Computational Epistemology for Quantifiers

Nina Gierasimczuk

Institute for Logic, Language and Computation
Universiteit van Amsterdam

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Outline

Plan

1. Problems
2. Quantifiers
   - Quantifiers of type $\langle 1 \rangle$
   - Quantifiers of type $\langle 1, 1 \rangle$
3. Computational Epistemology
4. Identifiability
5. General question

Problems

- Epistemological properties of quantifiers.
- Their influence on NL comprehension.
- Linking them to learnability features.
- Compare notions of decidability and identifiability.

Plan

Quantifiers

Lindström definition

Definition

A monadic generalized quantifier of type $\underbrace{(1, \ldots, 1)}_{n}$ is a class Q of structures of the form $\mathbf{M} = (M, A_1, \ldots, A_n)$, where A_i is a subset of M. Additionally, Q is closed under isomorphism.

Q of type $\langle 1 \rangle$

Monotonicity

Definition

$\mathsf{Q}_{\mathbf{M}}$ is MON↑ iff: if $A \subseteq A' \subseteq M$, then $\mathsf{Q}_{\mathbf{M}}(A)$ implies $\mathsf{Q}_{\mathbf{M}}(A')$.

Definition

$\mathsf{Q}_{\mathbf{M}}$ is MON↓ iff: if $A' \subseteq A \subseteq M$, then $Q_{\mathbf{M}}(A)$ implies $Q_{\mathbf{M}}(A')$.

Q of type $\langle 1 \rangle$

Extendability

Definition

Q of type $\langle 1 \rangle$ satisfies *EXT* iff for all models $\mathbf{M}$ and $\mathbf{M}'$:
$A \subseteq M \subseteq M'$ implies $\mathsf{Q}_{\mathbf{M}}(A) \iff \mathsf{Q}_{\mathbf{M}'}(A)$.

Quantifiers of type $\langle 1, 1 \rangle$

Preconditions

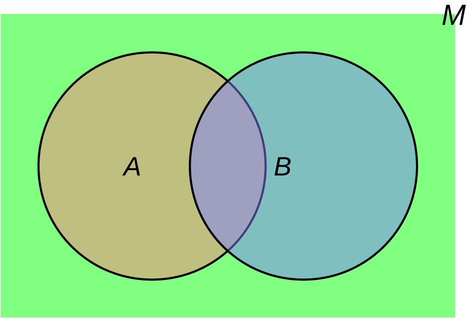
Restriction to CE quantifiers.

Definition

Let Q be of type $\langle 1, 1 \rangle$.
Then for all $\mathbf{M}$, $\mathbf{M}'$, all $A, B \subseteq M$, and $A', B' \subseteq M'$:
(ISOM) If $(M, A, B) \cong (M', A', B')$, then $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M}'}(A', B')$.
(CONS) $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M}}(A, A \cap B)$.
(EXT) If $M \subseteq M'$, then $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M}'}(A, B)$.

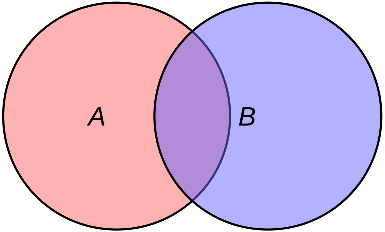
CE Quantifiers

(ISOM) If $(M, A, B) \cong (M', A', B')$, then $\mathsf{Q}_M(A, B) \Leftrightarrow \mathsf{Q}_{M'}(A', B')$



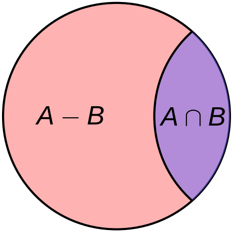
CE Quantifiers - EXT

(EXT) If $M \subseteq M'$, then $Q_M(A, B) \Leftrightarrow Q_{M'}(A, B)$



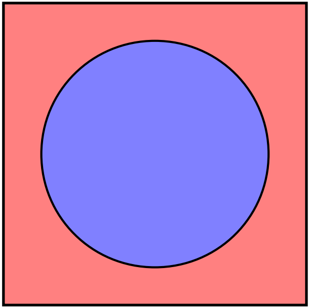
CE Quantifiers - CONS

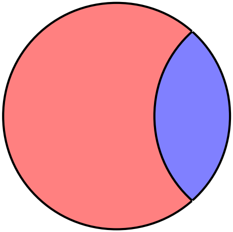
(CONS) $\mathsf{Q}_{\mathsf{M}}(A, B) \Leftrightarrow \mathsf{Q}_{\mathsf{M}}(A, A \cap B)$



CE QUANTIFIERS

The scope we are interested in for both $\langle 1 \rangle$ and $\langle 1, 1 \rangle$ cases.





