The imperative mood in update semantics

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Imperatives.

Compare

- Go!
- John had to go.
- You must go.

The last sentence is ambiguous between a *performative* and a *declarative* reading.

We want more than just to explain what it means for a command to be *in force*. How can we model the performative use?

Update semantics

Slogan: You know the meaning of a sentence if you know the change it brings about in the cognitive state of anyone who wants to incorporate the information conveyed by it.

- The meaning $[\varphi]$ of a sentence φ is an operation on cognitive states.

Let S be an cognitive state and φ a sentence with meaning $[\varphi]$. We write

$$S[\varphi]$$

for the cognitive state that results when S is updated with φ.

Key notions

Support Sometimes the information conveyed by φ will already be subsumed by S. In this case, we say that φ *is accepted* in S, or that S *supports* φ, and we write this as $S \models \varphi$. In simple cases this relation can be defined as follows:

- $S \models \varphi$ iff $S[\varphi] = S$

Logical validity An argument is *valid* if updating any state with the premises, yields a state that supports the conclusion.

- $\varphi_1, \ldots, \varphi_n \models \psi$ iff for every state S, $S[\varphi_1] \ldots [\varphi_n] \models \psi$.

Imperatives in update semantics

Basic idea: Let α be an expression which denotes an activity. Then the the imperative $\alpha!$ – if it is accepted – induces a *change of plans* in the cognitive state of the addressee.

For English, α is just an uninflected intransitive verb phrase.

Mixed moods

Eat that apple

Mixed moods

Eat that apple and you will choke.

Mixed moods

Eat that apple and you will choke.

Eat that apple or you will starve.

Mixed moods

Eat that apple and you will choke.

Eat that apple or you will starve.

Choke or starve and you will die.

Therefore: you will die.

I said

Basic idea: Let α be an expression which denotes an activity. Then the the imperative $\alpha!$ – if it is accepted – induces a *change of plans* in the cognitive state of the addressee.

Do imperative sentences have a subject?

Example

- Hey, you, get out of my way!
- Bello, sit!
- Everybody clap your hands!
- God, save the queen!

Claim: Imperatives have an addressee rather than a subject.

But then, how about

- Nobody go in there!

- Whoever wants to dance get himself a partner!

Logical puzzles

The cheapest logical theory of imperatives:

- If φ is a declarative sentence, then $!\varphi$ is a sentence. (Read $!\varphi$ as 'See to it that φ!')

- The logic of imperatives is given by: $!\varphi_1, \ldots, !\varphi_n \models !\psi$ iff $\varphi_1, \ldots, \varphi_n \models \psi$

This way, we get

$$!\varphi \models !(\varphi \vee \psi)$$

Ross's paradox

That is to say, the following argument would be valid.

$$\begin{array}{ll} & \textit{Post the letter!} \\ \hline \therefore & \textit{Post the letter or burn it!} \end{array}$$

A matter of Modus Tollens?

$$\varphi \rightarrow !\alpha, !\neg\alpha \models \neg\varphi$$

If it's raining outside, take an umbrella with you!
Don't take an umbrella with you!

$\therefore$ *It is not raining outside*

Semantics or Pragmatics?

Fact: Whether not not we, as the addressee, accept a given command depends heavily on the ‘authority’ of the speaker. It happens often that one authority overrules the other.

Question: Is this relevant to the *semantics* of imperatives? Or is this just a matter of pragmatics?

Commands and requests

Compare

- Go!
- Please, go!

Claim: Same semantics, different pragmatics.

Free choice permission

- In none of the standard systems of deontic logic we have:

  permitted $(p \vee q) \models$ *permitted* p.

- Yet, intuitively, *You may take an apple or a pear* implies

  You may take an apple.

Complex imperatives?

How about a disjunction of imperatives, a conjunction of imperatives, a negation of an imperative

versus

an imperative disjunction, an imperative conjunction, and imperative negation?

