Scalar Implicatures and Implicatures of Irrelevant Answers

Anton Benz

Center for General Linguistics, ZAS, Berlin

Seventh International Tbilisi Symposium on Language, Logic, and Computation
2007

Core Examples

Example 1 (Out of Petrol)

I is standing by an obviously immobilized car and is approached by *E*, after which the following exchange takes place:
I: I am out of patrol.
E: There is a garage round the corner. (*G*)
$+>$ The garage is open. (*H*)

Core Examples

Example 2 (Bus Ticket)

An email was sent to all employees that bus tickets for a joint excursion have been bought and are ready to be picked up. By mistake, no contact person was named. Hence, *I* asks one of the secretaries:

I: Where can I get the bus tickets for the excursion?

E: Ms. Müller is sitting in office 2.07. (M)

$+>$ Bus tickets are available from Ms. Müller. (H)

Outline

The Optimal Answer (OA) Model

Implicatures in the OA Model

Optimal Completion

Section I

The Optimal Answer (OA) Model

General Situation

$$
\begin{array}{ccccc}
\text{Expert } E & & I \text{ decides} & & \text{Evaluation} \\
\text{answers} & & \text{for action} & & \\
\downarrow & & \downarrow & & \downarrow \\
\bullet & \xrightarrow{A} & \bullet & \xrightarrow{a} & \bullet \\
\uparrow & & \uparrow & & \uparrow \\
\text{expectations} & & \text{expectations} & & \text{utility} \\
\text{of } E & & \text{of } I & & \text{function} \\
(\Omega, P_E) & & (\Omega, P_I) & & u(a, v)
\end{array}
$$

General Situation

We consider situations in which:

- A person I, called inquirer, has to solve a decision problem $\langle(\Omega, P), \mathcal{A}, u\rangle$.
- A person E, called expert, provides I with information that helps solving E's decision problem.
- P_E represents E's expectations about Ω at the time when E answers.

Decision Problems

Definition 3

A decision problem is a triple $\langle(\Omega, P), \mathcal{A}, u\rangle$ such that:

- (Ω, P) is a finite probability space,
- $\mathcal{A}$ a finite, non–empty set, and
- $u : \mathcal{A} \times \Omega \longrightarrow I\!R$ a function.

$\mathcal{A}$ is called the action set, and its elements actions. u is called a payoff or utility function.

Support Problems

Definition 4 (Support Problem)

$\sigma = \langle \Omega, P_E, P_I, \mathcal{A}, u \rangle$ is a support problem if

- (Ω, P_E) is a finite probability space, and
- $\langle (\Omega, P_I), \mathcal{A}, u \rangle$ a decision problem.

We assume:

$$\forall X \subseteq \Omega \; P_E(X) = P_I(X|K) \text{ for } K = \{v \in \Omega \mid P_E(v) > 0\}. \quad (1)$$

The Inquirer's Decision Situation

Optimising expected utilities of actions

The expected utility of an action a is defined by:

$$EU(a) = \sum_{v \in \Omega} P(v) \times u(a, v). \tag{2}$$

After learning A, the inquirer optimises the conditional expected utility:

$$EU_I(a|A) = \sum_{v \in \Omega} P_I(v|A) \times u(a, v). \tag{3}$$

Hence, he will choose his actions from the set:

$$\mathcal{B}(A) := \{a \in \mathcal{A} \mid \forall b \in \mathcal{A}\; EU_I(b|A) \leq EU_I(a|A)\}. \tag{4}$$

The Expert's Decision Situation

Optimising expected utilities of answers

If there exists for each answer A a unique optimal choice $a_A \in \mathcal{B}(A)$, then the expected utility of an answer is defined as:

$$EU_E(A) := \sum_{v \in \Omega} P_E(v) \times u(v, a_A) = EU_E(a_A). \tag{5}$$

If the inquier's choice is not unique, then let $h(\,.\,|A)$ be a probability distribution over $\mathcal{B}(A)$ representing the inquirer's choice:

$$h(a|A) > 0 \Rightarrow a \in \mathcal{B}(A). \tag{6}$$

The expert has to optimise:

$$EU_E(A) := \sum_{a \in \mathcal{B}(A)} h(a|A) \times EU_E(a). \tag{7}$$

The Set of Optimal Answers

with representation of Gricean maxims

Maxim of Quality: Be truthful!

This restricts the expert's answers to:

$$Adm_\sigma := \{A \subseteq \Omega \mid P_E(A) = 1\} \tag{8}$$

Hence, the set of optimal answers is provided by:

$$\mathrm{Op}_\sigma := \{A \in Adm_\sigma \mid \forall B \in Adm_\sigma \; EU_E(B) \leq EU_E(A)\}. \tag{9}$$

Section II

Implicatures in the OA Model

Implicatures

[Grice(1989), p. 86]

What is an implicature?

