



114 km

1° N 44° 47' 33.91" O

Image © 2007 TerraMetrics  
Image NASA

Streaming: 100%

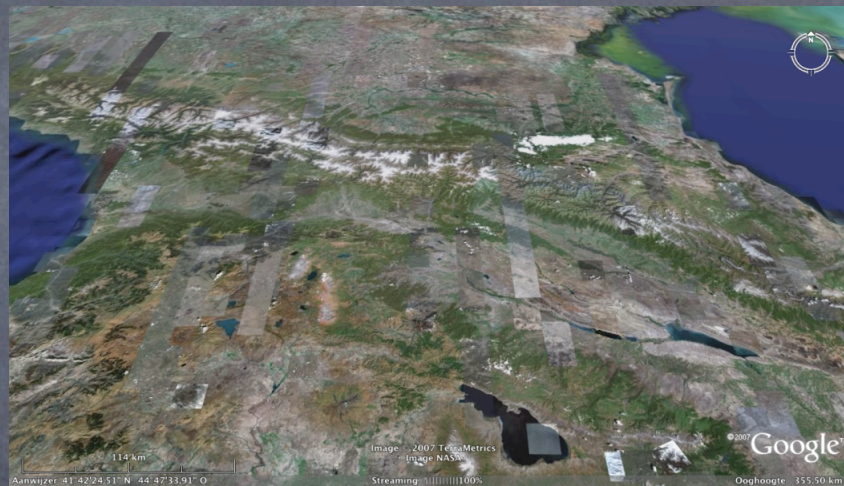
© 2007

Geo



# Inquisitive Semantics

Jeroen Groenendijk @ ILLC UvA



Seventh International Tbilisi Symposium  
on Language, Logic and Computation  
Tbilisi, Georgia, October 1-5, 2007



# Inquisitive Semantics

Mission Statement



# Inquisitive Semantics

## Mission Statement

- Meaning is information **exchange** potential



# Inquisitive Semantics

## Mission Statement

- ④ Meaning is information **exchange** potential
  - Information exchange is a dynamic process of raising and resolving **issues**



# Inquisitive Semantics

## Mission Statement

- ④ Meaning is information **exchange** potential
  - Information exchange is a dynamic process of raising and resolving **issues**
  - **Inquisitive meanings** directly reflect this



# Inquisitive Semantics

## Mission Statement

- ① Meaning is information **exchange** potential
  - Information exchange is a dynamic process of raising and resolving **issues**
  - **Inquisitive meanings** directly reflect this
  - They embody **both** information and issues



# Inquisitive Semantics

## Mission Statement

- ④ Meaning is information **exchange** potential
  - Information exchange is a dynamic process of raising and resolving **issues**
  - **Inquisitive meanings** directly reflect this
  - They embody **both** information and issues
  - When the notion of meaning changes, so does the logic that comes with it



# Inquisitive Semantics

## Mission Statement

- ① Meaning is information **exchange** potential
  - Information exchange is a dynamic process of raising and resolving **issues**
  - **Inquisitive meanings** directly reflect this
  - They embody **both** information and issues
  - When the notion of meaning changes, so does the logic that comes with it
  - In inquisitive logic the core notion is **licensing**



# Inquisitive Semantics

## Mission Statement

- ① Meaning is information **exchange** potential
  - Information exchange is a dynamic process of raising and resolving **issues**
  - **Inquisitive meanings** directly reflect this
  - They embody **both** information and issues
  - When the notion of meaning changes, so does the logic that comes with it
  - In inquisitive logic the core notion is **licensing**
  - Licensing is a notion of logical **relatedness**



# Overview



# Overview

1. Syntax and semantics for a language of  
inquisitive propositional logic



# Overview

1. Syntax and semantics for a language of inquisitive propositional logic
2. Disjunction responsible for inquisitiveness



# Overview

1. Syntax and semantics for a language of **inquisitive propositional** logic
2. **Disjunction** responsible for inquisitiveness
3. Conditional questions and other constructions



# Overview

1. Syntax and semantics for a language of **inquisitive propositional** logic
2. **Disjunction** responsible for inquisitiveness
3. Conditional questions and other constructions
4. Inquisitive meanings and **pictures** of them



# Overview

1. Syntax and semantics for a language of **inquisitive propositional** logic
2. **Disjunction** responsible for inquisitiveness
3. Conditional questions and other constructions
4. Inquisitive meanings and **pictures** of them
5. Inquisitive logic: the logical notion of **licensing**



# Overview

1. Syntax and semantics for a language of **inquisitive propositional** logic
2. **Disjunction** responsible for inquisitiveness
3. Conditional questions and other constructions
4. Inquisitive meanings and **pictures** of them
5. Inquisitive logic: the logical notion of **licensing**
6. Inquisitive semantics meets Grice on disjunction





# Syntax of propositional language $L$





# Syntax of propositional language $L$

means  
definition





# Syntax of propositional language $L$

$PV$  set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$





# Syntax of propositional language $L$

PV set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$

2.  $\perp \in L$





# Syntax of propositional language $L$

PV set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$

2.  $\perp \in L$

3. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \rightarrow \psi) \in L$





# Syntax of propositional language $L$

PV set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$

2.  $\perp \in L$

3. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \rightarrow \psi) \in L$

4. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \wedge \psi) \in L$





# Syntax of propositional language $L$

PV set of propositional variables

1. If  $p \in PV$ , then  $p \in L$
2.  $\perp \in L$
3. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \rightarrow \psi) \in L$
4. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \wedge \psi) \in L$
5. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \vee \psi) \in L$





# Syntax of propositional language $L$

PV set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$

2.  $\perp \in L$

gives classical logic

3. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \rightarrow \psi) \in L$

4. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \wedge \psi) \in L$

5. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \vee \psi) \in L$





# Syntax of propositional language $L$

PV set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$

2.  $\perp \in L$

gives classical logic

3. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \rightarrow \psi) \in L$

4. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \wedge \psi) \in L$

5. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \vee \psi) \in L$





# Syntax of propositional language $L$

PV set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$

2.  $\perp \in L$

gives classical logic

3. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \rightarrow \psi) \in L$

4. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \wedge \psi) \in L$

5. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \vee \psi) \in L$

gives inquisitiveness





# Syntax of propositional language L

PV set of propositional variables

1. If  $p \in PV$ , dan  $p \in L$

2.  $\perp \in L$

gives classical logic



$\neg\varphi =_{\text{def}} (\varphi \rightarrow \perp)$

3. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \rightarrow \psi) \in L$

4. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \wedge \psi) \in L$

5. If  $\varphi \in L$  and  $\psi \in L$ , then  $(\varphi \vee \psi) \in L$

gives inquisitiveness



# Notation Conventions ? !



# Notation Conventions ? !


$$!\varphi =_{\text{def}} \neg\neg\varphi$$



# Notation Conventions ? !

  $!\varphi =_{\text{def}} \neg\neg\varphi$

  $?\varphi =_{\text{def}} (\varphi \vee \neg\varphi)$



# Notation Conventions ? !

!  $\varphi =_{\text{def}} \neg\neg\varphi$

?  $\varphi =_{\text{def}} (\varphi \vee \neg\varphi)$

• Syntax allows for things like



# Notation Conventions ? !

!  $\varphi =_{\text{def}} \neg\neg\varphi$

?  $\varphi =_{\text{def}} (\varphi \vee \neg\varphi)$

• Syntax allows for things like

•  $(p \vee q), !(p \vee q), ?(p \vee q)$



# Notation Conventions ? !

!  $\varphi$  =<sub>def</sub>  $\neg\neg\varphi$

?  $\varphi$  =<sub>def</sub>  $(\varphi \vee \neg\varphi)$

• Syntax allows for things like

•  $(p \vee q), !(p \vee q), ?(p \vee q)$

•  $(p \rightarrow ?q), (?p \vee ?q), (?p \wedge ?q), (p \wedge ?q)$



# Notation Conventions ? !

!  $\varphi =_{\text{def}} \neg\neg\varphi$

?  $\varphi =_{\text{def}} (\varphi \vee \neg\varphi)$

- Syntax allows for things like
- $(p \vee q), !(p \vee q), ?(p \vee q)$
- $(p \rightarrow ?q), (?p \vee ?q), (?p \wedge ?q), (p \wedge ?q)$
- $\neg?p, !?p, !!p, ??p, (?p \rightarrow q), (?p \rightarrow ?q)$





# Inquisitive Semantics





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$
2.  $\langle i, j \rangle \not\models \perp$





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$

5.  $\langle i, j \rangle \models (\varphi \vee \psi)$  iff  $\langle i, j \rangle \models \varphi$  or  $\langle i, j \rangle \models \psi$





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

symmetry

$\langle i, j \rangle \models \varphi$  iff  $\langle j, i \rangle \models \varphi$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$

5.  $\langle i, j \rangle \models (\varphi \vee \psi)$  iff  $\langle i, j \rangle \models \varphi$  or  $\langle i, j \rangle \models \psi$





# Inquisitive Semantics

$i$  and  $j$  valuations for  $p \in PV$ ,  $i(p) \in \{0,1\}$

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

symmetry

$\langle i, j \rangle \models \varphi$  iff  $\langle j, i \rangle \models \varphi$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

three pairs matter

$\langle i, j \rangle \langle i, i \rangle \langle j, j \rangle$

if  $\iota \models \varphi$ , then  $\iota \models \psi$

4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$

5.  $\langle i, j \rangle \models (\varphi \vee \psi)$  iff  $\langle i, j \rangle \models \varphi$  or  $\langle i, j \rangle \models \psi$



# Inquisitiveness and Informativeness



# Inquisitiveness and Informativeness

📌  $\varphi$  is **inquisitive** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \models \varphi \text{ and } \langle i, j \rangle \not\models \varphi$$



# Inquisitiveness and Informativeness

📌  $\varphi$  is **inquisitive** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \models \varphi \text{ and } \langle i, j \rangle \not\models \varphi$$

📌  $\varphi$  is **informative** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \not\models \varphi$$



# Inquisitiveness and Informativeness

📌  $\varphi$  is **inquisitive** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \models \varphi \text{ and } \langle i, j \rangle \not\models \varphi$$

📌  $\varphi$  is **informative** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \not\models \varphi$$

📌  $\varphi$  is an **assertion** iff  $\varphi$  is not inquisitive



# Inquisitiveness and Informativeness

- 📌  $\varphi$  is **inquisitive** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \models \varphi \text{ and } \langle i, j \rangle \not\models \varphi$$