Examples

Shut up or leave!
Shut up! ... or... leave! (??)

John, stand here and Mary, stand there!
John, stand here or Mary, stand there! (??)

But then:
My advice to you is: Keep together. Either everybody stay or everybody leave.

States

Fix a finite set of atomic sentences, and a finite set of atomic infinitives.

(i) a *state* S is a set of possibilities;

(ii) a *possibility* is a pair $\langle w, \pi \rangle$ with w a world, and π a plan;

(iii) a *world* is a function w that assigns to every atomic sentence one of the truth values $\mathbf{true}$ or $\mathbf{false}$;

(iv) a *plan* is a set of to-do lists; a *to-do list* is a set of pairs $\langle a, \mathbf{x} \rangle$, with a an atomic infinitive and $\mathbf{x} \in \{\mathbf{do}, \mathbf{don't}\}$.

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Special States

- the *minimal state* consists of all pairs $\langle w, \{\emptyset\}\rangle$
- a state S is *absurd* iff either (i) $S = \emptyset$, or (ii) there is some world $\langle w, \pi\rangle \in S$ such that π is not *executable*.

Updating a state S with a descriptive sentence

If φ is a formula of propositional logic:

$$S[\varphi] = \{\langle w, \pi \rangle \in S \mid \varphi \text{ is true in } w\}$$

Updating a state S with an epistemic possibility

Let φ is a formula of propositional logic.

If $S[\varphi] \neq \emptyset$, then

$$S[might\, \varphi] = S$$

Otherwise,

$$S[might\, \varphi] = \emptyset$$

Sentences of the form $might\, \varphi$ provide an invitation to perform a *test* on S rather than to incorporate some new information in it.

Order matters

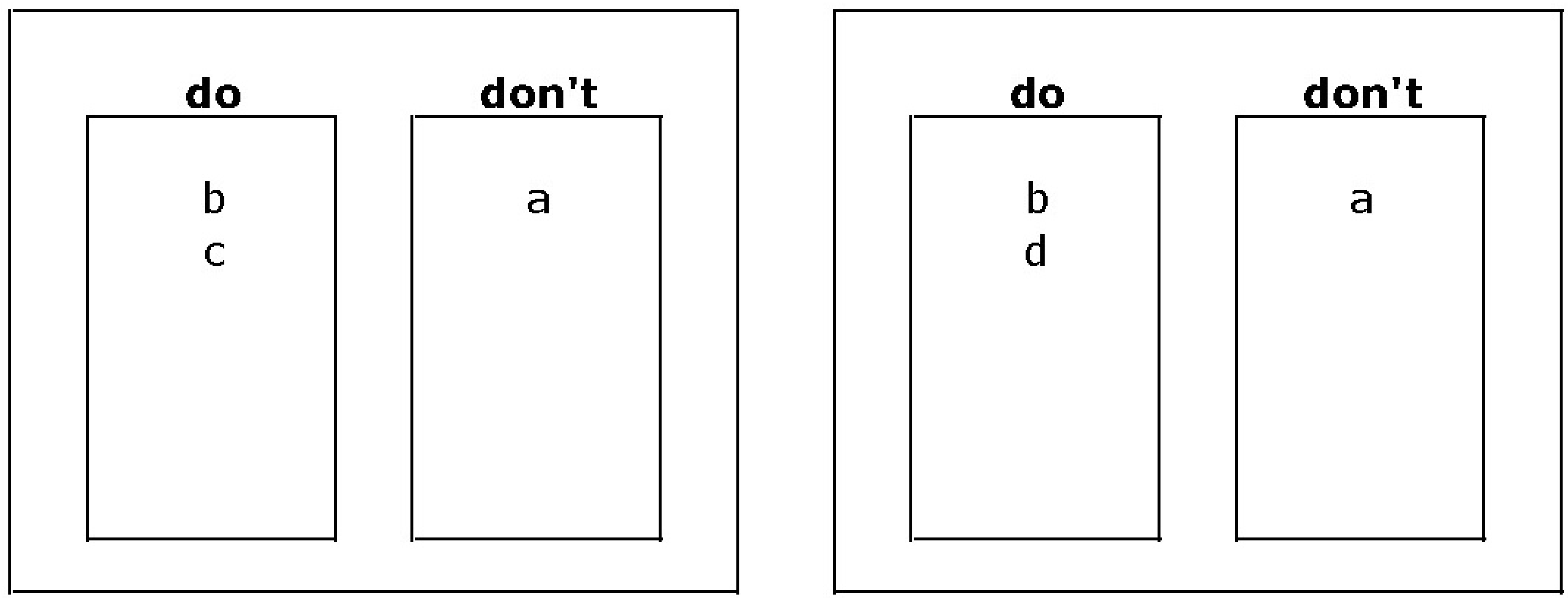
Let S_0 be the minimal state.