"... what is implicated is what is required that one assume a speaker to think in order to preserve the assumption that he is observing the Cooperative Principle (and perhaps some conversational maxims as well), ..."

Definition of Implicatures

We write $A +\!\!> H$ if *an utterance of A implicates proposition H*.

Definition 5 (Implicature)

Let $\sigma = \langle \Omega, P_E, P_I, \mathcal{A}, u \rangle$ be a given support problem, $\sigma \in \hat{\mathcal{S}} \subseteq \mathcal{S}$. For $A, H \in \mathcal{P}(\Omega)$, $A \in \mathrm{Op}_\sigma$ we define:

$$A +\!\!> H :\Leftrightarrow \ \forall \hat{\sigma} \in [\sigma]_{\hat{\mathcal{S}}} : \ A \in \mathrm{Op}_{\hat{\sigma}} \rightarrow P_E^{\hat{\sigma}}(H) = 1, \qquad (10)$$

with $[\sigma]_{\hat{\mathcal{S}}}$ the set of all support problems that only differ in P_E from σ.

Special Case

Expert knows optimal Action

Let $O(a)$ be the set of all worlds where a is an optimal action:

$$O(a) := \{w \in \Omega \mid \forall b \in \mathcal{A}\, u(a, w) \geq u(b, w)\}. \qquad (11)$$

As a special case, we find:

Lemma 6

Let $\hat{\mathcal{S}}$ be the set of all support problems with $\exists a \in \mathcal{A}\, P_E(O(a)) = 1$. Let $\sigma \in \hat{\mathcal{S}}$, $A, H \subseteq \Omega$, $A \in \mathrm{Op}_\sigma$, and $A^* := \{w \in A \mid P_I(w) > 0\}$. Then:

$$A +\!> H \ \text{iff}\ \forall a \in \mathcal{B}(A) :\ A^* \cap O(a) \subseteq H. \qquad (12)$$

Special Case

Application

Example 7

A: I am out of petrol.

B: There is a garage round the corner. (G)

$+>$ The garage is open. (H)

Ω	G	H	go-to-g	search
w_1	$+$	$+$	1	ε
w_2	$+$	$-$	0	ε
w_3	$-$	$-$	0	ε

- Assumption: $EU_I(\text{go-to-g}|G) > \varepsilon$.
- $O(\text{go-to-g}) = \{w_1\} = H$.
- Hence, $G +> H$.

Section III

Optimal Completion

Special Case

All expert types possible

Let $\hat{\mathcal{S}}$ be such that:

1. $\forall\theta \subseteq \Omega\, \exists\sigma \in \hat{\mathcal{S}} : \theta = \{v \in \Omega \mid P_E^{\sigma}(v) > 0\}$,
2. $A \subsetneq B \Rightarrow EU_E(A) > EU_E(B)$.

Then:

$$\text{For all } H \subseteq \Omega : A +\!> H \textit{ iff } H = A^*.$$

Consequences

- Implicatures arise if not all speaker's information states are possible.
- Implicatures are only defined for

$$\{F \mid \exists\sigma\, F \in \mathrm{Op}_\sigma\}$$

Question

Are there natural principles that allow to generate implicatures for Fs not in $\{F \mid \exists\sigma\, F \in \mathrm{Op}_\sigma\}$?

An Example

Schalar Implicatures

1. All of the boys came to the party. ($F_{\forall}$)
2. Some of the boys came to the party. ($F_{\exists}$)
3. Some but not all of the boys came to the party. ($F_{\exists\neg\forall}$)
4. Not all of the boys came to the party. ($F_{\neg\forall}$)
5. None of the boys came to the party. ($F_{\neg\exists}$)

$$F_{\exists},\ F_{\neg\forall}\ \ +>\ \ F_{\exists\neg\forall}$$

Observation

1. $\{F \mid \exists\sigma\, F \in \mathrm{Op}_\sigma\} = \{F_\forall, F_{\neg\exists}, F_{\exists\neg\forall}\}$.
2. $F_\exists,\ F_{\neg\forall} \notin \{F \mid \exists\sigma\, F \in \mathrm{Op}_\sigma\}$.
3. Some and Not All are sub-forms of Some but not All:

$$F_\exists,\ F_{\neg\forall}\ \lhd\ F_{\exists\neg\forall}$$

Hypothesis

Unused non-optimal sub-forms inherit the interpretation of their optimal super-forms.

Principle of Optimal Completion

Bus Tickets

Example 8

I: Where can I get the bus tickets for the excursion?