- 📌  $\varphi$  is **informative** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \not\models \varphi$$

- 📌  $\varphi$  is an **assertion** iff  $\varphi$  is not inquisitive

- 📌  $\varphi$  is an **question** iff  $\varphi$  is not informative



# Inquisitiveness and Informativeness

- $\varphi$  is **inquisitive** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \models \varphi \text{ and } \langle i, j \rangle \not\models \varphi$$

- $\varphi$  is **informative** iff for some  $i \in I$  and  $j \in I$ :

$$\langle i, i \rangle \models \varphi \text{ and } \langle j, j \rangle \not\models \varphi$$

- $\varphi$  is an **assertion** iff  $\varphi$  is not inquisitive

- $\varphi$  is an **question** iff  $\varphi$  is not informative

- $\varphi$  is a **hybrid** iff  $\varphi$  is inquisitive and  $\varphi$  is informative



# Assertions



# Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$
2.  $\langle i, j \rangle \not\models \perp$



# Assertions

means  
fact

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

☒  $p$  and  $\perp$  are not inquisitive,  $p$  and  $\perp$  **assertions**



# Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

☒  $p$  and  $\perp$  are not inquisitive,  $p$  and  $\perp$  **assertions**

☒  $p$  is informative



# Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

☒  $p$  and  $\perp$  are not inquisitive,  $p$  and  $\perp$  **assertions**

☒  $p$  is informative

☒  $\perp$  is not informative



# Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

☑  $p$  and  $\perp$  are not inquisitive,  $p$  and  $\perp$  **assertions**

☑  $p$  is informative

☑  $\perp$  is not informative

Next things to show:



# Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

☒  $p$  and  $\perp$  are not inquisitive,  $p$  and  $\perp$  **assertions**

☒  $p$  is informative

☒  $\perp$  is not informative

Next things to show:

☒ If  $\psi$  is an assertion, then  $(\varphi \rightarrow \psi)$  is an assertion



# Implication



# Implication

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$



# Implication

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

- If  $\psi$  is an assertion

then  $(\varphi \rightarrow \psi)$  is an assertion



# Implication

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

- If  $\psi$  is an assertion, then among  $\{i, j\}^2$  only  $\langle i, i \rangle$  and  $\langle j, j \rangle$  matter, hence:

then  $(\varphi \rightarrow \psi)$  is an assertion



# Implication

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

• If  $\psi$  is an assertion, then among  $\{i, j\}^2$  only  $\langle i, i \rangle$  and  $\langle j, j \rangle$  matter, hence:

>  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff if  $\langle i, i \rangle \models \varphi$ , then  $\langle i, i \rangle \models \psi$   
and if  $\langle j, j \rangle \models \varphi$ , then  $\langle j, j \rangle \models \psi$

then  $(\varphi \rightarrow \psi)$  is an assertion



# Implication

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :  only  $\langle i, i \rangle$  if  $i=j$   
if  $\iota \models \varphi$ , then  $\iota \models \psi$

• If  $\psi$  is an assertion, then among  $\{i, j\}^2$  only  $\langle i, i \rangle$  and  $\langle j, j \rangle$  matter, hence:

>  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff if  $\langle i, i \rangle \models \varphi$ , then  $\langle i, i \rangle \models \psi$   
and if  $\langle j, j \rangle \models \varphi$ , then  $\langle j, j \rangle \models \psi$

□  $\langle i, i \rangle \models (\varphi \rightarrow \psi)$  iff if  $\langle i, i \rangle \models \varphi$ , then  $\langle i, i \rangle \models \psi$

then  $(\varphi \rightarrow \psi)$  is an assertion



# Implication

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :  only  $\langle i, i \rangle$  if  $i=j$   
if  $\iota \models \varphi$ , then  $\iota \models \psi$

• If  $\psi$  is an assertion, then among  $\{i, j\}^2$  only  $\langle i, i \rangle$  and  $\langle j, j \rangle$  matter, hence:

>  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff if  $\langle i, i \rangle \models \varphi$ , then  $\langle i, i \rangle \models \psi$   
and if  $\langle j, j \rangle \models \varphi$ , then  $\langle j, j \rangle \models \psi$

□  $\langle i, i \rangle \models (\varphi \rightarrow \psi)$  iff if  $\langle i, i \rangle \models \varphi$ , then  $\langle i, i \rangle \models \psi$

>  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff  $\langle i, i \rangle \models (\varphi \rightarrow \psi)$   
and  $\langle j, j \rangle \models (\varphi \rightarrow \psi)$

☑ If  $\psi$  is an assertion, then  $(\varphi \rightarrow \psi)$  is an assertion



Sofar only Assertions



# Sofar only Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$
2.  $\langle i, j \rangle \not\models \perp$
3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :  
if  $\iota \models \varphi$ , then  $\iota \models \psi$



# Sofar only Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$



# Sofar only Assertions

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$



If  $\varphi$  and  $\psi$  assertions, then  $(\varphi \wedge \psi)$  is an assertion



# Negation, Assertion



# Negation, Assertion

$$\neg\varphi =_{\text{def}} (\varphi \rightarrow \perp)$$



# Negation, Assertion

$$\neg \varphi =_{\text{def}} (\varphi \rightarrow \perp)$$

$$\langle i, j \rangle \models \neg \varphi \text{ iff } \langle i, i \rangle \not\models \varphi \text{ and } \langle j, j \rangle \not\models \varphi$$



# Negation, Assertion

$$\neg\varphi =_{\text{def}} (\varphi \rightarrow \perp)$$

$$\langle i, j \rangle \models \neg\varphi \text{ iff } \langle i, i \rangle \not\models \varphi \text{ and } \langle j, j \rangle \not\models \varphi$$

 For all  $\varphi$ :  $\neg\varphi$  is an assertion



# Negation, Assertion

$$\neg\varphi =_{\text{def}} (\varphi \rightarrow \perp)$$

$$\langle i, j \rangle \models \neg\varphi \text{ iff } \langle i, i \rangle \not\models \varphi \text{ and } \langle j, j \rangle \not\models \varphi$$

- ☑ For all  $\varphi$ :  $\neg\varphi$  is an assertion
- ☑ For all  $\varphi$ :  $\neg\neg\varphi$ , and hence  $!\varphi$  is an assertion



# Entailment



# Entailment

📌  $\varphi_1, \dots, \varphi_n \models \psi$  iff for all  $\iota \in I^2$  :

if  $\iota \models \varphi_1$ , and  $\dots$ , and  $\iota \models \varphi_n$ , then  $\iota \models \psi$



# Entailment

📌  $\varphi_1, \dots, \varphi_n \models \psi$  iff for all  $\iota \in I^2$  :

if  $\iota \models \varphi_1$ , and  $\dots$ , and  $\iota \models \varphi_n$ , then  $\iota \models \psi$

☑ For  $\psi$  is an assertion, we obtain classical logic



# Entailment

📌  $\varphi_1, \dots, \varphi_n \models \psi$  iff for all  $\iota \in I^2$  :

if  $\iota \models \varphi_1$ , and  $\dots$ , and  $\iota \models \varphi_n$ , then  $\iota \models \psi$

☑ For  $\psi$  is an assertion, we obtain classical logic

☑ If  $\varphi$  is an assertion, then  $\neg\neg\varphi \models \varphi$ ,  $!\varphi \models \varphi$



# Entailment

📌  $\varphi_1, \dots, \varphi_n \models \psi$  iff for all  $\iota \in I^2$  :

if  $\iota \models \varphi_1$ , and  $\dots$ , and  $\iota \models \varphi_n$ , then  $\iota \models \psi$

☑ For  $\psi$  is an assertion, we obtain classical logic

☑ If  $\varphi$  is an assertion, then  $\neg\neg\varphi \models \varphi$ ,  $!\varphi \models \varphi$

☑ If  $\psi$  is an assertion, then  $\varphi \models \psi$  iff  $!\varphi \models \psi$



# Inquisitive Semantics



# Inquisitive Semantics

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$
2.  $\langle i, j \rangle \not\models \perp$
3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :  
if  $\iota \models \varphi$ , then  $\iota \models \psi$
4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$



# Inquisitive Semantics

1.  $\langle i, j \rangle \models p$  iff  $i(p) = 1$  and  $j(p) = 1$

2.  $\langle i, j \rangle \not\models \perp$

3.  $\langle i, j \rangle \models (\varphi \rightarrow \psi)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models \varphi$ , then  $\iota \models \psi$

4.  $\langle i, j \rangle \models (\varphi \wedge \psi)$  iff  $\langle i, j \rangle \models \varphi$  and  $\langle i, j \rangle \models \psi$

5.  $\langle i, j \rangle \models (\varphi \vee \psi)$  iff  $\langle i, j \rangle \models \varphi$  or  $\langle i, j \rangle \models \psi$



Disjunctions can be Hybrid



# Disjunctions can be Hybrid

- ☒  $(p \vee q)$  is inquisitive and informative



# Disjunctions can be Hybrid

☑  $(p \vee q)$  is inquisitive and informative

□ Let  $i(p) = 1, i(q) = 0$



# Disjunctions can be Hybrid

☒  $(p \vee q)$  is inquisitive and informative

☐ Let  $i(p) = 1, i(q) = 0$

☐ And  $j(p) = 0, j(q) = 1$



# Disjunctions can be Hybrid

☑  $(p \vee q)$  is inquisitive and informative

□ Let  $i(p) = 1, i(q) = 0$

□ And  $j(p) = 0, j(q) = 1$

⦿  $\langle i, i \rangle \models (p \vee q)$ , because  $\langle i, i \rangle \models p$



# Disjunctions can be Hybrid

☑  $(p \vee q)$  is inquisitive and informative

□ Let  $i(p) = 1, i(q) = 0$

□ And  $j(p) = 0, j(q) = 1$

•  $\langle i, i \rangle \models (p \vee q)$ , because  $\langle i, i \rangle \models p$

•  $\langle j, j \rangle \models (p \vee q)$ , because  $\langle j, j \rangle \models q$



# Disjunctions can be Hybrid

☑  $(p \vee q)$  is inquisitive and informative

□ Let  $i(p) = 1, i(q) = 0$

□ And  $j(p) = 0, j(q) = 1$

🌀  $\langle i, i \rangle \models (p \vee q)$ , because  $\langle i, i \rangle \models p$

🌀  $\langle j, j \rangle \models (p \vee q)$ , because  $\langle j, j \rangle \models q$

