p \vee \mathrm{O}q$ in which expressions lacking (supposedly) truth value are joined by a truth connective with other expressions.

Problems that arise in a number of common systems of deontic logic can be illustrated by the discussions concerning the so called Ross paradox (Ross 1941, Hansen 2006)—a problem which has been the subject of many debates over the past sixty plus years. The debates usually focus on the question of the validity of the inference schema

$$\text{RP} \quad \frac{\mathrm{O}p}{\mathrm{O}(p \vee q)}$$

which seems to be subverted by interpretations like

RP* You should mail this letter.
—————————————
You should mail this letter or burn it.

I want to show that the problems around this paradox can be grasped and, in effect, solved if we properly parcel out the area of logical studies that concern prescriptive discourse. I also suggest that normative language games proposed by David Lewis together with his conception of scorekeeping in the games can provide a suitable starting point for the parceling (c.f. Lewis 1979a, Lewis 1979b).

I also propose a language game inspired by Lewis' proposal. The only players in the game are *the Prescriber*, *the Doer* and *the Kibitzer*. The Prescriber's moves consist in issuing commands and permissions to the Doer, whose moves consist in making what the Prescriber requires. The Kibitzer's moves are his descriptions of the normative situation. The steps of the game are indexed by the order in which the Prescriber issues the prescriptions. Situations (or possible worlds) conforming to the Prescriber's commands and permissions together create *the sphere of permissibility* [SP]. At the start of the game, the sphere of permissibility does not differ from *the sphere of accessibility*, i.e. the space of all possible worlds that come into consideration as alternatives to the actual world.

It is quite clear that the roles of the players determine what kind of moves are in their repertoire. The Prescriber addresses the Doer only with sentences that are to be interpreted *prescriptively.* To manifest that clearly, we can suppose that he only uses sentences in the imperative mood and permissive sentences employing the phrase "you may...". The Kibitzer, on the other hand, has in his repertoire only sentences *describing* the normative situation, i.e. statements to the effect what the Doer is obliged (must), is forbidden (must not) or is allowed (may) to do. (He is not in position to issue any commands or give his own advice.)

Using the perspective of the game we can distinguish different theories that can be developed within deontic logic. They differ a) in their focus on different kinds of moves in the language game, b) in their conceiving the language game either as static (all prescriptions are issued simultaneously) or as dynamic (typically new prescriptions 'surpass' old ones), and c) in their explanatory ambitions. Classifications of the theories from the area can, of course, be more or less fine grained. Here I propose a classification that divides theories falling within the scope of deontic logic into five different categories:

DL1 Logical study of the language of the Prescriber conceived as static

DL2 Logical study of the language of the Kibitzer conceived as static

DL3 Logical study of the language of the Prescriber conceived as dynamic

DL4 Logical study of the language of the Kibitzer conceived as dynamic

DL5 Logical study of the principles determining how prescriptions issued by the Prescriber shape the sphere of permissibility.

In my paper, I will explain how the individual theories differ. I will also try to show that if individual systems from the history of deontic logic (that consists to a large extent only of the 'static systems') are classified as belonging to a particular category of this classification we can avoid some controversies and assess their merits and vices in a more qualified way.

I will also argue that old problems like the Ross paradox will appear much more tractable. In particular, I will try to show that in logical systems belonging to categories DL1 and DL3 the inference schema corresponding to RP clearly *is not* a good candidate for a valid inference pattern. The prescriptions that give the addressee (the Doer) a chance to choose between two or more different ways of 'approved action' are indispensable constituents of prescriptive communication. The Prescriber who issues commands like "Mail this letter or burn it!" opens for the Doer the chance to decide in which way he will comply with the obligation. In result the Doer is (implicitly) allowed to mail the letter as well as to burn it (i.e. if the Kibitzer says in the situation to the Doer, "You may mail the letter and you may burn the letter", he describes the normative situation correctly). Thus it is quite clear that RP cannot be accepted as a correct inference pattern in this context—the conclusion of the inference is not valid (in force) whenever the premise is valid (in force).

