

# A Logic for Assertion Networks

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# A Logic for Assertion Networks

## 1 Background

- Assertion network

## 2 Logic for Assertion network

- Graph operations
- A logical language

## 3 Conclusions

- Final notes

# A real life situation

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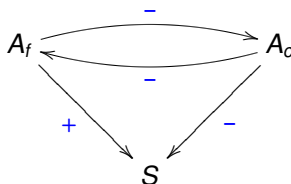
- Agents and facts represented by vertices ( $V$ ).
- Agents' opinions represented by labelled edges ( $l : E \rightarrow \{+, -\}$ ).
- Directed labelled graph (DLG)  $G = (V, E, l)$ .

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- $S$  - sun is shining

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- $\lambda_s$  stable value of  $s \in (V \cup E)$  if  $\lim_{i \rightarrow \infty} H_i(s) = \lambda_s$

$$\text{St}_H(G, s) = \lambda_s$$

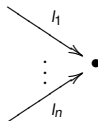


# A particular $\Psi$

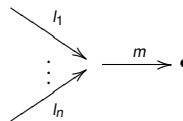
Focus on belief changes in opinions (edges) and facts (terminal vertices).



non-terminal vertices

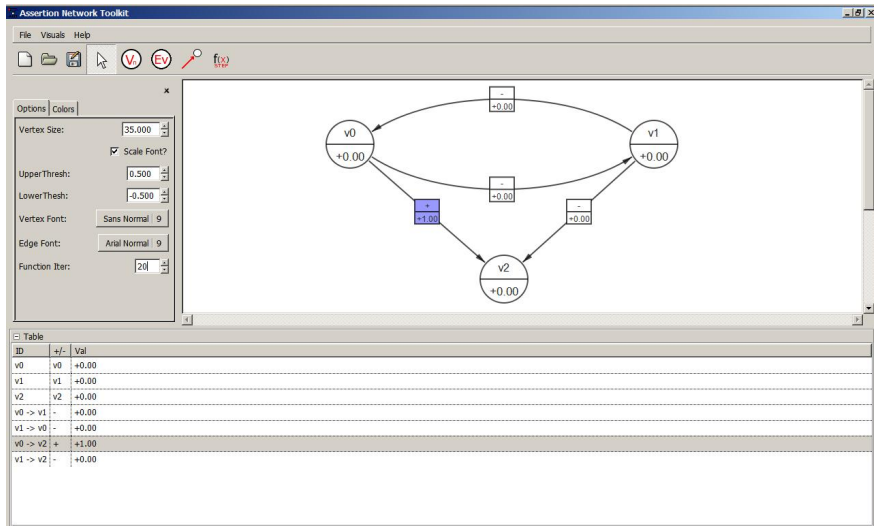


terminal vertices

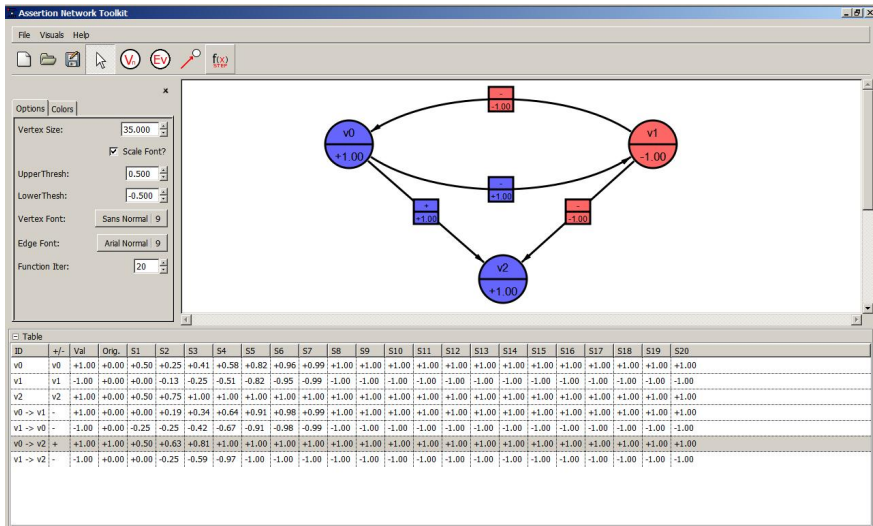


edges

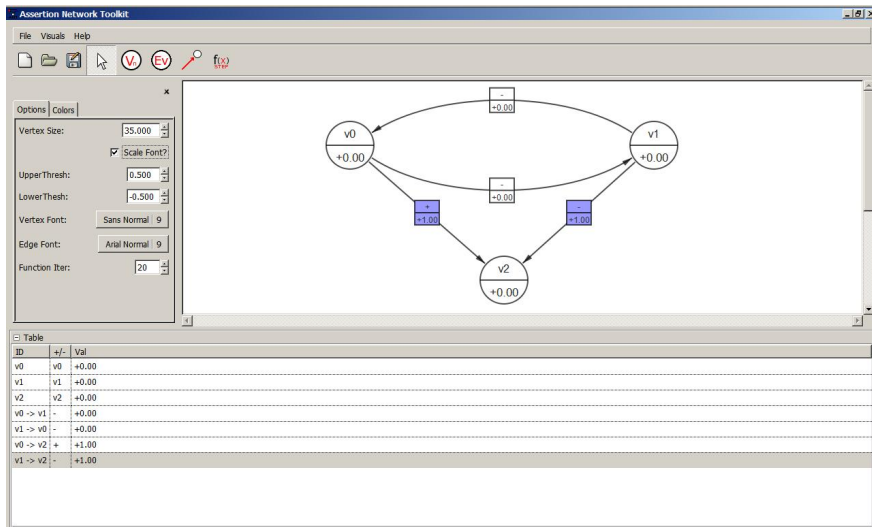
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# Reference

- Sujata **Ghosh**, Benedikt **Löwe**, and Erik **Scorelle**. Belief flow in assertion networks. In Uta Priss, Simon Polovina, and Richard Hill, editors, *Proceedings of the 15th International Conference on Conceptual Structures (ICCS 2007), Sheffield, UK*, volume 4604 of *LNAI*, pages 401–414, Heilderberg, July 2007. Springer-Verlag.