Q OF TYPE $\langle 1, 1 \rangle$

MONOTONICITY

DEFINITION

Q of type $\langle 1, 1 \rangle$ is MON$\uparrow$ iff:
If $A \subseteq M$ and $B \subseteq B' \subseteq M$, then $\mathsf{Q}_{\mathbf{M}}(A, B) \Rightarrow \mathsf{Q}_{\mathbf{M}}(A, B')$.

Q of type $\langle 1, 1 \rangle$

Persistence

Definition

Q of type $\langle 1, 1 \rangle$ is PER iff:
If $A \subseteq A' \subseteq M$ and $B \subseteq M$, then $\mathsf{Q}_{\mathbf{M}}(A, B) \Rightarrow \mathsf{Q}_{\mathbf{M}}(A', B)$.

EXAMPLES

Determiner	MON $\uparrow$	EXT (for $\langle 1 \rangle$)	PER (for $\langle 1, 1 \rangle$)
All	+	-	-
No	-	-	-
Some	+	+	+
At least *n*	+	+	+
At most *n*	-	+	-
Exactly *n*	-	+	-

TABLE: Quantifiers and their properties

Monotonicity & linguistics

- Does monotonicity influence NL comprehension?
- Does monotonicity influence NL learning?
- Monotonicity and inference patterns (B. Geurts).
- Proposal: focus on persistence.

Plan

Logic of reliable inquiry - Kevin Kelly

- Inspired by learning theory.
- Similar framework.
- Verification/falsification in computational setting.

Logic of reliable inquiry - Kevin Kelly

ε — infinite string of data;
$\varepsilon|n$ — finite initial segment of ε through the position $n-1$;
h — hypothesis;
C — correctness relation;
$C(\varepsilon, h)$ iff h is correct w.r.t. ε;
α — an assessment method;
OUT conjectures 1, 0, !.

Certainty in reliable inquiry

Definition

α produces b with certainty on (h, ε) iff there is an n s.t.:

1. $\alpha(h, \varepsilon | n) = !$, and
2. $\alpha(h, \varepsilon | n + 1) = b$, and
3. for each $m < n$, $\alpha(h, \varepsilon | m) \neq !$.

Certainty in reliable inquiry

Definition

α verifies h with certainty on ε (with respect to C) iff α produces 1 with certainty on $(h, \varepsilon) \Leftrightarrow C(\varepsilon, h)$.

Definition

α refutes h with certainty on ε (with respect to C) iff α produces 0 with certainty on $(h, \varepsilon) \Leftrightarrow \neg C(\varepsilon, h)$.

Definition

Decidability with certainty is simply verifiability and refutability with certainty at the same time.

EXAMPLES

- At least six bikes are broken. - Verifiable with certainty
- An even number of bikes is broken. - Verifiable in the limit

EPISTEMOLOGICAL PROPERTIES OF Q O.T. $\langle 1 \rangle$

- $1-1$ enumeration of elements of the universe.
- Assignment of χ_A to each of them.
- Infinite sequence of 0s and 1s.
- In each step checking if finite initial segment satisfies a hypothesis (quantifier sentence).

EPISTEMOLOGICAL PROPERTIES OF Q O.T. $\langle 1 \rangle$

PROPOSITION

Let Q be FO quantifier of type $\langle 1 \rangle$*.*
Q is MON $\uparrow$ *and EXT iff it is verifiable with certainty.*

PROPOSITION

Let Q be FO quantifier of type $\langle 1 \rangle$*.*
$\neg$*Q is verifiable with certainty iff Q is falsifiable with certainty.*

Epistemological properties of Q o.t. $\langle 1, 1 \rangle$

Proposition

Let Q be FO CE quantifier of type $\langle 1, 1 \rangle$.
Q is persistent iff it is verifiable with certainty.

Proposition

Let Q be FO CE quantifier of type $\langle 1, 1 \rangle$.
$\neg$Q is falsifiable with certainty iff Q is verifiable with certainty.