🌀  $\langle i, j \rangle \not\models (p \vee q)$ , because  $\langle i, j \rangle \not\models p$  and  $\langle i, j \rangle \not\models q$



# Disjunctions can be Hybrid



$(p \vee q)$  is inquisitive and informative

□ Let  $i(p) = 1, i(q) = 0$

□ And  $j(p) = 0, j(q) = 1$



•  $\langle i, i \rangle \models (p \vee q)$ , because  $\langle i, i \rangle \models p$

•  $\langle j, j \rangle \models (p \vee q)$ , because  $\langle j, j \rangle \models q$

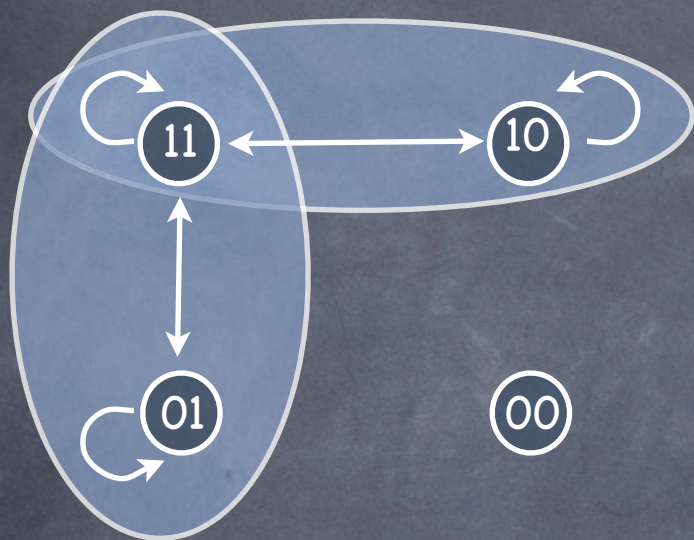
•  $\langle i, j \rangle \not\models (p \vee q)$ , because  $\langle i, j \rangle \not\models p$  and  $\langle i, j \rangle \not\models q$



Picture of Meaning ( $p \vee q$ )

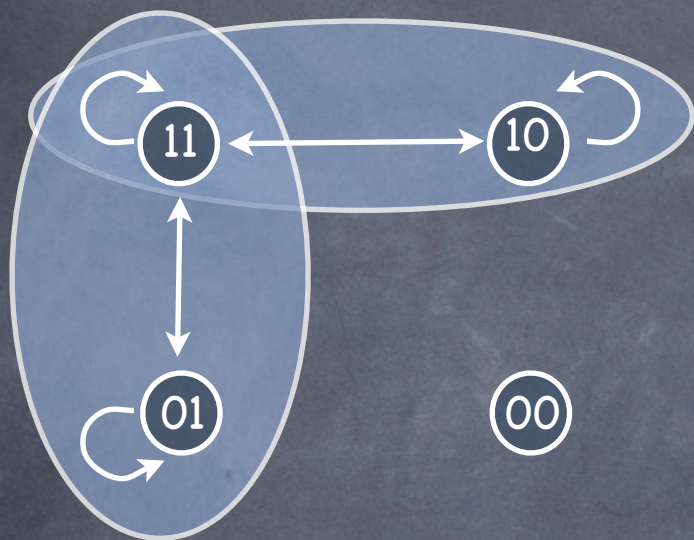


# Picture of Meaning ( $p \vee q$ )





# Picture of Meaning ( $p \vee q$ )



$i(p)=1$     $i(q)=0$

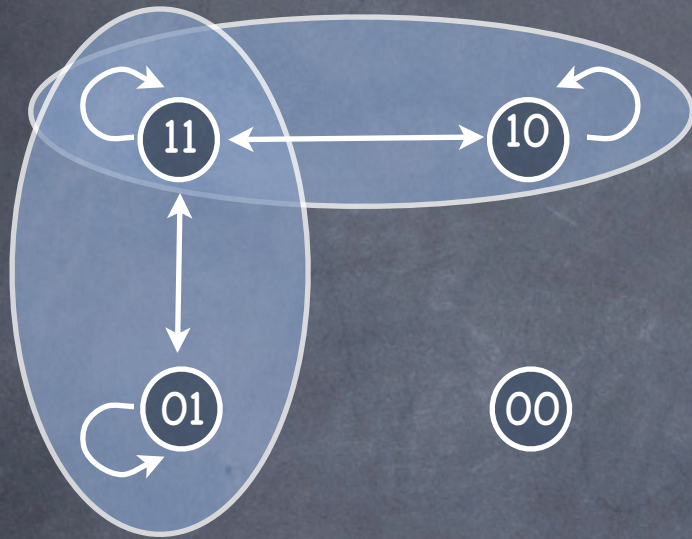
10

i

world



# Picture of Meaning ( $p \vee q$ )



$$i(p)=1 \quad i(q)=0$$

10

i

world

i

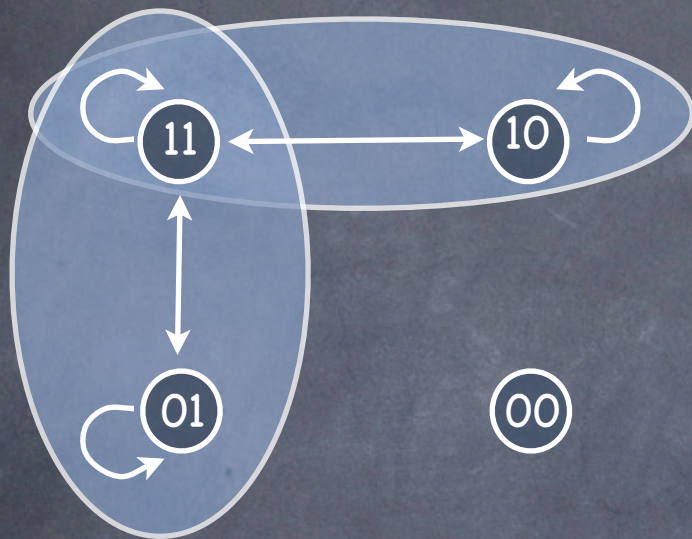
→ j

j

$$\langle i, j \rangle \models (p \vee q)$$



# Picture of Meaning ( $p \vee q$ )



possibility for ( $p \vee q$ )

$$i(p)=1 \quad i(q)=0$$

10

$i$

world

$i$

$\longrightarrow$

$j$

$i$

$j$

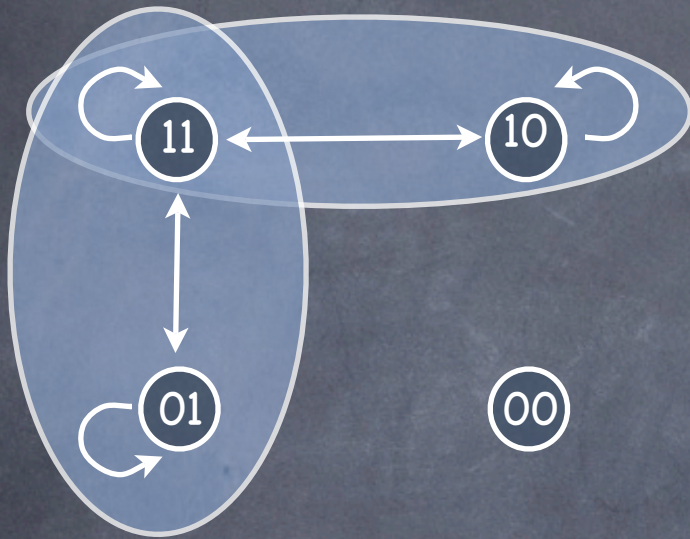
$$\langle i, j \rangle \models (p \vee q)$$



largest set of mutually  
related worlds

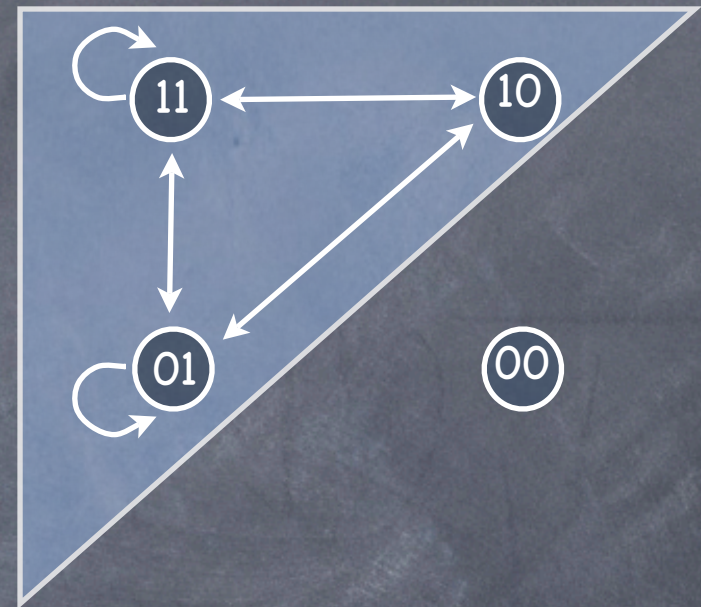
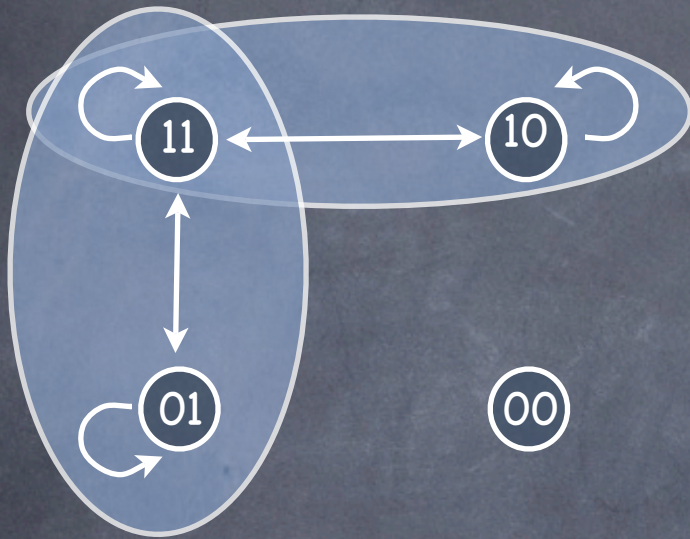