1. *E*: Ms. Müller is sitting in office 2.07. (F_M)
2. *E*: Bus tickets are available from Ms. Müller. (F_H)
3. *E*: Bus tickets are available from Ms. Müller. She is sitting in office 2.07. (F_{MH})

- $F_M \lhd F_{MH}$,
- $F_M \notin \{F \mid \exists\sigma\, F \in \mathrm{Op}_\sigma\}$,
- $F_{MH} \in \{F \mid \exists\sigma\, F \in \mathrm{Op}_\sigma\}$.

What can justify Optimal Completion?

Robustness

Language interpretation should be robust against mistakes by the speaker.

Definition 9 (Trembling Hand)

A trembling hand prefect equilibrium of a finite strategic game is a mixed strategy profile σ such that there exists a sequence $(\sigma^k)_{k=0}^{\infty}$ of completely mixed strategy profiles which converge to σ such that σ_i is a best response to each σ_i^k [Osborne & Rubinstein(1994), Def. 248.1].

Which Trembles Count?

Not all mistakes equally probable:

1. If speaker is expert, he knows whether A(all), A(some but not all), or A(none);
2. Hence, A(some, if not all) is sub-optimal and entails A(all);

⇒ Assumption: A(some, if not all) will not occur as mistake for A(all).

Only sub-forms of optimal forms count:

1. A(some) is sub-form of A(some but not all) and A(some, if not all);
2. If the speaker is expert, then A(some, if not all) is sub-optimal;

⇒ A(some) can only be optimally completed to A(some but not all).

Optimal Completion

Definition 10

We say that, for a support problem σ, a form E can be optimally completed to form F, $oc(\sigma, E, F)$, iff

1. $F \in \mathrm{Op}_\sigma$,
2. $E \notin \mathrm{Op}_\sigma$,
3. $P_E^\sigma([\![E]\!]) = P_E^\sigma([\![F]\!]) = 1$,
4. $E \lhd F$.

Putting Noise into Speakers' strategies

An epsilon downward approximation of a mixed speaker's strategy s is a probability distribution s^ϵ on $\sigma \times \mathcal{F}$ such that:

1. $s^\epsilon(.|\sigma)$ is defined for
   - optimal forms in σ,
   - sub-forms E of optimal forms F such that:
     - E is true in σ,
     - F is the only optimal super-form of E.
2. for optimal forms F,
$$s^\epsilon(F|\sigma) = (1-\epsilon)s(F|\sigma)$$
3. for all sub-optimal forms E of F,
$$s^\epsilon(E|\sigma) = \epsilon\, n^{-1}\, s(F|\sigma),$$
   with n the number of forms that can only be optimally completed to F.

Downward Perfection

Definition 11 (Downward Perfect Equilibrium)

A strategy pair (s, h) is a downward (trembling hand) prefect Bayesian equilibrium iff (s, h) is a perfect Bayesian equilibrium and if for every sequence $(s^{\epsilon_n})_{n>0}$ of epsilon downward approximations of s with $\epsilon_n \to 0$, there exists an N such that for all $n \geq N$ h is a perfect Bayesian best response to s^{ϵ_n}.

The Principle of Optimal Completion

The hearer can interpret a non-optimal utterance E in σ iff

1. $\exists\sigma \in [\sigma]\exists F \in \mathcal{F}\, oc(\sigma, E, F)$,
2. $\forall\sigma, \sigma' \in [\sigma]\forall F, F' \in \mathcal{F}$:

$$oc(\sigma, E, F) \wedge oc(\sigma', E, F') \rightarrow F = F'.$$

Then, utterance E implicates A, $E +> A$, iff

$$\forall\sigma \in [\sigma]\forall F \in \mathcal{F}\, (oc(\sigma, E, F) \rightarrow P^{\sigma}_{E}(A) = 1),$$

which is equivalent to

$$\forall\sigma \in [\sigma]\forall F \in \mathcal{F}\, (oc(\sigma, E, F) \rightarrow F +> A),$$

- A. Benz (2006): *Utility and Relevance of Answers*; in: A. Benz, G. Jäger, R. v. Rooij (eds.): *Game Theory and Pragmatics*, Palgrave, Mcmillan.
- A. Benz, R. v. Rooij (2007): *Optimal assertions and what they implicate: a uniform game theoretic approach*; *Topoi - An International Review of Philosophy* 26(1), pp. 63-78.
- H.P. Grice (1989): *Studies in the Way of Words*; Harvard University Press, Cambridge Massechusetts.
- D. Lewis (1969): *Convention*; first published by Harvard University Press 1969; reissued by Blackwell Publishers Ltd., Oxford, 2002.
- M.J. Osborne, A. Rubinstein (1994): *A Course in Game Theory*; The MIT Press.
- P. Parikh (2001): *The Use of Language*; CSLI Lecture Notes, Stanford, CA.