Examples

Determiner	verifiable	falsifiable	MON $\uparrow$	EXT (for $\langle 1 \rangle$)	PER (for $\langle 1, 1 \rangle$)
All	-	+	+	-	-
No	-	+	-	-	-
Some	+	-	+	+	+
At least n	+	-	+	+	+
At most n	-	+	-	+	-
Exactly n	-	-	-	+	-

TABLE: Quantifiers and their properties

Plan

IDENTIFIABILITY GAME

- Class of objects is chosen (e.g. class of grammars).
- Player 1 picks out one object from the class (e.g. G).
- Player 1 generates positive instances of this object, repetitions allowed (e.g. words from a language of G).
- Player 2 knows about the class, but he does not know which object is chosen.
- Player 2 has to guess which object Player 1 has in mind.

Learning the semantics of natural language

Identifiability from text in use

- Class of quantifiers is chosen.
- Player 1 picks one of them (Q)
- Player 2 is presented finite worlds in which Q is true.
- Player 2 has to identify Q.

Number triangle representation

Assuming CE, we can represent all relevant models in the form of number triangle.

				(0,0)				
			(1,0)		(0,1)			
		(2,0)		(1,1)		(0,2)		
	(3,0)		(2,1)		(1,2)		(0,3)	
(4,0)		(3,1)		(2,2)		(1,3)		(0,4)
...								...

Number triangle representation

- Graphic representation of a class of CE quantifiers.
- In particular: PER.

$$
\begin{array}{ccccccccccc}
 & & & & & - & & & & & \\
 & & & & - & & + & & & & \\
 & & & - & & + & & + & & & \\
 & & - & & + & & + & & + & & \\
 & - & & + & & + & & + & & + & \\
\ldots & & & & & & & & & & \ldots
\end{array}
$$

Number triangle representation

- Graphic representation of a class of CE quantifiers.
- In particular: PER.

$$
\begin{array}{ccccccccc}
 & & & & - & & & & \\
 & & & - & & \boxed{+} & & & \\
 & & - & & + & & + & & \\
 & - & & + & & + & & + & \\
- & & + & & + & & + & & + \\
\ldots & & & & & & & & \ldots
\end{array}
$$

Number triangle representation

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- In particular: PER.

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Number triangle representation

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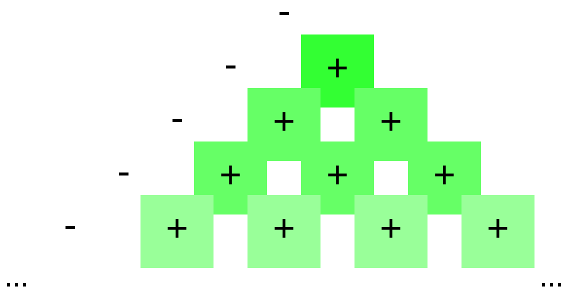
- In particular: PER.



TIEDE'S RESULT

THEOREM

The class of FO PER Q is identifiable from text.