# Picture of Meaning $(p \vee q)$ and $!(p \vee q)$



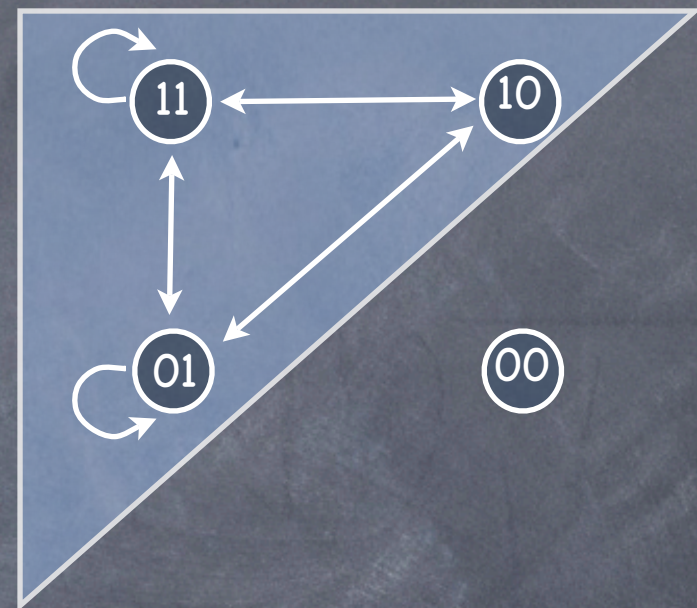
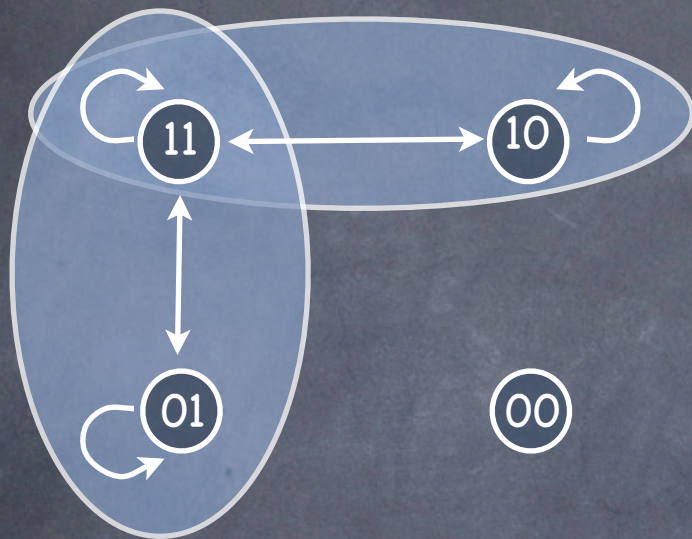


# Picture of Meaning $(p \vee q)$ and $!(p \vee q)$





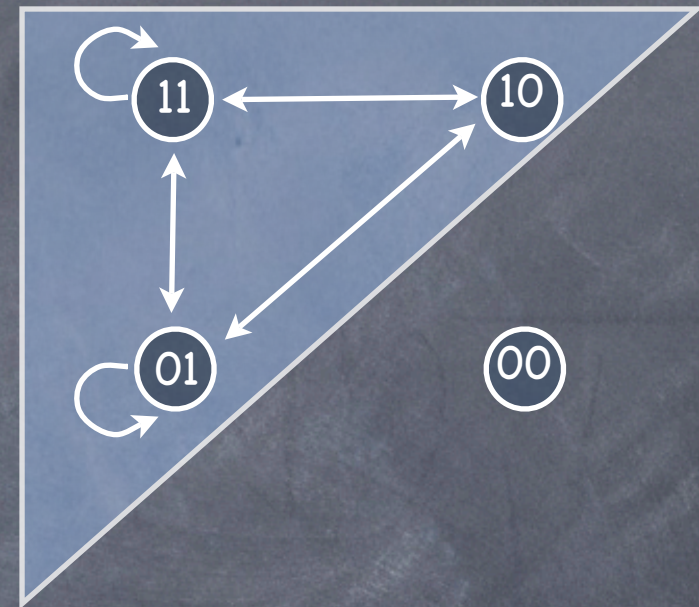
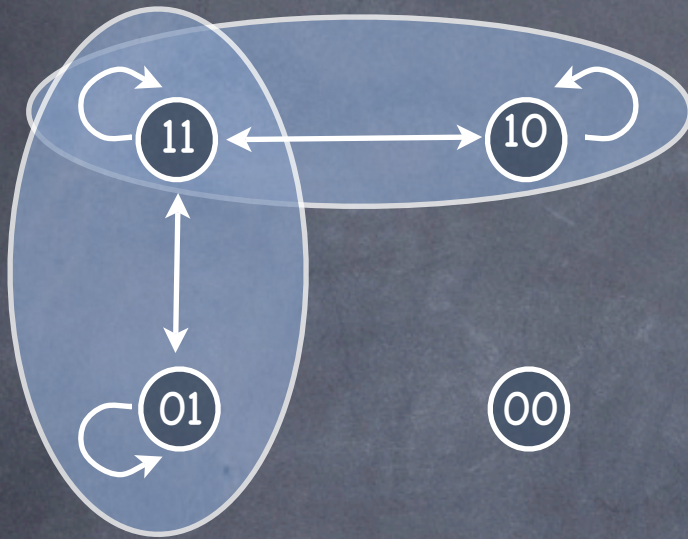
# Picture of Meaning $(p \vee q)$ and $!(p \vee q)$



☒  $(p \vee q) \models !(p \vee q)$



# Picture of Meaning $(p \vee q)$ and $!(p \vee q)$



☒  $(p \vee q) \models !(p \vee q)$

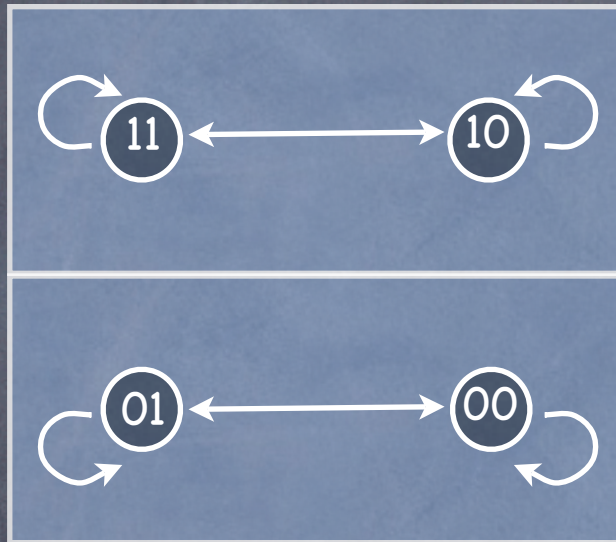
☒  $!(p \vee q) \not\models (p \vee q)$



Picture of Meaning ?  $p = (p \vee \neg p)$



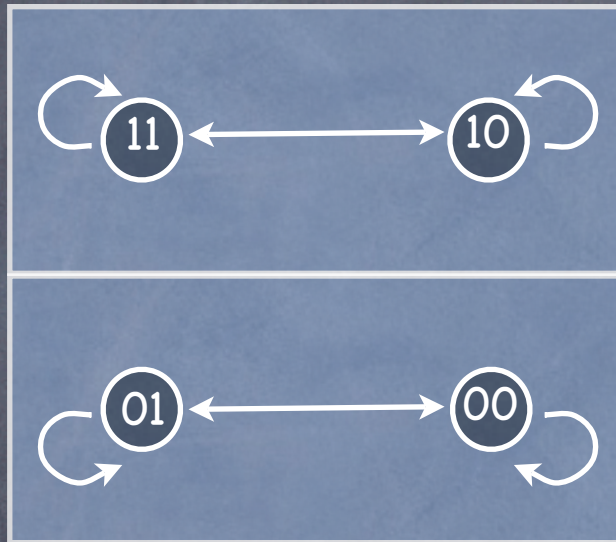
Picture of Meaning ? $p = (p \vee \neg p)$