The situation is different with logical systems belonging to categories DL2 and DL 4. While considering the question whether the sentence of Kibitzer's language, "You ought to (you must) mail the letter or burn it", is entailed by the premise, "You ought to (you must) mail the letter", we should appreciate the fact that Kibitzer's sentences (utterances) of this kind have two somewhat different readings—a weak one and a strong one. It is natural to take the formula $\mathrm{O}(p \vee q)$ as a standard formal representation of the weak reading. In such a case, there is nothing paradoxical on validity of RP. We shouldn't, however, forget that sentences like "You ought to mail the letter or burn it" can be interpreted in a stronger way. Such an interpretation takes them as statements reporting a normative situation in which a certain free-choice prescription has been issued by the Prescriber. The logical form of the normative statement then can be represented by the formula $\mathrm{O}(A \vee B) \wedge \mathrm{P}A \wedge \mathrm{P}B$ or even by the formula $\mathrm{O}(A \vee B) \wedge \mathrm{P}A \wedge \mathrm{P}\neg A \wedge \mathrm{P}B \wedge \mathrm{P}\neg B$ (if we go for extremely strong interpretation).

In the rest of the paper, I will focus on issues that have been neglected by scholars interested in deontic logic (with rare exceptions like Makinson 1999), namely on those issues on which theories belonging to the category DL5 are focused. I will present examples of simple language games in which we encounter specific problems (some of them are reminiscent of problems scrutinized by belief revision theories) and outline a proposal concerning their solution which is based on my previously published work (e.g. Childers-Svoboda 2003, Svoboda 2003).

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Generalized Quantifier Distribution and Semantic Complexity

Camilo Thorne and Jakub Szymanik

Introduction

Quantification is an essential feature of natural languages. It is used to specify the (vague) number or quantity of objects satisfying a certain property. Quantifier expressions are built from *noun phrases* (whether definite or indefinite, names or pronouns) and *determiners* resulting in expressions such as "a subject", "more than half of the men", "the queen of England", "John", "some", "five" or "every".

More recently, interest has arised regarding *semantic complexity*, viz., the complexity of reasoning with (and understanding) natural language quantifiers —a problem of interest for both cognitive science and computational linguistics. One model that has been proposed to study natural language semantic complexity is to consider the computational properties that arise from formal semantic analysis, see e.g., [19, 3, 23].

Generalized Quantifier Theory (see, e.g., [17]) makes it also possible to distinguish *tractable* from *intractable* quantifiers, see, e.g., [4, 25]. Following some work in cognitive science, e.g., [7, 20] one would expect that speakers (due to their restricted cognitive resources) are naturally biased towards low complexity expressions (tractable or lower) [25, 15].

Related work by the first author in [27] shows that, when one considers the satisfiability problem of English sentences (specifically, of its fragments possessing a first-order semantics, see, e.g., [18]), then tractable combinations of first-order English constructs occur exponentially more frequently than intractable (or undecidable) ones.

This paper extends such work to the domain of quantifiers and contributes to the semantic complexity debate by focusing on the verification problem and on how the ensuing computational properties affect their distribution in corpora. Specifically, we show for a selected set of corpora that quantifier distribution is skewed towards tractable quantifiers.

Semantic Complexity of Generalized Quantifiers

Table 7: Base tractable FO, proportional, and aggregate quantifiers studied in this paper. The other quantifiers mentioned in Section (7) can be defined from these by complementation and d uality.

Quantifier	Model Class	D.C.	Example
some	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times \mathbb{D}_{\mathcal{I}}$ and $A^{\mathcal{I}} \cap B^{\mathcal{I}} \neq \emptyset\}$	$\mathbf{AC}^0$	some men are happy
at least k	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times \mathbb{D}_{\mathcal{I}}$ and $\lvert A^{\mathcal{I}} \cap B^{\mathcal{I}}\rvert \geq k\}$	$\mathbf{AC}^0$	at least 5 men are happy
most	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times \mathbb{D}_{\mathcal{I}}$ and $\lvert A^{\mathcal{I}} \cap B^{\mathcal{I}}\rvert > \lvert A^{\mathcal{I}} \setminus B^{\mathcal{I}}\rvert\}$	**L**	most trains are safe
$> p/k$	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times \mathbb{D}_{\mathcal{I}}$ and $\lvert A^{\mathcal{I}} \cap B^{\mathcal{I}}\rvert \geq p \cdot (\lvert A\rvert/k)\}$	**L**	more than $2/3$ of planets are lifeless
total α *of*	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times \mathbb{R}$ and $\mathbf{sum}(\mu_\alpha(A^{\mathcal{I}})) \in B^{\mathcal{I}}\}$	**L**	the total surface of shopping centers is hard to measure
number of	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times \mathbb{N}$ and $\lvert A^{\mathcal{I}}\rvert \in B^{\mathcal{I}}\}$	**L**	the number of weeks in a year is even
α*-est*	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times (\mathbb{R} \cup \mathbb{D}_{\mathcal{I}})$ and $\mu_\alpha(\{d\}) = \mathbf{max}(\mu_\alpha(A^{\mathcal{I}})) \in B^{\mathcal{I}}\}$	**L**	the highest mountain in Peru is the Huascaran
average α *of*	$\{\mathcal{I} \mid (A^{\mathcal{I}}, B^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times \mathbb{R}$ and $\mathbf{avg}(\mu_\alpha(A^{\mathcal{I}})) \in B^{\mathcal{I}}\}$	**L**	the average height of mountains in Peru is 5,000 metres