Plan

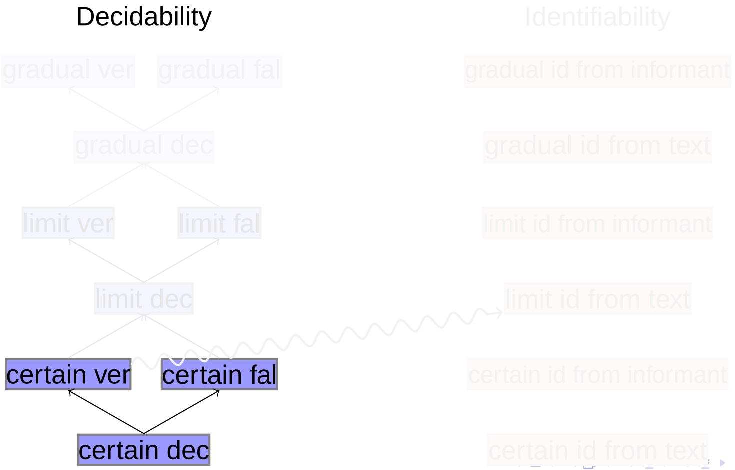
General question

Relation between Ver/Fal hierarchy and identifiability

Decidability   Identifiability

General question

Relation between Ver/Fal hierarchy and identifiability



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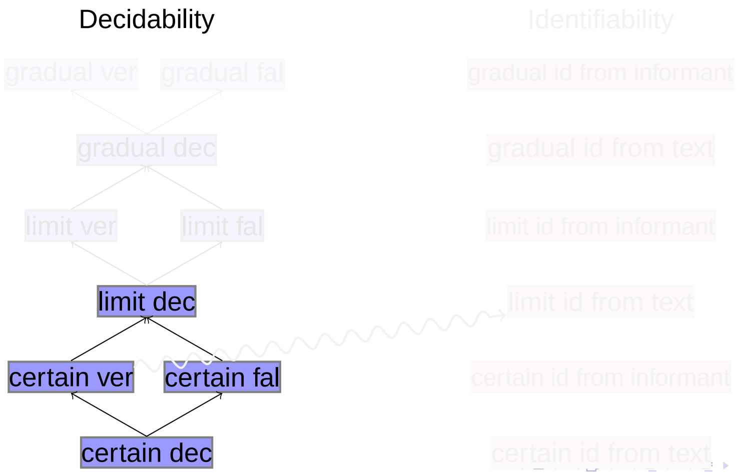

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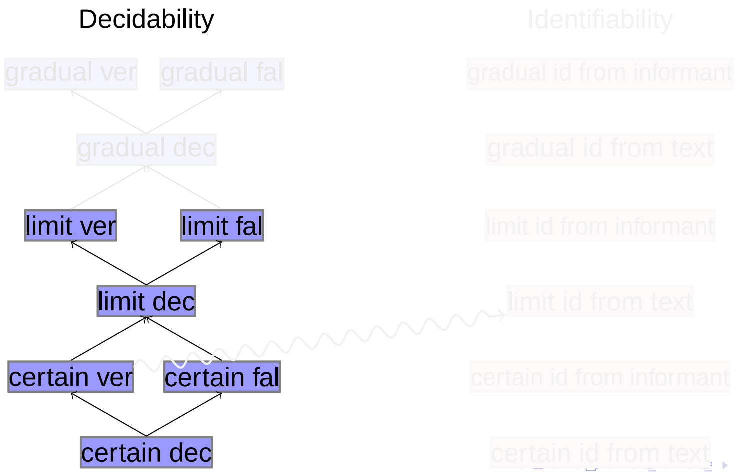

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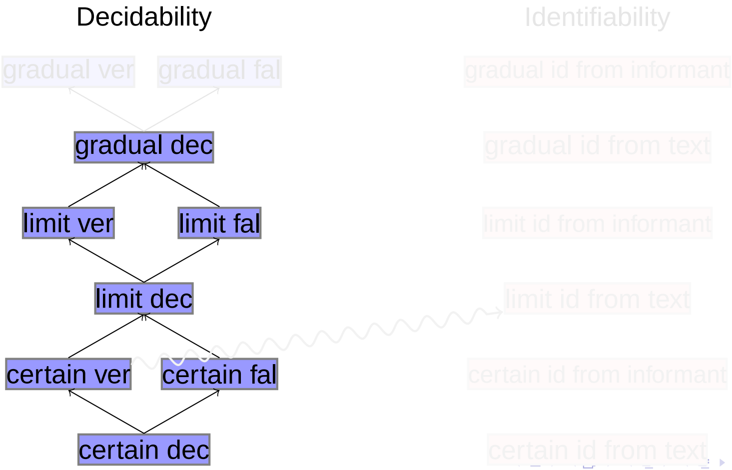

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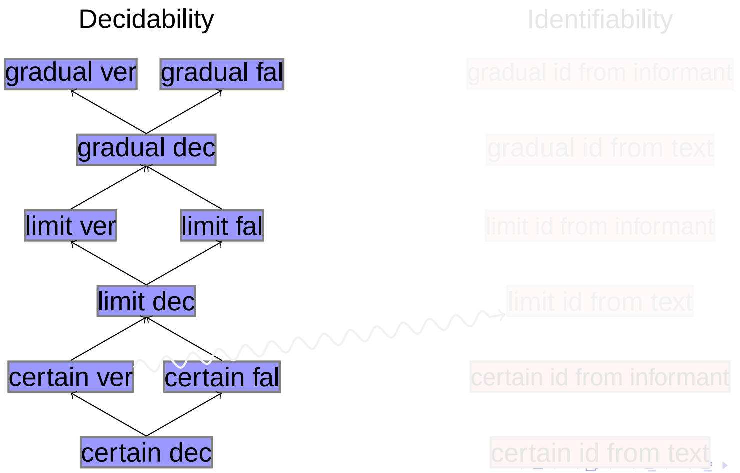
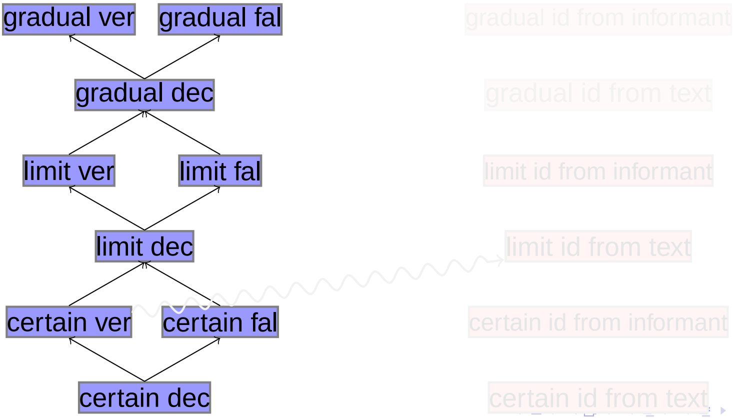