Proper question:  
inquisitive, not informative



Picture of Meaning ? $p = (p \vee \neg p)$

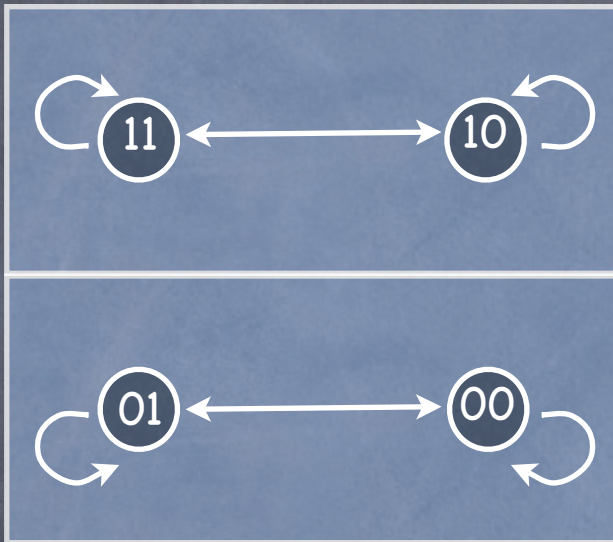


$\neq (p \vee \neg p)$

Proper question:  
inquisitive, not informative



# Picture of Meaning ? $p = (p \vee \neg p)$



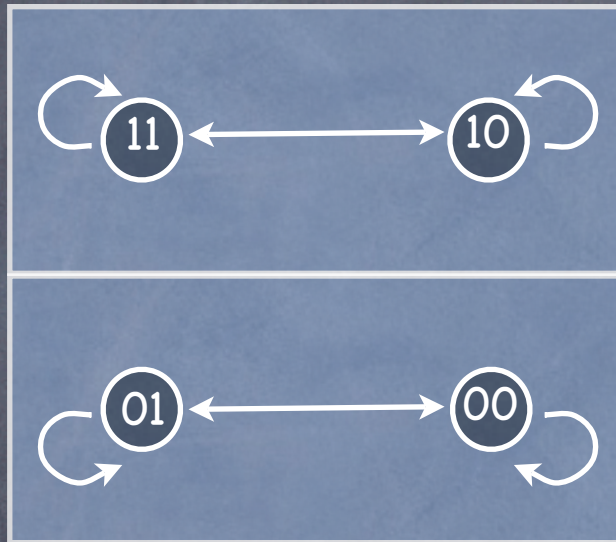
$\not\models (p \vee \neg p)$

$\models \neg(p \vee \neg p)$

Proper question:  
inquisitive, not informative



# Picture of Meaning ?p = (p $\vee$ $\neg$ p)



$$\models (p \vee \neg p)$$

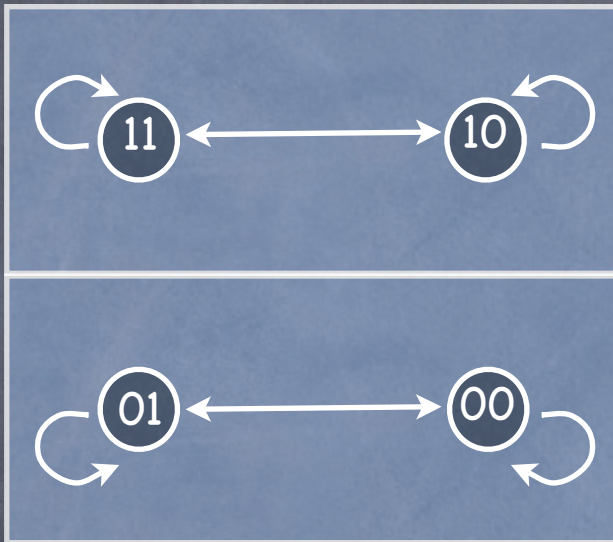
$$\models !(p \vee \neg p)$$

$$\models !?p$$

Proper question:  
inquisitive, not informative



# Picture of Meaning $?p = (p \vee \neg p)$



$$\models (p \vee \neg p)$$

$$\models !(p \vee \neg p)$$

$$\models !?p$$

$$\neg ?p \models \perp$$

Proper question:  
inquisitive, not informative



Why pairs of indices



# Why pairs of indices

- With  $\langle i, j \rangle \models ?p$  you **compare**  $i(p)$  and  $j(p)$



# Why pairs of indices

- With  $\langle i, j \rangle \models ?p$  you **compare**  $i(p)$  and  $j(p)$
- What the semantics does:



# Why pairs of indices

- With  $\langle i, j \rangle \models ?p$  you **compare**  $i(p)$  and  $j(p)$
- What the semantics does:
  - $\langle i, j \rangle \models ?p$  iff  $i(p) = j(p)$



# Why pairs of indices

• With  $\langle i, j \rangle \models ?p$  you **compare**  $i(p)$  and  $j(p)$

• What the semantics does:

□  $\langle i, j \rangle \models ?p$  iff  $i(p) = j(p)$

□ Note that this means:

Relative to  $i$  and  $j$   $?p$  is **not** an issue



# Why pairs of indices

- With  $\langle i, j \rangle \models ?p$  you **compare**  $i(p)$  and  $j(p)$
- What the semantics does:
  - $\langle i, j \rangle \models ?p$  iff  $i(p) = j(p)$
  - Note that this means:  
Relative to  $i$  and  $j$   $?p$  is **not** an issue
- The remarkable thing is not that we need pairs of indices, but that **that is all we need!**



Conditional question ( $p \rightarrow ?q$ )



## Conditional question $(p \rightarrow ?q)$

•  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models p$ , then  $\iota \models ?q$



## Conditional question $(p \rightarrow ?q)$

•  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models p$ , then  $\iota \models ?q$

□ Since for all  $\langle i, i \rangle$  it holds that  $\langle i, i \rangle \models ?q$

The only  $\iota \in \{i, j\}^2$  that matters is  $\langle i, j \rangle$



## Conditional question $(p \rightarrow ?q)$

•  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models p$ , then  $\iota \models ?q$

□ Since for all  $\langle i, i \rangle$  it holds that  $\langle i, i \rangle \models ?q$

The only  $\iota \in \{i, j\}^2$  that matters is  $\langle i, j \rangle$

>  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff if  $\langle i, j \rangle \models p$ , then  $\langle i, j \rangle \models ?q$



## Conditional question $(p \rightarrow ?q)$

•  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models p$ , then  $\iota \models ?q$

□ Since for all  $\langle i, i \rangle$  it holds that  $\langle i, i \rangle \models ?q$

The only  $\iota \in \{i, j\}^2$  that matters is  $\langle i, j \rangle$

>  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff if  $\langle i, j \rangle \models p$ , then  $\langle i, j \rangle \models ?q$

iff if  $i(p) = j(p) = 1$ , then  $i(q) = j(q)$



## Conditional question $(p \rightarrow ?q)$

•  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff for all  $\iota \in \{i, j\}^2$ :

if  $\iota \models p$ , then  $\iota \models ?q$

□ Since for all  $\langle i, i \rangle$  it holds that  $\langle i, i \rangle \models ?q$

The only  $\iota \in \{i, j\}^2$  that matters is  $\langle i, j \rangle$

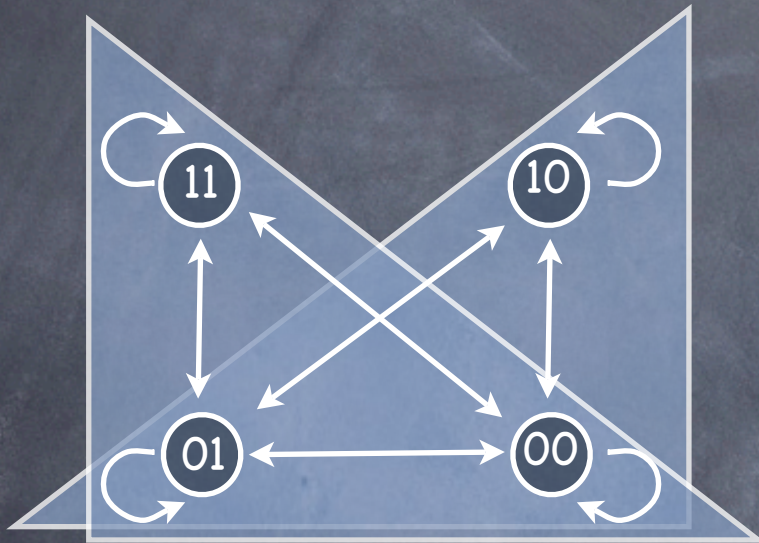
>  $\langle i, j \rangle \models (p \rightarrow ?q)$  iff if  $\langle i, j \rangle \models p$ , then  $\langle i, j \rangle \models ?q$

iff if  $i(p) = j(p) = 1$ , then  $i(q) = j(q)$

>  $\langle i, j \rangle \not\models (p \rightarrow ?q)$  iff  $i(p) = j(p) = 1$  and  $i(q) \neq j(q)$

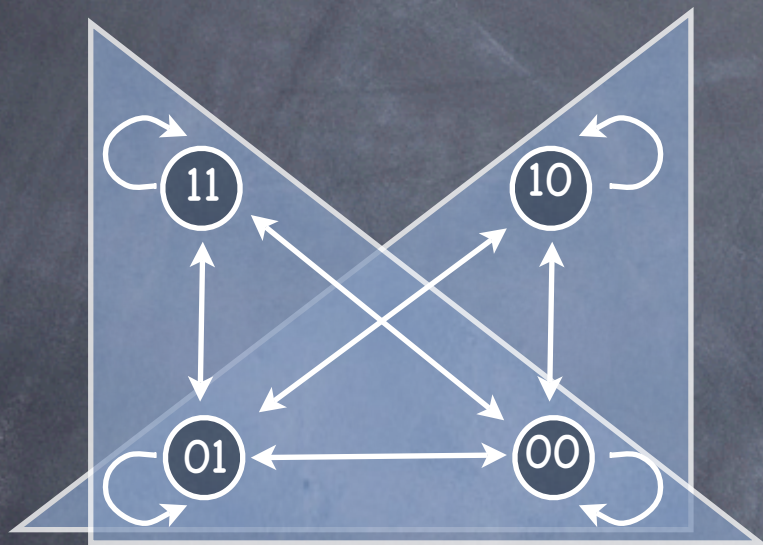


Conditional question ( $p \rightarrow ?q$ )





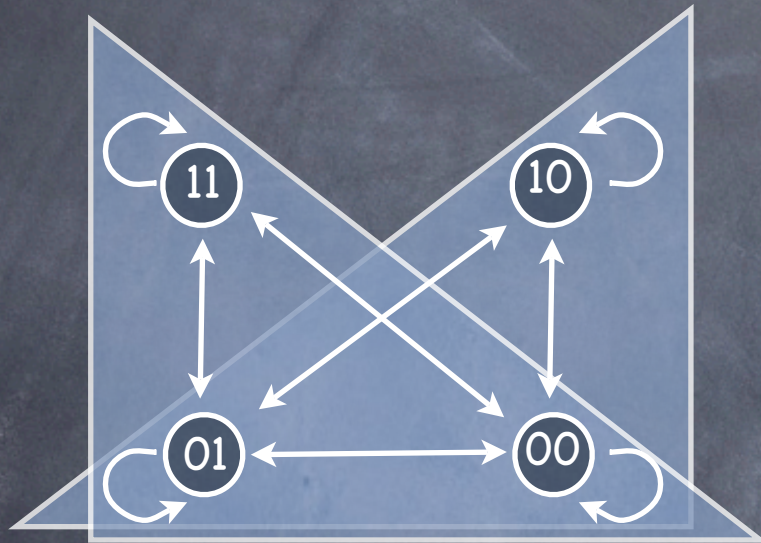
Conditional question ( $p \rightarrow ?q$ )



Question: inquisitive  
not informative

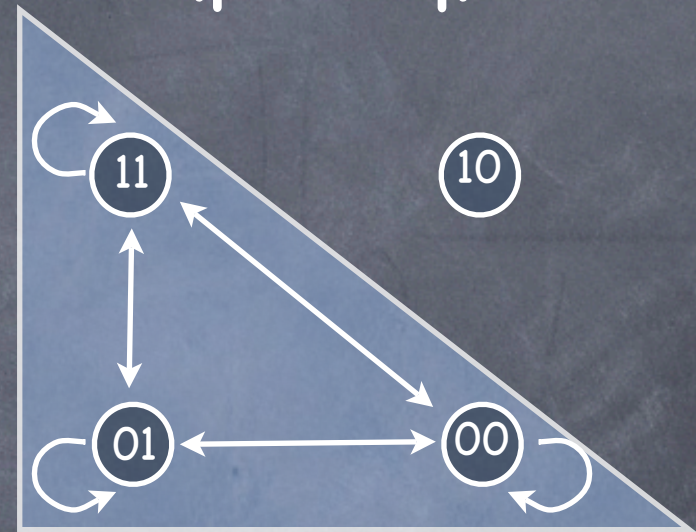


Conditional question ( $p \rightarrow ?q$ )