Generalized quantifier theory models the meaning of determiners with first order (FO) and higher order (HO) logics augmented by generalized quantifiers. We can think about generalized quantifiers as relations between subsets of a given universe. For instance, in a given model $\mathcal{I} = (\mathbb{D}_{\mathcal{I}}, \cdot^{\mathcal{I}})$ the statement "most As are B" says that $|A^{\mathcal{I}} \cap B^{\mathcal{I}}| > |A^{\mathcal{I}} \backslash B^{\mathcal{I}}|$, where $A^{\mathcal{I}}, B^{\mathcal{I}} \subseteq \mathbb{D}_{\mathcal{I}}$. Going a step further we can take a generalized quantifier Q to be a functional relation associating with each model $\mathcal{I}$ a relation between relations on its universe, $\mathbb{D}_{\mathcal{I}}$. This is actually equivalent to their standard definition from model theory, where generalized quantifiers are simply classes of models:

Definition 36. Let $t = (n_1, \ldots, n_k)$ be a k-tuple of positive integers. A *generalized quantifier* of type t is a class Q of models of a vocabulary $\tau_t = \{R_1, \ldots, R_k\}$, such that R_i is n_i-ary for $1 \leqslant i \leqslant k$, and Q is closed under isomorphisms, i.e. if $\mathcal{I} \in Q$ and $\mathcal{I}'$ is isomorphic to $\mathcal{I}$, then $\mathcal{I}' \in Q$.

Each generalized quantifier Q of type t over vocabulary $\tau_t = \{R_1, \ldots, R_k\}$ gives rise to a so-called (FO or HO) *query* $Q(R_1, \ldots, R_k)$, viz., a (FO or HO) formula (possibly grounded or closed) over τ_t, such that $\mathcal{I} \in Q$ iff $\mathcal{I} \models Q(R_1, \ldots, R_k)$, for all interpretations over τ_t (viz., $Q(R_1, \ldots, R_k)$ *expresses* Q). The complexity of a quantifier can be determined by considering the logical verification problem that arises:

Definition 37. The *model checking* problem is the following decision problem. **Input:** A finite model $\mathcal{I}$ over $\tau_t = \{R_1, \ldots, R_k\}$, and a generalized quantifier Q of type t. **Question:** Does $\mathcal{I} \models Q(R_1, \ldots, R_k)$?

When considering (finite) model checking we are interested in its complexity w.r.t. the size of the model, that is, in *data complexity* [10]. The data complexity of model checking induces a partition into *tractable* and *intractable* generalized quantifiers. Respectively: quantifiers, for which model-checking is *at most* **P**, and quantifiers that are exponential, viz., for which model-checking is *at least* **NP**-hard.

Tractable Quantifiers.

First-order. First-order quantifiers Q of type t over $\tau_t = \{R_1, \ldots, R_n\}$ are quantifiers which give rise to FO queries $Q(R_1, \ldots, R_n)$. They are the best known and most thoroughly studied, and also those with the lowest complexity: model checking in FO (with identity) is in $\mathbf{AC}^0$. See Table 7.

Proportional. More interesting are *proportional* quantifiers. The former, studied extensively in the literature, are *most* ("most men") and $> p/k$ ("more than one third of men"), and their relatives.

Aggregate. Less known but equally interesting are *aggregate* quantifiers such as: *total* α *of* or, simply, *sum* ("the total surface of"); *number of* or, simply, *count* ("the number of men"); *average* α *of* or, simply, *avg* ("the average weight"); and α*-est* or, simply, *min* or *max* ("the hottest day", "the hottest summer"). These quantifiers involve computing a (integer, rational or real) number over the relations of the quantifiers via a (second-order) *aggregate function* such as **sum**[1]; α stands for a *metric* (denoted by a *metric adjective*), viz., a function $\mu_\alpha(\cdot)$ mapping relations A to sets (so-called "aggregates") $\mu_\alpha(A)$ of numbers relatively to attribute α over which aggregation is performed. For instance, once such metric might be, e.g., height or weight. See Table 7.