- $S_0[might\, p]\,[\neg p] \neq \emptyset$

- $S_0[\neg p]\,[might\, p] = \emptyset$

- The logic is nonmonotonic: $might\, p \models might\, p$, but $might\, p, \neg p \not\models might\, p$.

This is a picture of a plan



Some notions

- A to-do list s is *consistent* iff there is no a such that both $\langle a, \mathbf{do} \rangle \in s$ and $\langle a, \mathbf{don't} \rangle \in s$.

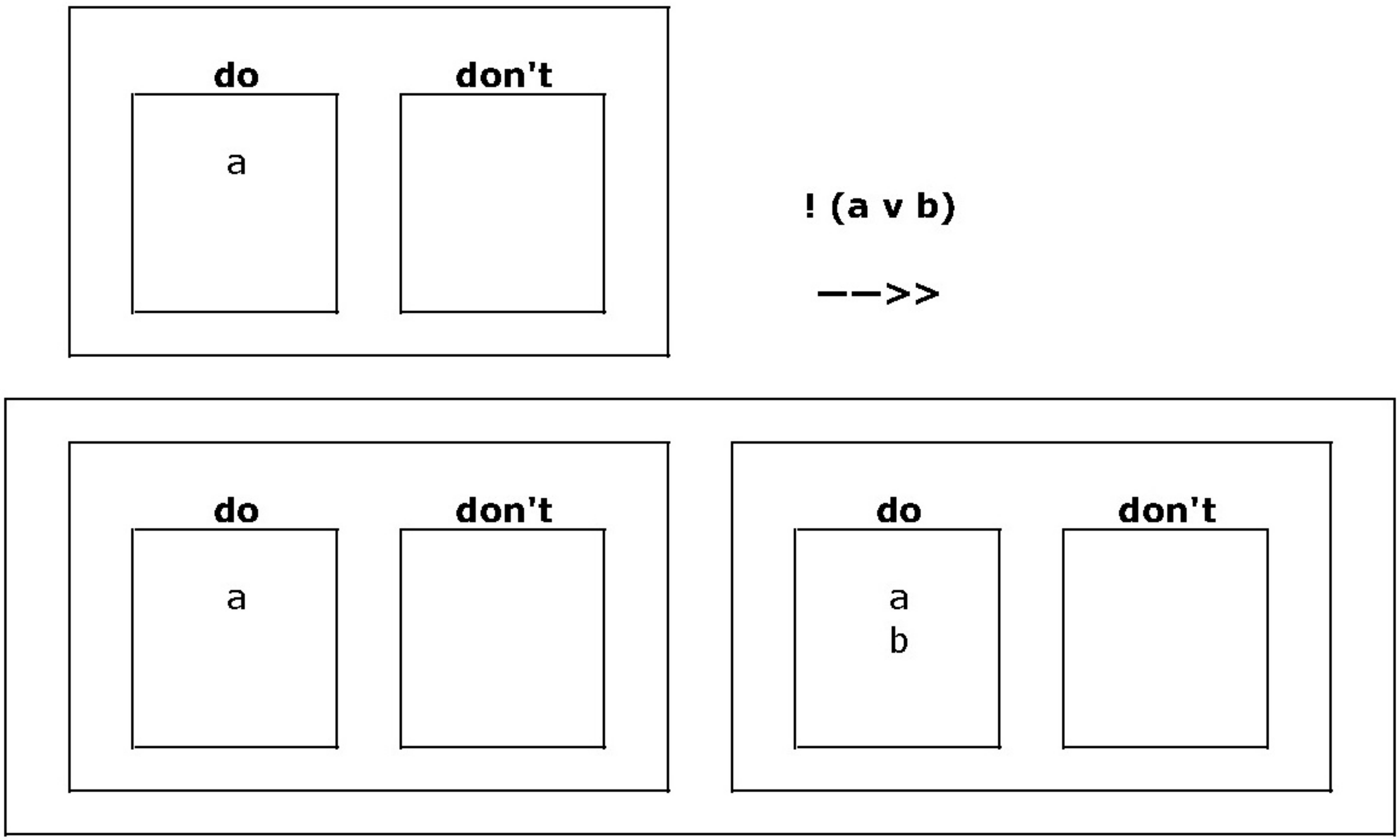
- A plan is *executable* iff it contains at least one consistent to-do list.