Question: inquisitive  
not informative

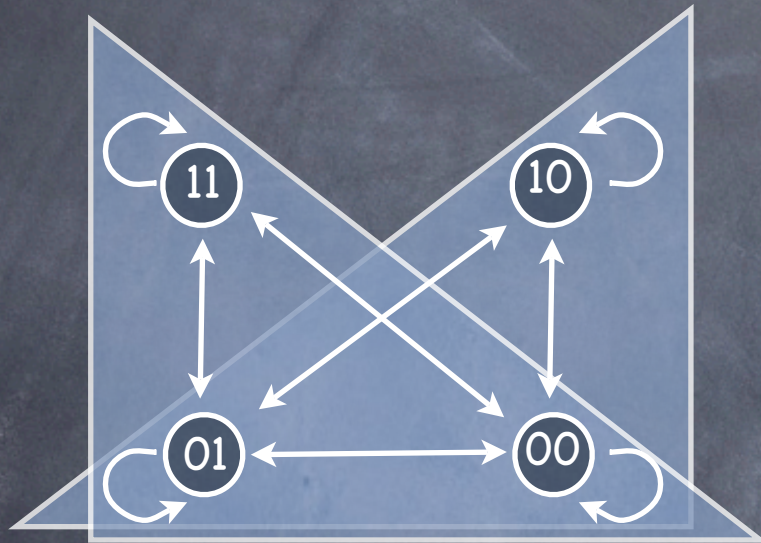
Assertive conditional  
( $p \rightarrow q$ )



☑ ( $p \rightarrow q$ )  $\models$  ( $p \rightarrow ?q$ )

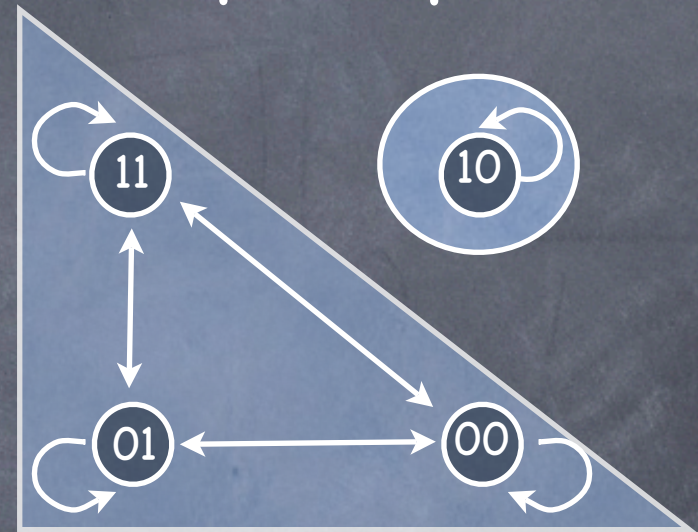


# Conditional question ( $p \rightarrow ?q$ )



Question: inquisitive  
not informative

# Questioned conditional $?(p \rightarrow q)$



- ☒  $(p \rightarrow q) \models (p \rightarrow ?q)$
- ☒  $?(p \rightarrow q) \models (p \rightarrow ?q)$



# Interplay disjunction and implication



# Interplay disjunction and implication

- $((p \vee q) \rightarrow ?r)$  is a conditional question:



# Interplay disjunction and implication

•  $((p \vee q) \rightarrow ?r)$  is a conditional question:

☑ The semantics predicts it has 4 answers:



# Interplay disjunction and implication

🌀  $((p \vee q) \rightarrow ?r)$  is a conditional question:

☑ The semantics predicts it has 4 answers:

$$((p \vee q) \rightarrow r)$$



# Interplay disjunction and implication

🌀  $((p \vee q) \rightarrow ?r)$  is a conditional question:

☑ The semantics predicts it has 4 answers:

$$((p \vee q) \rightarrow r)$$

$$((p \vee q) \rightarrow \neg r)$$



# Interplay disjunction and implication

🌀  $((p \vee q) \rightarrow ?r)$  is a conditional question:

☑ The semantics predicts it has 4 answers:

$$((p \vee q) \rightarrow r)$$

$$((p \vee q) \rightarrow \neg r)$$

$$(p \rightarrow r) \wedge (q \rightarrow \neg r)$$



# Interplay disjunction and implication

🌀  $((p \vee q) \rightarrow ?r)$  is a conditional question:

☑ The semantics predicts it has 4 answers:

$$((p \vee q) \rightarrow r)$$

$$((p \vee q) \rightarrow \neg r)$$

$$(p \rightarrow r) \wedge (q \rightarrow \neg r)$$

$$(p \rightarrow \neg r) \wedge (q \rightarrow r)$$



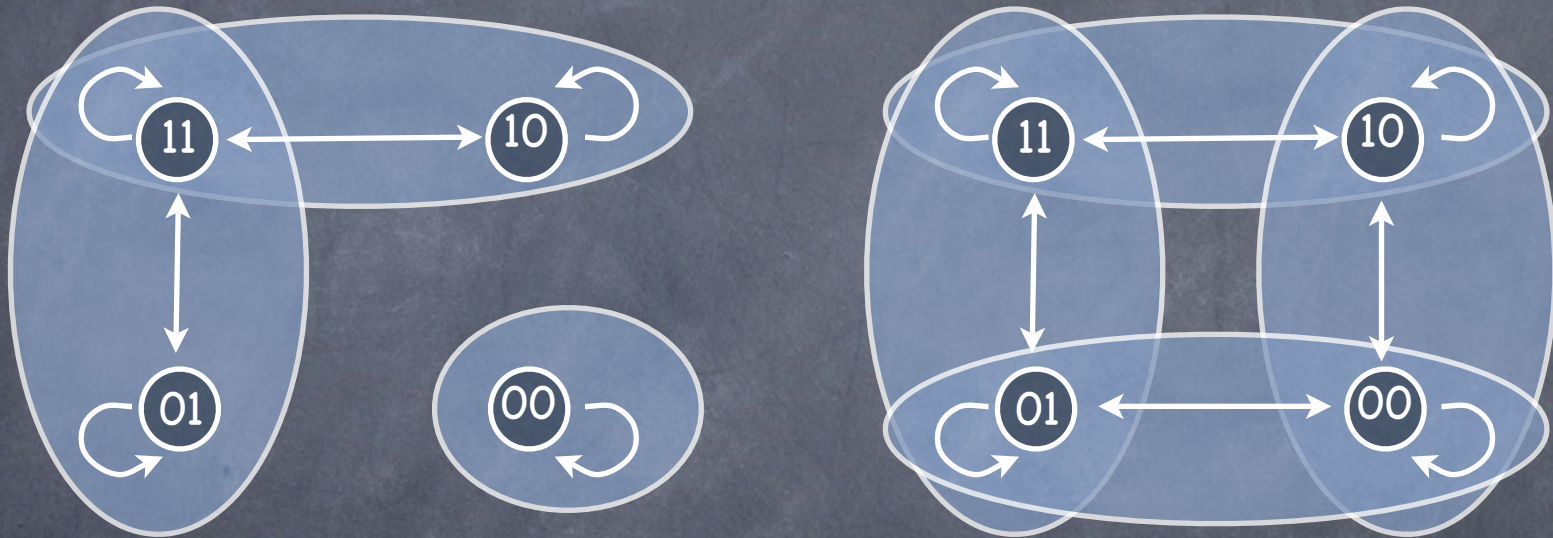
Alternative question  $?(p \vee q)$

Choice question  $(?p \vee ?q)$



Alternative question  $?(p \vee q)$

Choice question  $(?p \vee ?q)$



Questions: inquisitive, not  
informative



# Meanings

Reflexive Closure of Symmetric Relation



# Meanings

## Reflexive Closure of Symmetric Relation

• The **meaning** of  $\varphi$ :  $\langle \varphi \rangle^I = \{ \iota \mid \iota \in I^2 \text{ \& } \iota \models \varphi \}$



# Meanings

## Reflexive Closure of Symmetric Relation

📌 The **meaning** of  $\varphi$ :  $\langle \varphi \rangle^I = \{ \iota \mid \iota \in I^2 \text{ \& } \iota \models \varphi \}$

☑  $\langle \varphi \rangle^I$  is a symmetric and reflexive relation  
on a subset of the set of all indices



# Meanings

## Reflexive Closure of Symmetric Relation

📌 The **meaning** of  $\varphi$ :  $\langle \varphi \rangle^I = \{ \iota \mid \iota \in I^2 \text{ \& } \iota \models \varphi \}$

☑  $\langle \varphi \rangle^I$  is a symmetric and reflexive relation  
on a subset of the set of all indices

☐ For all  $\varphi$ , for all  $i$  and  $j$ :



# Meanings

## Reflexive Closure of Symmetric Relation

📌 The **meaning** of  $\varphi$ :  $\langle \varphi \rangle^I = \{ \iota \mid \iota \in I^2 \text{ \& } \iota \models \varphi \}$

☑  $\langle \varphi \rangle^I$  is a symmetric and reflexive relation  
on a subset of the set of all indices

☐ For all  $\varphi$ , for all  $i$  and  $j$ :

1.  $\langle i, j \rangle \models \varphi$  iff  $\langle j, i \rangle \models \varphi$



# Meanings

## Reflexive Closure of Symmetric Relation

📌 The **meaning** of  $\varphi$ :  $\langle \varphi \rangle^I = \{ \iota \mid \iota \in I^2 \text{ \& } \iota \models \varphi \}$

☑  $\langle \varphi \rangle^I$  is a symmetric and reflexive relation  
on a subset of the set of all indices

□ For all  $\varphi$ , for all  $i$  and  $j$ :

1.  $\langle i, j \rangle \models \varphi$  iff  $\langle j, i \rangle \models \varphi$

2. If  $\langle i, j \rangle \models \varphi$  then  $\langle i, i \rangle \models \varphi$  and  $\langle j, j \rangle \models \varphi$



# Meanings

## Reflexive Closure of Symmetric Relation

📌 The **meaning** of  $\varphi$ :  $\langle \varphi \rangle^I = \{ \iota \mid \iota \in I^2 \text{ \& } \iota \models \varphi \}$