[1]Technically speaking, in order to capture aggregate quantifiers Definition 36 would need to be extended to models equipped with some measure function and numerical sort.

As proportional quantifiers can be verified using the same algorithm, albeit with some minor variations, it follows that both are in **L** [26][2].

Intractable Quantifiers

Intractable quantifiers can be derived from tractable ones via various model-theoretic operations, usually marking the transition from monadic to polyadic quantification. Three such operations have been defined in the literature: branching and Ramseyfication [25] as well as various form of type-lifting from distributive to collective quantification [11]. In the present paper we focus on Ramseyfication, that turns a monadic quantifier of type $(1, 1)$ into a polyadic quantifier of type $(1, 2)$:

Definition 38 (Ramseyfication)**.** Let Q be a quantifier of type $(1, 1)$. The *Ramseyfication* of Q is the following quantifier R_Q of type $(1, 2)$: $R_Q = \{\mathcal{I} \mid (A^{\mathcal{I}}, R^{\mathcal{I}}) \subseteq \mathbb{D}_{\mathcal{I}} \times (\mathbb{D}_{\mathcal{I}} \times \mathbb{D}_{\mathcal{I}})$ s.t. exists $X \subseteq A^{\mathcal{I}}$ s.t. $Q(A^{\mathcal{I}}, X)$ and for all $x, y \in X, (x, y) \in R^{\mathcal{I}}\}$.

Ramseyfication can be conveyed by some English sentences with the reciprocal expression "each other" under a default (strong) interpretation [6, 25, 21]. It intuitively states that the models of the resulting Ramseyfied quantifiers are graphs with connected components. Intractability arises when its application gives rise to a so-called "clique" quantifier: if Q is the quantifier *at leastk* or $> p/k$, then R_Q is **NP**-complete [25]. The downward monotone counterparts, like *at most k* ("at most five men"), $< k$ ("less than 5 men") or $< p/k$ ("fewer than one third of men"), are also intractable.

Notice however that Ramseyfication does not always produce intractable quantifiers. When applied to an "Aristotelian" quantifier (i.e., *some* or *all*) or the "bounded" proportional quantifier *most*, it yields **P** quantifiers [25].

Pattern-based Corpus Analysis

In this section we summarize our analysis regarding the occurence of generalized quantifiers (FO, proportional and reciprocal, aggregate) in English question and sentence corpora. We identified such quantifiers indirectly, via part-of-speech (POS) patterns that approximate their surface forms. We considered Penn Treebank/Brown corpus POSs[3]. We considered: **i.** The base quantifiers from Table 7, plus the FO *all* ("all men", "everybody", "everything", "everyone"), $< k$ and $> k$ ("more than 5 men"), *exacly k* ("5 men", "exactly 3 bridges"); the proportional $> k/100$ ("more than 10% of") and $< p/k$; and the reciprocal *recip* Ramseyfier "each other". **i.** The tractable *most+recip*, *all+recip*, *some+all* Ramsey quantifiers, and the intractable $< k$*+recip*, $> k$*+recip* and $> p/k$*+recip* Ramsey quantifiers.

Due to reasons of space, we will not specify completely our patterns here, but instead illustrate them with examples. To the *some* the pattern "*.* (someone/pn|somebody/pn | something/pn|some/dti|a/at|many/ap) .**" (i.e., a simple regular expression) was associated. To the reciprocal *recip* (i.e., "each other"), the pattern "*.* each/dt other/ap .**" was associated. Finally, for quantifiers such as *some+recip*, we checked for sentences that match *at the same time* the two regular expressions of their constituent quantifiers.

Using such patterns we observed the frequency of **i.** generalized quantifiers, and **i.** Ramseyfied and non-Ramseyfied counting and proportional quantifiers, to see whether such distribution is skewed towards low complexity quantifiers.

[2]Note that $\mathbf{AC}^0 \subseteq \mathbf{L} \subseteq \mathbf{P}$; $\mathbf{AC}^0$ denotes the class of problems solvable via circuits of constant depth, and **L** the class of problems solvable in logarithmic space.

[3]For the POS tagging a a 3-gram tagger, with 2-gram and unigram backoffs, trained over the (POS annotated) Brown corpus, and with 80% accuracy.

Table 8: Corpora used in this study.