- π' *is at least as strong as* π iff if for every consistent $s \in \pi$ there is some consistent $s' \in \pi'$ such that $s \subseteq s'$.

Updating plans

$$
\begin{array}{rrcl}
\textit{atomic}: & \pi\!\uparrow\! a & = & \{s' \mid s' = s \cup \{\langle a, \mathbf{do}\rangle\} \text{ for some } s \in \pi\} \\
 & \pi\!\downarrow\! a & = & \{s' \mid s' = s \cup \{\langle a, \mathbf{don't}\rangle\} \text{ for some } s \in \pi\} \\
\neg: & \pi\!\uparrow\!\neg\alpha & = & \pi\!\downarrow\!\alpha \\
 & \pi\!\downarrow\!\neg\alpha & = & \pi\!\uparrow\!\alpha \\
\wedge: & \pi\!\uparrow\!(\alpha \wedge \beta) & = & \pi\!\uparrow\!\alpha\!\uparrow\!\beta \\
 & \pi\!\downarrow\!(\alpha \wedge \beta) & = & \pi\!\downarrow\!\alpha \cup \pi\!\downarrow\!\beta \\
\vee: & \pi\!\uparrow\!(\alpha \vee \beta) & = & \pi\!\uparrow\!\alpha \cup \pi\!\uparrow\!\beta \\
 & \pi\!\downarrow\!(\alpha \vee \beta) & = & \pi\!\downarrow\!\alpha\!\downarrow\!\beta
\end{array}
$$

Concerning Ross's Paradox



Permission

Note first: in many circumstances in which somebody gets permission to do something some prohibition is lifted.

Example: when you come to visit me at my place, you are not supposed to take a beer from the fridge without first asking permission. When I give you permission to take a beer, this prohibition to take a beer to is lifted.

This suggests that we can think of updating with a permission to do α as retracting α from the forbidden actions.