☑  $\langle \varphi \rangle^I$  is a symmetric and reflexive relation  
on a subset of the set of all indices

□ For all  $\varphi$ , for all  $i$  and  $j$ :

1.  $\langle i, j \rangle \models \varphi$  iff  $\langle j, i \rangle \models \varphi$

2. If  $\langle i, j \rangle \models \varphi$  then  $\langle i, i \rangle \models \varphi$  and  $\langle j, j \rangle \models \varphi$

Note: for assertions this runs in both directions



# Meanings

## Reflexive Closure of Symmetric Relation

📌 The **meaning** of  $\varphi$ :  $\langle \varphi \rangle^I = \{ \iota \mid \iota \in I^2 \text{ \& } \iota \models \varphi \}$

☒  $\varphi \models \psi$  iff  $\langle \varphi \rangle^I \subseteq \langle \psi \rangle^I$

☒  $\langle \varphi \rangle^I$  is a symmetric and reflexive relation  
on a subset of the set of all indices

☐ For all  $\varphi$ , for all  $i$  and  $j$ :

1.  $\langle i, j \rangle \models \varphi$  iff  $\langle j, i \rangle \models \varphi$

2. If  $\langle i, j \rangle \models \varphi$  then  $\langle i, i \rangle \models \varphi$  and  $\langle j, j \rangle \models \varphi$

Note: for assertions this runs in both directions



# Data and Issues



# Data and Issues

- The meaning of  $\varphi$  is a symmetric and reflexive relation on a subset of the set of indices  $I$



# Data and Issues

- The meaning of  $\varphi$  is a symmetric and reflexive **relation** on a **subset** of the set of indices  $I$ 
  - Which subset of  $I$  is determined by the **data** provided by  $\varphi$



# Data and Issues

- The meaning of  $\varphi$  is a symmetric and reflexive **relation** on a **subset** of the set of indices  $I$ 
  - Which subset of  $I$  is determined by the **data** provided by  $\varphi$
  - The relation determines the **issues** raised by  $\varphi$



# Data and Issues

- The meaning of  $\varphi$  is a symmetric and reflexive **relation** on a **subset** of the set of indices  $I$ 
  - Which subset of  $I$  is determined by the **data** provided by  $\varphi$
  - The relation determines the **issues** raised by  $\varphi$

When  $i$  and  $j$  are **not related** in the meaning of  $\varphi$ , some **difference** between them **matters**, according to  $\varphi$



# Data and Issues

- The meaning of  $\varphi$  is a symmetric and reflexive **relation** on a **subset** of the set of indices  $I$ 
  - Which subset of  $I$  is determined by the **data** provided by  $\varphi$
  - The relation determines the **issues** raised by  $\varphi$   of indifference

When  $i$  and  $j$  are **not related** in the meaning of  $\varphi$ , some **difference** between them **matters**, according to  $\varphi$



# Possibilities and Pictures



# Possibilities and Pictures

• Let  $\langle \varphi \rangle^I$  be the meaning of  $\varphi$

So,  $\langle \varphi \rangle^I$  is a reflexive and symmetric relation on some subset of  $I$



# Possibilities and Pictures

• Let  $\langle \varphi \rangle^I$  be the meaning of  $\varphi$

So,  $\langle \varphi \rangle^I$  is a reflexive and symmetric relation on some subset of  $I$

1.  $P$  is a **possibility** for  $\varphi$  iff

(i)  $P \subseteq I$  and for all  $i, j \in P : \langle i, j \rangle \in \langle \varphi \rangle^I$



# Possibilities and Pictures

• Let  $\langle \varphi \rangle^I$  be the meaning of  $\varphi$

So,  $\langle \varphi \rangle^I$  is a reflexive and symmetric relation on some subset of  $I$

1.  $P$  is a **possibility** for  $\varphi$  iff

(i)  $P \subseteq I$  and for all  $i, j \in P : \langle i, j \rangle \in \langle \varphi \rangle^I$

(ii) There is no  $P' : P \subset P'$  and (i) holds for  $P'$



# Possibilities and Pictures

• Let  $\langle \varphi \rangle^I$  be the meaning of  $\varphi$

So,  $\langle \varphi \rangle^I$  is a reflexive and symmetric relation on some subset of  $I$

1.  $P$  is a **possibility** for  $\varphi$  iff

(i)  $P \subseteq I$  and for all  $i, j \in P : \langle i, j \rangle \in \langle \varphi \rangle^I$

(ii) There is no  $P' : P \subset P'$  and (i) holds for  $P'$

2.  $\pi(\varphi)$  is a **picture** of  $\varphi$  iff  $\pi(\varphi)$  is the set of possibilities for  $\varphi$



# Possibilities and Pictures

• Let  $\langle \varphi \rangle^I$  be the meaning of  $\varphi$

So,  $\langle \varphi \rangle^I$  is a reflexive and symmetric relation on some subset of  $I$

Largest set of worlds which are all related to each other

1.  $P$  is a **possibility** for  $\varphi$  iff

(i)  $P \subseteq I$  and for all  $i, j \in P : \langle i, j \rangle \in \langle \varphi \rangle^I$

(ii) There is no  $P' : P \subset P'$  and (i) holds for  $P'$

2.  $\pi(\varphi)$  is a **picture** of  $\varphi$  iff  $\pi(\varphi)$  is the set of possibilities for  $\varphi$

Set of propositions



# Possibilities and Pictures

Let  $\langle \varphi \rangle^I$  be the meaning of  $\varphi$

☒  $\varphi \models \psi$  iff every  $P \in \pi(\varphi)$  there is some  $P' \in \pi(\psi)$  such that  $P \subseteq P'$

Largest set of worlds which are all related to each other

1.  $P$  is a **possibility** for  $\varphi$  iff

(i)  $P \subseteq I$  and for all  $i, j \in P : \langle i, j \rangle \in \langle \varphi \rangle^I$

(ii) There is no  $P' : P \subset P'$  and (i) holds for  $P'$

2.  $\pi(\varphi)$  is a **picture** of  $\varphi$  iff  $\pi(\varphi)$  is the set of possibilities for  $\varphi$

Set of propositions



# Overlapping Possibilities



# Overlapping Possibilities

- 📌  $\varphi$  is **classical** iff no two possibilities in the picture of  $\varphi$  overlap



# Overlapping Possibilities

- 📌  $\varphi$  is **classical** iff no two possibilities in the picture of  $\varphi$  overlap
- 👁️ By definition a possibility for  $\varphi$  cannot be included in another possibility



# Overlapping Possibilities

- $\varphi$  is **classical** iff no two possibilities in the picture of  $\varphi$  overlap
- By definition a possibility for  $\varphi$  cannot be included in another possibility
- But two different possibilities **may overlap**



# Overlapping Possibilities

- $\varphi$  is **classical** iff no two possibilities in the picture of  $\varphi$  overlap
- By definition a possibility for  $\varphi$  cannot be included in another possibility
- But two different possibilities **may overlap**
  - If **no** two possibilities for  $\varphi$  **overlap**,  $\pi(\varphi)$  is a **partition** of a subset of  $I$



# Overlapping Possibilities

- $\varphi$  is **classical** iff no two possibilities in the picture of  $\varphi$  overlap
- By definition a possibility for  $\varphi$  cannot be included in another possibility
- But two different possibilities **may overlap**
  - If **no** two possibilities for  $\varphi$  **overlap**,  $\pi(\varphi)$  is a **partition** of a subset of  $I$
  - and  $\langle \varphi \rangle^I$  is an **equivalence relation** on a subset of  $I$



# Overlapping Possibilities

📌  $\varphi$  is **classical** iff no two possibilities in the picture of  $\varphi$  overlap

👁 By definition a possibility for  $\varphi$  cannot be included in another possibility

👁 But two different possibilities **may overlap**

**NEW!**

□ If **no** two possibilities for  $\varphi$  **overlap**,  
 $\pi(\varphi)$  is a **partition** of a subset of  $I$

□ and  $\langle \varphi \rangle^I$  is an **equivalence relation** on a subset of  $I$



# Inquisitive Logic



# Inquisitive Logic

- Logical notion of **licensing**



# Inquisitive Logic

- Logical notion of **licensing**

- A notion of contextual relatedness



# Inquisitive Logic

- Logical notion of **licensing**

- A notion of contextual relatedness
- Judges whether a sentence fits the context



# Inquisitive Logic

## • Logical notion of **licensing**

- A notion of contextual relatedness
- Judges whether a sentence fits the context
- Information and issues the sentence embodies should be logically related to the contextual issues



# Licensing



# Licensing



A stimulus  $\psi$  **licenses** a response  $\varphi$ ,  $\psi \propto \varphi$  iff



# Licensing



A stimulus  $\psi$  **licenses** a response  $\varphi$ ,  $\psi \propto \varphi$  iff

every possibility in the picture of  $\varphi$   
is the union of some possibilities in the  
picture of **? $\psi$**



# Licensing



A stimulus  $\psi$  **licenses** a response  $\varphi$ ,  $\psi \propto \varphi$  iff

every possibility in the picture of  $\varphi$   
is the union of some possibilities in the  
picture of **? $\psi$**

□ Note: licensing considers the **question behind**  
the stimulus



# Licensing

📌 A stimulus  $\psi$  **licenses** a response  $\varphi$ ,  $\psi \propto \varphi$  iff

every possibility in the picture of  $\varphi$   
is the union of some possibilities in the  
picture of **? $\psi$**

□ Note: licensing considers the **question behind**  
the stimulus

🌀 The notion of licensing intends to  
characterize when a sentence  $\varphi$  is **logically**  
**related** to a sentence  $\psi$



Licensing allows critical responses

Top of the iceberg!



# Licensing allows critical responses

Top of the iceberg!

- Licensing allows for assertions to be confirmed, denied, doubted, and accepted



# Licensing allows critical responses

Top of the iceberg!

👁️ Licensing allows for assertions to be confirmed, denied, doubted, and accepted

☑️  $! \varphi$  licenses  $! \varphi$ ,  $\neg \varphi$ ,  $? ! \varphi$ , and  $\top$  (silence)



# Licensing allows critical responses

Top of the iceberg!

- Licensing allows for assertions to be confirmed, denied, doubted, and accepted

☒  $! \varphi$  licenses  $! \varphi$ ,  $\neg \varphi$ ,  $? ! \varphi$ , and  $\top$  (silence)

☐ and up to equivalence nothing else



# Licensing allows critical responses

Top of the iceberg!