# This work

## Goal

### **Goal:**

***to define a logic to reason about assertion networks***

# This work

## Restrictions

Some restrictions for this initial work:



# This work

## Restrictions

Some restrictions for this initial work:

- Finite graphs.

# This work

## Restrictions

Some restrictions for this initial work:

- Finite graphs.
- Initial opinions just for facts ( $s \notin T \Rightarrow H(s) = 0$ ).

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- New agents with opinion about existing agents.
- A distinguished non-terminal node (an already considered agent).
- New nodes and edges representing new agents and their opinions.

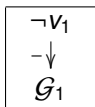
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$\mathcal{G}_1 = (G_1, v_1)$  and  $\mathcal{G}_2 = (G_2, v_2)$  two pointed DLG:



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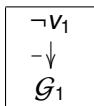
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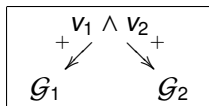
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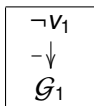
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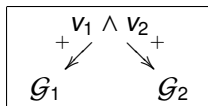
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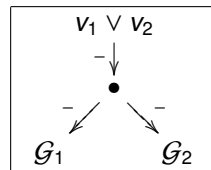
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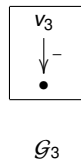
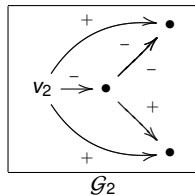
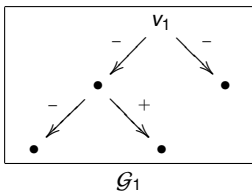


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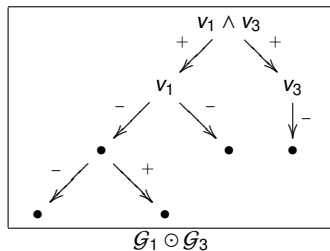
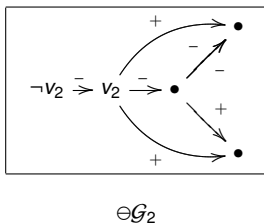
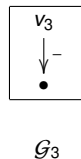
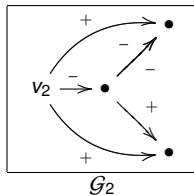
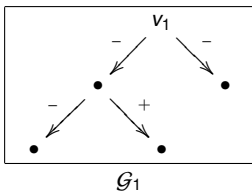