Corpus	**Size**	**Domain**	**Type**
Brown	19,741 sent.	Open (news)	Decl.
Geoquery	364 ques.	Geographical	Int.
Clinical ques.	12,189 ques.	Clinical	Int.
TREC 2008	436 ques.	Open	Int.

Table 9: Ramseyfied quantifier (raw) frequencies.

Corpus	$> k+$ *recip*	$> p/k+$ *recip*	*most+* *recip*	*some+* *recip*	*all+* *recip*	$< k+$ *recip*
Brown	1	1	2	2	2	16
TREC 2008	0	0	0	0	0	0
Geoquery	0	0	0	0	0	0
Clinical ques.	0	0	0	0	0	0
total	1	1	2	2	2	16

Corpora. To obtain a representative sample, we considered corpora of multiple domains and with sentences of arbitrary type (declarative and interrogative). We considered: **(i)** a subset (A: press articles) of the Brown corpus[4]; **(ii)** a subset (Geoquery880) of the Geoquery corpus[5]; **(iii)** a corpus of clinical questions[6]; and **(iv)** a sample from the TREC 2008 corpus[7]. Table 8 summarizes their main features.

Power Law Behavior, Skewness and χ^2 Tests. We seeked to infer a power law or Zipfean relation between *quantifier frequency* $fr(Q)$ and *quantifier complexity rank* $rk(Q)$, viz.,

$$fr(Q) = a/rk(Q)^{-b}, \quad (0.0.13)$$

where rank refers to how "easy" Q is (i.e., how low its data complexity is). To approximate the parameters a and b it is customary to run a least squares linear regression, since (0.0.13) is equivalent to a linear model on the log-log scale. The R^2 coefficient measures how well the observations fit the inferred power law equation.

Power laws are exponential, non-normal and skewed distributions where the topmost (w.r.t. rank) 20% outcomes of a variable concentrate 80% of the probability mass or frequency. They are widespread in natural language data [1].

To validate our models, we run a χ^2 test (at $p = 0.01$ significance) w.r.t. a uniform null hypothesis and measured the overall skewness of the distribution.

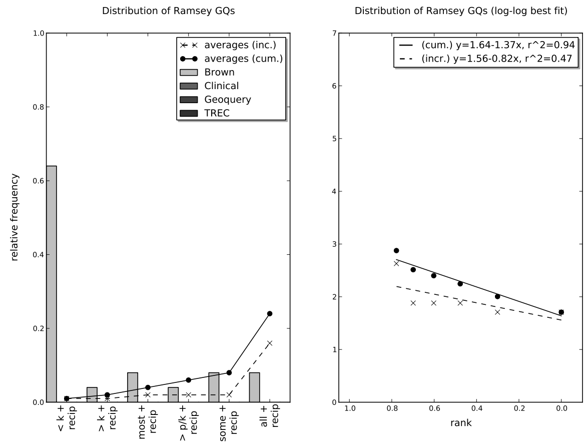

Figure 8: Ramsey Quantifier distribution and log-log power law regression.

Table 10: Summary of test results.

Model/Test	**Recip. GQs**	**GQs**
P. law $fr(Q)$	$36.00/rk(Q)^{-0.82}$	$2.88/rk(Q)^{-4.52}$
Skewness	1.76	1.98
χ^2 value	530.81	$183,815,415,173.11$
p value, df.	1.78, 5	0.0, 13
R^2 coeff.	0.46	0.81

Results and Interpretation

The distributions observed are summarized by Figures 8 and 9. Tables 9 and 4 summarize, resp., the contingency raw frequency tables from which the Figures were generated. The reader will find on the left of each figure the relative average and cumulative frequency plots for the quantifiers considered, and to the right the plots of the log-log regressions. In Table 10 we spell

[4] `http://nltk.googlecode.com/svn/trunk/nltk_data/index.xml`
[5] `http://www.cs.utexas.edu/users/ml/nldata/geoquery.html`
[6] `http://clinques.nlm.nih.gov`
[7] `http://trec.nist.gov`

Table 4: Observed quantifier (raw) frequencies.