- Licensing allows for assertions to be confirmed, denied, doubted, and accepted

☒  $! \varphi$  licenses  $! \varphi$ ,  $\neg \varphi$ ,  $? ! \varphi$ , and  $\top$  (silence)

☐ and up to equivalence nothing else

To show that  $\top$  corresponds to acceptance requires some Gricean reasoning



# Licensing and partial answerhood



# Licensing and partial answerhood

Partial answerhood to a question is a clear case of logical relatedness



# Licensing and partial answerhood

Partial answerhood to a question is a clear case of logical relatedness

☒  $? \psi \propto ! \varphi$  corresponds to partial answerhood



# Licensing and partial answerhood

Partial answerhood to a question is a clear case of logical relatedness

☑  $?ψ \propto !φ$  corresponds to partial answerhood

👁 **Every possibility** in the picture of  $!φ$  is the union of some possibilities in the picture of  **$?ψ$**  (is what the definition tells us)



# Licensing and partial answerhood

Partial answerhood to a question is a clear case of logical relatedness

☒  $? \psi \propto ! \varphi$  corresponds to partial answerhood

👁 **Every possibility** in the picture of  $! \varphi$  is the union of some possibilities in the picture of  **$? \psi$**  (is what the definition tells us)



👁 **The** possibility in the picture of  $! \varphi$  is the union of some possibilities in the picture of  **$? \psi$**



# Classical licensing and entailment



# Classical licensing and entailment

☑ If  $\varphi$  and  $\psi$  are **classical**, then  
 $\psi \propto \varphi$  iff  $? \psi \models ? \varphi$

Licensing reducible  
to entailment

□ For classical  $? \psi$  and  $? \varphi$

$? \psi \models ? \varphi$  means:  $? \varphi$  is a subquestion of  $? \psi$



# Classical licensing and entailment

✓ If  $\varphi$  and  $\psi$  are **classical**, then  
 $\psi \propto \varphi$  iff  $? \psi \models ? \varphi$

Licensing reducible  
to entailment

Does not hold for  
non-classical case

□ For classical  $? \psi$  and  $? \varphi$

$? \psi \models ? \varphi$  means:  $? \varphi$  is a subquestion of  $? \psi$

□ For classical  $? \psi$  and  $! \varphi$

$? \psi \models ? ! \varphi$  means  $! \varphi$  is a partial answer  
to  $? \psi$



# Licensing and conditional questions



# Licensing and conditional questions

•  $(p \rightarrow ?q) \propto (p \rightarrow q)$  and  $(p \rightarrow ?q) \propto (p \rightarrow \neg q)$



# Licensing and conditional questions

•  $(p \rightarrow ?q) \propto (p \rightarrow q)$  and  $(p \rightarrow ?q) \propto (p \rightarrow \neg q)$

□  $(p \rightarrow q)$  and  $(p \rightarrow \neg q)$  are answers to  $(p \rightarrow ?q)$



# Licensing and conditional questions

•  $(p \rightarrow ?q) \propto (p \rightarrow q)$  and  $(p \rightarrow ?q) \propto (p \rightarrow \neg q)$

□  $(p \rightarrow q)$  and  $(p \rightarrow \neg q)$  are answers to  $(p \rightarrow ?q)$

• However:  $(p \rightarrow ?q) \not\equiv ?(p \rightarrow q)$ , (the other way around!)



# Licensing and conditional questions

•  $(p \rightarrow ?q) \propto (p \rightarrow q)$  and  $(p \rightarrow ?q) \propto (p \rightarrow \neg q)$

□  $(p \rightarrow q)$  and  $(p \rightarrow \neg q)$  are answers to  $(p \rightarrow ?q)$

• However:  $(p \rightarrow ?q) \not\models ?(p \rightarrow q)$ , (the other way around!)

☑ For non-classical sentences

$\psi \propto \varphi$  iff  $? \psi \models ? \varphi$  does not always hold



# Licensing and conditional questions

•  $(p \rightarrow ?q) \propto (p \rightarrow q)$  and  $(p \rightarrow ?q) \propto (p \rightarrow \neg q)$

□  $(p \rightarrow q)$  and  $(p \rightarrow \neg q)$  are answers to  $(p \rightarrow ?q)$

• However:  $(p \rightarrow ?q) \not\models ?(p \rightarrow q)$ , (the other way around!)

☒ For non-classical sentences

$\psi \propto \varphi$  iff  $? \psi \models ? \varphi$  does not always hold

□  $? \psi \models ? ! \varphi$  no longer means that  $! \varphi$  is a (partial) answer to  $? \psi$



# Licensing and atomic questions



# Licensing and atomic questions

👁 **Not:**  $?p \propto (q \rightarrow ?p)$       and      **Not:**  $?p \propto (?p \vee ?q)$



# Licensing and atomic questions

👁 **Not:**  $?p \propto (q \rightarrow ?p)$       and      **Not:**  $?p \propto (?p \vee ?q)$

- $?p$  is an 'atomic' question and has no subquestions



# Licensing and atomic questions

👁 **Not:**  $?p \propto (q \rightarrow ?p)$       and      **Not:**  $?p \propto (?p \vee ?q)$

□  $?p$  is an 'atomic' question and has no subquestions

👁 But:  $?p \models (q \rightarrow ?p)$       and       $?p \models (?p \vee ?q)$



# Licensing and atomic questions

👁 **Not:**  $?p \propto (q \rightarrow ?p)$       and      **Not:**  $?p \propto (?p \vee ?q)$

□  $?p$  is an 'atomic' question and has no subquestions

👁 But:  $?p \models (q \rightarrow ?p)$       and       $?p \models (?p \vee ?q)$

☑ For non-classical sentences

$\psi \propto \varphi$  iff  $? \psi \models ? \varphi$  does not always hold



# Licensing and atomic questions

👁 **Not:**  $?p \propto (q \rightarrow ?p)$       and      **Not:**  $?p \propto (?p \vee ?q)$

□  $?p$  is an 'atomic' question and has no subquestions

👁 But:  $?p \models (q \rightarrow ?p)$       and       $?p \models (?p \vee ?q)$

☑ For non-classical sentences

$\psi \propto \varphi$  iff  $? \psi \models ? \varphi$  does not always hold

□  $? \psi \models ? \varphi$  no longer means that  $? \varphi$  is a subquestion of  $? \psi$



# Licensing and atomic questions

$$\boxed{\checkmark} \quad (?p \vee ?q) \propto ?p$$

👁 **Not:**  $?p \propto (q \rightarrow ?p)$       and      **Not:**  $?p \propto (?p \vee ?q)$

□  $?p$  is an 'atomic' question and has no subquestions

👁 But:  $?p \models (q \rightarrow ?p)$       and       $?p \models (?p \vee ?q)$

$\boxed{\checkmark}$  For non-classical sentences

$\psi \propto \varphi$  iff  $? \psi \models ? \varphi$  does not always hold

□  $? \psi \models ? \varphi$  no longer means that  $? \varphi$  is a subquestion of  $? \psi$



# Conclusion



# Conclusion

- Inquisitive semantics gives rise to a new inquisitive logic



# Conclusion

- Inquisitive semantics gives rise to a new inquisitive logic
- Entailment no longer suffices



# Conclusion

- Inquisitive semantics gives rise to a new inquisitive logic
- Entailment no longer suffices
- Licensing is the new crucial notion



# Conclusion

- Inquisitive semantics gives rise to a new inquisitive logic
- Entailment no longer suffices
- Licensing is the new crucial notion
- It embodies a notion of logical relatedness which rules the logical coherence of dialogue



# Conclusion

- Inquisitive semantics gives rise to a new inquisitive logic
- Entailment no longer suffices
- Licensing is the new crucial notion
- It embodies a notion of logical relatedness which rules the logical coherence of dialogue
- Unlike entailment, licensing is concerned with the communicative function of language



# Grice on Disjunction

In 'Indicative Conditionals', Grice (1989:68), as cited in Simons (2000)



# Grice on Disjunction

In 'Indicative Conditionals', Grice (1989:68), as cited in Simons (2000)

A standard (if not **the** standard) employment of "or" is in the specification of possibilities (one of which is supposed by the speaker to be realized, although he does not know which one), each of which is relevant in the same way to a given topic. 'A or B' is characteristically employed to give a partial answer to some [wh]-question, to which each disjunct, if assertible, would give a fuller, more specific, more satisfactory answer.



# Grice on Disjunction

In 'Indicative Conditionals', Grice (1989:68), as cited in Simons (2000)

A standard (if not **the** standard) employment of "or" is in the **specification of possibilities** (one of which is supposed by the speaker to be realized, although he does not know which one), each of which is relevant in the same way to a given topic. 'A or B' is characteristically employed to give a partial answer to some [wh]-question, to which each disjunct, if assertible, would give a fuller, more specific, more satisfactory answer.



# Grice on Disjunction

In 'Indicative Conditionals', Grice (1989:68), as cited in Simons (2000)

A standard (if not **the** standard) employment of "or" is in the **specification of possibilities** (one of which is **inquisitive semantics of "or"** realized, although he does not know which one), each of which is relevant in the same way to a given topic. 'A or B' is characteristically employed to give a partial answer to some [wh]-question, to which each disjunct, if assertible, would give a fuller, more specific, more satisfactory answer.



# Grice on Disjunction

In 'Indicative Conditionals', Grice (1989:68), as cited in Simons (2000)

A standard (if not **the** standard) employment of "or" is in the **specification of possibilities** (one of which is **speculative** in the **inquisitive semantics of "or"** realized, although he does not know which one), **each** of which is **relevant** in the same way **to a given topic**. 'A or B' is characteristically employed to give a partial answer to some [wh]-question, to which each disjunct, if assertible, would give a fuller, more specific, more satisfactory answer.



# Grice on Disjunction

In 'Indicative Conditionals', Grice (1989:68), as cited in Simons (2000)

A standard (if not **the** standard) employment of "or" is in the **specification of possibilities** (one of which is **speculative** inquisitive semantics of "or" realized, although he does not know which one), **each** of which is **relevant** in the same way **to a given topic**. **required by licensing** typically employed to give a partial answer to some [wh]-question, to which each disjunct, if assertible, would give a fuller, more specific, more satisfactory answer.



# Grice on Disjunction

In 'Indicative Conditionals', Grice (1989:68), as cited in Simons (2000)

met by inquisitive semantics

specification of possibilities

inquisitive semantics of "or"

each  
to a given topic

relevant  
required by licensing