$\mathcal{G}_1 \oplus \mathcal{G}_2$

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| 0         | 0                            | 0                                    |
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| $H(w_1)$ | $H(w_2)$ | $St_H(\mathcal{G}_1)$ | $St_H(\mathcal{G}_2)$ | $St_H(\mathcal{G}_1 \odot \mathcal{G}_2)$ | $St_H(\mathcal{G}_1 \oplus \mathcal{G}_2)$ |
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| (0,1]    | (0,1]    | 1                     | 1                     | 1                                         | 1                                          |
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# Syntax

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## Definition (Syntax)

Given a set  $\Phi$  of atomic propositions (agents),  $\mathcal{L}_{ANT}$  is the smallest set of *agent terms* containing  $\Phi$  and closed under  $\neg, \wedge, \vee$ .

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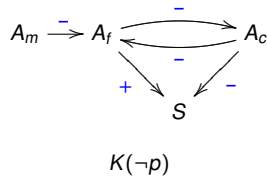
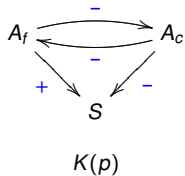
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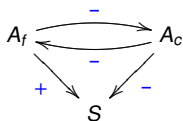
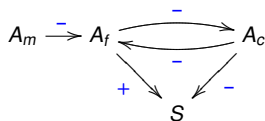
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*Believing in  $\varphi$  forces the observer to not disbelieve in  $\psi$*

# Examples

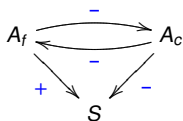
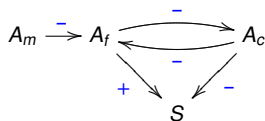


# Examples


 $K(p)$ 

 $K(\neg p)$ 

| $H(S)$    | $St_H(K(\neg p))$ | $St_H(K(p))$ |
|-----------|-------------------|--------------|
| $(0, 1]$  | -1                | 1            |
| 0         | 0                 | 0            |
| $[-1, 0)$ | 1                 | -1           |

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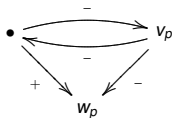

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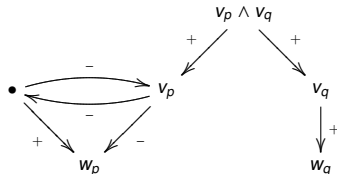
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●  $\neg p \not\models_K p$

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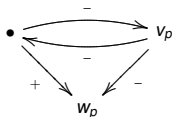
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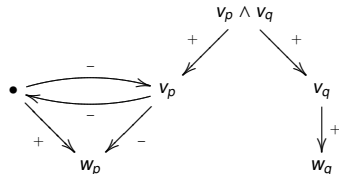

 $K(p)$ 

 $K(q)$ 

 $K(p \wedge q) = K(p) \odot K(q)$



# Examples


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 $K(q)$ 

 $K(p \wedge q) = K(p) \odot K(q)$ 

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| (0,1]    | (0,1]    | 0                     | -1           |
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| [-1,0)   | (0,1]    | 1                     | 1            |
| [-1,0)   | 0        | 0.5                   | 1            |
| [-1,0)   | [-1,0)   | 0                     | 1            |

We have

$$\bullet p \wedge q \models_K p$$

Similarly:

$$\bullet p \models_K p \vee q$$

$$\bullet (\neg p \wedge \neg q) \models_K \neg(p \vee q)$$

$$\bullet \neg(p \vee q) \models_K (\neg p \wedge \neg q)$$

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# Summary

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- DLGs for representing communication situations.
- Assertion network semantics.
- A logical language modelling AN.
- Simple graph operations defined and analyzed.

# Further work



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- Another graph operations (relating facts instead of agents)

# Further work

- Alternative definitions of  $\Psi$ .
- Generalization of the whole scenario (too much restrictions has been imposed).
- Another graph operations (relating facts instead of agents)
- A more universal entailment relation rather than the very contextual one given here.

# Thanks