Corpus	*at least k*	$< k$	*most*	$> k$	$> p/k$	*recip*	$> k\%$	*sum*	*count*	*avg*	*max/min*	*all*	*exactly k*	*some*
Brown	192	4	1532	540	38	101	2	1	354	17	4368	202587	90811	81693
TREC 2008	0	0	0	0	0	0	0	0	0	0	13	192	490	222
Geoquery	2	0	0	0	0	0	0	0	1	0	18	380	447	660
Clinical ques.	12	0	28	12	0	0	0	0	9	2	889	10712	11629	20780
total	206	4	1560	552	38	101	2	1	364	19	5288	213871	103377	103355

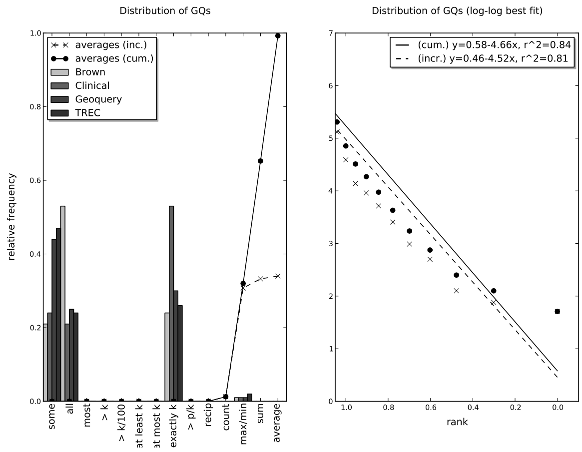

Figure 9: Quantifier distribution and log-log power law regression.

out the power law/Zipfean relations inferred, plus the model validation results.

As expected by both the theory and the similar results regarding language fragments, $\mathbf{AC}^0$ quantifiers occur more frequently than $\mathbf{L}$ (i.e.proportional and aggregate), and their distribution seems to follow a power law (with a high —$R^2 = 0.81$— correlation coefficient) as Figure 9 and Table 10 show. Table 10 shows that this bias is statistically significant: their distribution significantly differs from uniform or random distributions (the null hypothesis rejected by the test), since $p < 0.01$. Their distribution shows also a high measure of skewness. Furthermore, a comparison of Table 9 with Table (7) shows that such tractable quantifiers occur exponentially more often that intractable (viz., intractable Ramseyfied) quantifiers. This fact is further substantiated by the comparatively very low (raw and relative) frequency of *recip* (see again Figure 9 and Table (7)).

Regarding Ramseyfied quantifiers themselves, viz., tractable vs. intractable Ramseyfied quantifiers, the results were less conclusive. The distribution shows a bias towards intractable Ramseyfications, but does not exhibit (see Figure 8) a conclusive power law/Zipfean relation; the mean relative frequency regression yielded a rather low ($R^2 = 0.46$) correlation coefficient. Furthermore, the bias was not significant enough to reject the null hypothesis (i.e., a uniform distribution), even if skewness remains high; see Table 10. These inconclusive results might be due to the general "sparseness" of such quantifiers: as Table (7) shows, Ramseyfications are in general rare in natural language data. It is difficult to gather sufficient data to observe clear statistical trends among them, although the data we collected strongly suggests that they are (altogether) less frequent than non-Ramseyfied $\mathbf{AC}^0$ and $\mathbf{L}$ FO, proportional and aggregate quantifiers.

These results seem however to show altogether—as suggested by [19, 15]—that: even though an everyday fragment of natural language allows some intractable constructions, it consists mostly of tractable expressions. Moreover, the computationally easier the expression the more often it is used in everyday communication.

Conclusions

We have studied the distribution of FO, proportional, aggregate and Ramsey generalized in corpora, to observe if their distribution is correlated to their semantic (data) complexity. In particular, to observe if, as expected by the theory of and related results in semantic complexity, such distribution is significantly skewed towards tractable as opposed to intractable quantifiers.

Our results suggest that this is indeed the case. Furthermore, the distribution of $\mathbf{AC}^0$ vs. $\mathbf{L}$ quantifiers seems to follow a power law. We obtained less conclusive results regarding tractable and intractable Ramsey quantifiers due to their overall "sparseness".

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Exhaustivity through the maxim of relation

Matthijs Westera

Exhaustivity In response to the question in ((1)), the utterance in ((2)), with neutral (i.e., final fall) intonation, is interpreted *exhaustively*: that John doesn't like the other colours.

(1) Which colours (among red, blue, ...) does John like? $(p \vee q \vee (p \wedge q))$

(2) He likes blue. (p) $\rightsquigarrow$ *he doesn't like red* $(\neg q)$

Exhaustivity has usually been considered a prime example of a Gricean [5] *conversational implicature*. But how exactly this conversational implicature comes about has remained a topic of debate (e.g. [10, 1]. For simplicity in what follows, I assume the translations into propositional logic as given in parentheses in ((1),(2)). These rely on, without loss of generality, (i) treating 'which' as an existential quantifier (since replacing ((1)) by 'there are colours that John likes' yields the same pattern), and (ii) assuming that blue and red are the only colours.