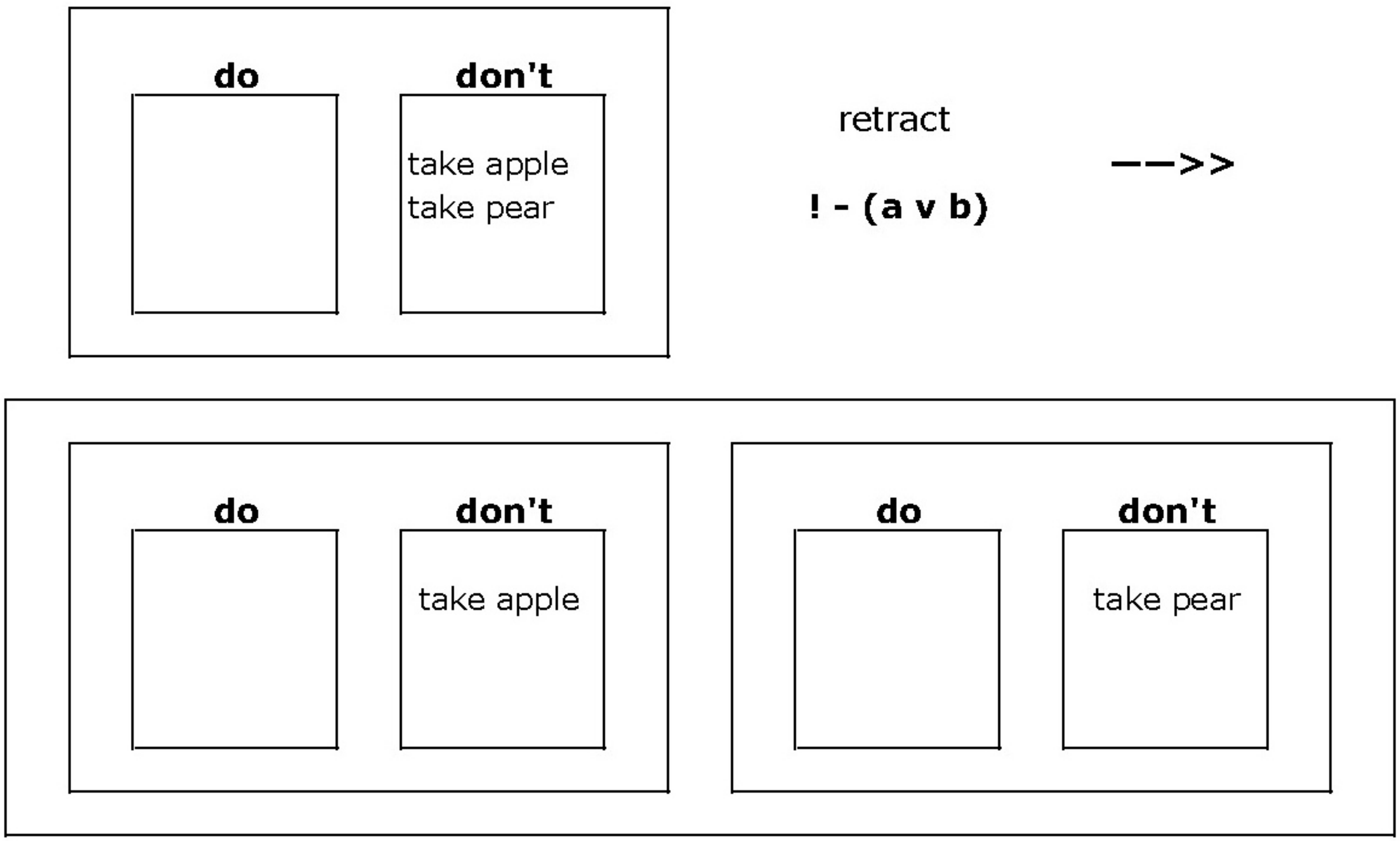
Updating a state S with a permission

$\langle w, \pi \rangle \in S[may\, \alpha]$ iff there is some π' such that

(a) $\langle w, \pi' \rangle \in S$ and

(b) π is the strongest weakening of π' such that $\pi{\uparrow}\alpha$ is executable.

Free Choice permission



Updating a state S with an imperative

$\langle w, \pi \rangle \in S[!\alpha]$ iff there are some π' and π'' such that

(a) $\langle w, \pi' \rangle \in S$

(b) π'' is the strongest weakening of π' such that $\pi'' \uparrow \alpha$ is executable.

(c) $\pi = \pi'' \uparrow \alpha$

Mixed moods

Stop or I'll shoot

To deal with mixed moods our models have to be enriched with a *future* so that we can encompass the results the actions of our agents.