The main challenge In order to explain exhaustivity as a conversational implicature, we must explain how it follows from the assumption that the speaker is *cooperative*.

Now, it is relatively easy to derive from cooperativity the *quantity implicature* that the speaker in ((2)) does not have the belief that Mary came to the party (for otherwise the speaker should have said so). The main challenge has been to somehow strengthen this into the claim that the speaker believes that Mary didn't come, a strengthening, from $\neg\Box q$ to $\Box\neg q$, known as the *epistemic step* [10]. All so-called 'Gricean' accounts of exhaustivity take the epistemic step by invoking some version of the assumption that the speaker is *maximally informed* regarding the question under discussion, i.e., that for each alternative, the speaker knows either that it is true or that it is false. This derivation is given in ((3)), where just for brevity I use modal logic (any standard epistemic or doxastic interpretation suffices).

(3)
$$\begin{array}{ll} \neg\Box q & \text{(quantity implicature)} \\ \underline{\Box q \vee \Box\neg q} \;\therefore & \text{(maximal informedness)} \\ \Box\neg q & \text{(exhaustivity)} \end{array}$$

However, the informedness assumption does *not* follow from the cooperativeness assumption [1]. Hence, existing 'Gricean' theories all capture exhaustivity not as a case of conversational implicature, but as a case of *underspecification.* Like postulating lexical ambiguity, this is an unsatisfying last resort. But it is also inaccurate, because exhaustivity is implicated even in the explicit absense of a contextual informedness assumption, as can be seen by prefixing ((1)) with the disclaimer "I'm probably asking the wrong person, but...".

Solution The solution is based on the following intuition: the response ((2)) is *not sufficiently related* to the question ((1)), because it leaves the possibility that Mary will come *unattended.* In comparison, the equally informative response in ((4)) does not leave the Mary-possibility unattended, and does not implicate exhaustivity:

(4) John will come, or John and Mary / At least John will come. $(p \vee (p \wedge q))$[1]

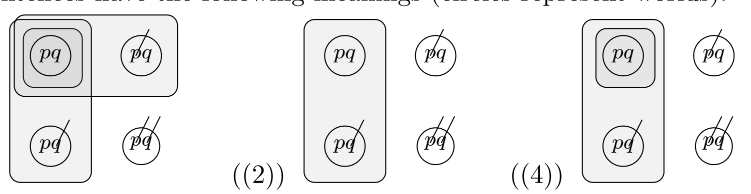
The difference between ((2)) and ((4)) lies in the possibilities they draw attention to, as captured by *attentive semantics* [9], in which meanings are sets of sets of worlds, i.e., sets of classical propositions. The meaning of a formula ϕ, $[\phi]$, is conceived of as the set of possibilities it draws attention to, and its *informative content* $|\phi|$ is given by $\bigcup[\phi]$. For s a set of worlds, a proposition A *restricted to* s is $A_s = \{a \cap s \mid a \in A, a \cap s \neq \varnothing\}$.

Definition 39 (Semantics of relevant fragment)**.**
$[p] = \{\{w \mid w(p) = \mathbf{true}\}\}$ $\quad [\neg\phi] = \{\overline{|\phi|}\}$
$[\phi \wedge \psi] = ([\phi] \cup [\psi])_{|\phi|\cap|\psi|}$ $\quad [\phi \vee \psi] = ([\phi] \cup [\psi])_{|\phi|\cup|\psi|}$ $(= [\phi] \cup [\psi])$

Entailment asks that the premiss is at least as informative *and attentive* as the conclusion:

Definition 40 (Entailment)**.** $[\phi]$ entails $[\psi]$, $[\phi] \models [\psi]$, iff $|\phi| \subseteq |\psi|$ and $[\psi]_{|\phi|} \subseteq [\phi]$

The example sentences have the following meanings (circles represent worlds):



((1)) ((2)) ((4))

To capture the intuition that ((4)), but not ((2)), is sufficiently related to the question, we adopt the following fairly standard maxim of relation:

[1]This translation is in line with [3] on 'at least'.

Definition 41 (Maxim of Relation)**.** A speaker with information state s (set of worlds) should utter ϕ in response to ψ, only if $[\phi]_s \models [\psi]$.

This is a stripped-down version of Roberts [8] *contextual entailment*, which requires entailment restricted to the *common ground*. Since the common ground contains the speaker's knowledge, my maxim logically follows from Roberts' (that is, if we would use the same semantics and entailment).[2]

The richer the semantics, the sparser entailment. In a classical semantics, ((2)) would entail ((1)), and no relation implicatures would be predicted. The same holds for, e.g., basic inquisitive semantics [6], as well as [8]. But with attentive semantics, rich enough to capture attentive content, this is no longer the case. For although ((2)) is more informative than ((1)), it is *less attentive*. On the other hand, ((4)) does entail ((1)), because it is both more informative and at least as attentive.

Therefore, for ((2)) to comply with the maxim of relation (i.e., to entail the question relative to the speaker's information), the speaker must know either:

1. that, if John goes, Mary goes too (because $[p]_{|\neg p \vee q|} = [p \wedge q] \models [p \vee q \vee (p \wedge q)]$); or
2. that, if John goes, Mary won't go (because $[p]_{|\neg p \vee \neg q|} = [p \wedge \neg q] \models [p \vee q \vee (p \wedge q)]$).

Of these, only the second is compatible with the quantity and quality implicatures, as shown in ((5)), and this yields exhaustivity (as before, using modal logic only for brevity):

(5)

$\Box p$	(quality implicature)
$\neg\Box q$	(quantity implicature)
$\underline{\Box(p \rightarrow q) \vee \Box(p \rightarrow \neg q)}$	(relation implicature)
$\Box\neg q$	(exhaustivity)

In sum, when attentive content is taken into account, the relation implicature of ((2)) enables the epistemic step.[3] In comparison, ((4)) complies with the maxim of relation as it is (because it entails ((1))), hence it lacks a relation implicature, and no exhaustivity is implicated.

Discussion The epistemic step is overcome by adopting a semantics that is fine-grained enough to distinguish ((2)) from ((4)). Because only the former implicates exhaustivity, a theory of exhaustivity will need such a fine granularity anyway. As a consequence, entailment automatically becomes sparser, and the maxim of relation, as one finds it in the literature, becomes more powerful. In particular, it becomes sensitive to the possibilities that a response leaves unattended. This result generalizes to other cases of exhaustivity involving disjunction and existential quantification, and it suggests that (i) pragmatic reasoning is indeed sensitive to attentive content and (ii) exhaustivity is a conversational implicature. While I have used attentive semantics, the result generalizes to other (otherwise reasonable) semantics that lack the absorption laws, i.e., that distinguish ((2)) from ((4)) (e.g., [4] *truth-maker semantics*, [2] *unrestricted inquisitive semantics*).

The epistemic step has been the main source of critique against 'Gricean' approaches to exhaustivity, but not the only one. The recent *localist-globalist* debate centers on exhaustivity implicatures that seem to arise from embedded positions [1]:

(6) Which books did every student read?
- Every student read Othello or King Lear. $\rightsquigarrow$ *no student read both.*

[2]We think Roberts' notion in fact conflates relatedness with a strong 'transparency' requirement that *all* dialogue participants know how an utterance is related to a question. A similar comparison could be drawn with [7] *pragmatic answerhood* (restricted to the *hearer*'s information state).

[3]It also shows that Roberts' requirement that all dialogue participants already *know* how an utterance is related, is too strong. In need only be required that the others *can figure out* how the utterance is related.

The difficulty has been to derive the implicature 'no student read both', rather than the weaker 'not every student read both' that is typically generated by 'Gricean' approaches. Here, too, adopting a richer semantics provides a solution: because attentive content more closely resembles sub-sentential structure than informative content, it enables the maxims to see 'inside' sentences to some extent. Indeed, the theory outlined here already predicts the correct exhaustivity for the response in ((6)). Crucially, it improves over existing localist approaches in that the reasoning is still post-compositional/globalist.

Existing approaches to exhaustivity are typically formulated in terms of *alternatives*, treating exhaustivity as an answer to the question 'why did the speaker not give the more informative alternative, $p \wedge q$?'. The reason why these approaches fail to take the epistemic step is that mere ignorance is sufficient reason for not giving the more informative answer. In comparison, the present theory treats exhaustivity as an answer to the question 'why did the speaker not give the more *attentive* alternative, $p \vee (p \wedge q)$?'. Because ignorance is insufficient reason for omitting attentive content, something stronger is implicated: exhaustivity.[4]

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[4]While this rephrasing in terms of alternatives can be insightful for comparisons to existing approaches, it provides only an indirect view on the Gricean reasoning that is going on - the notion of 'more attentive alternative' does not occur in the maxims.