General question

Relation between Ver/Fal hierarchy and identifiability

Decidability

Identifiability



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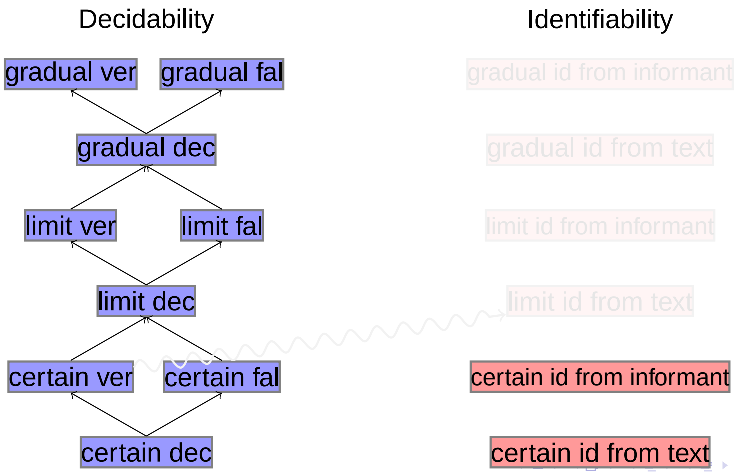

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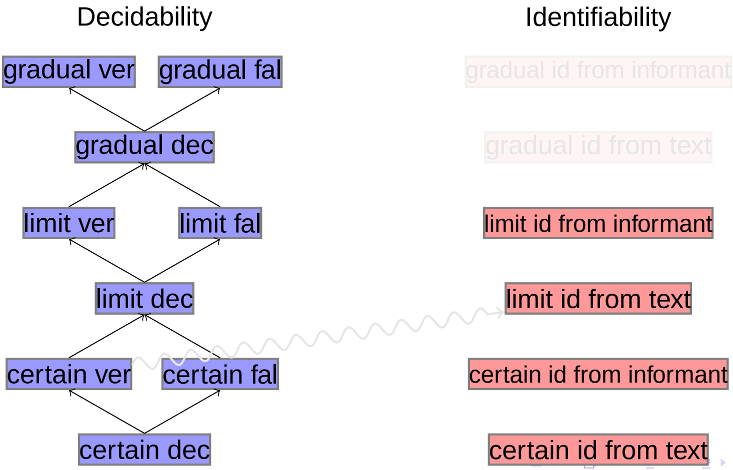

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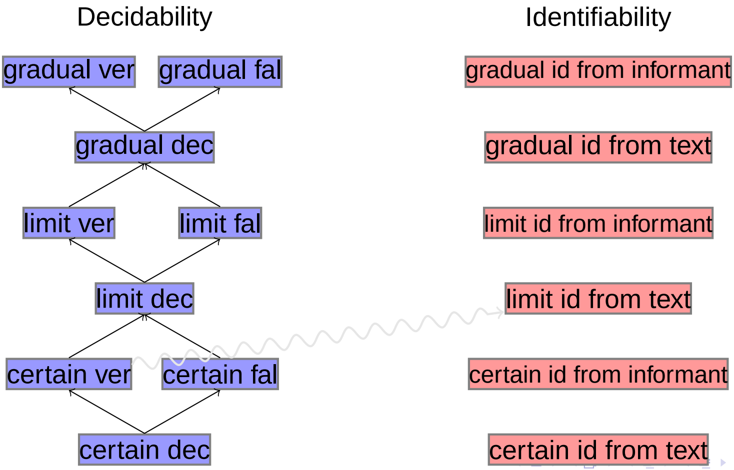

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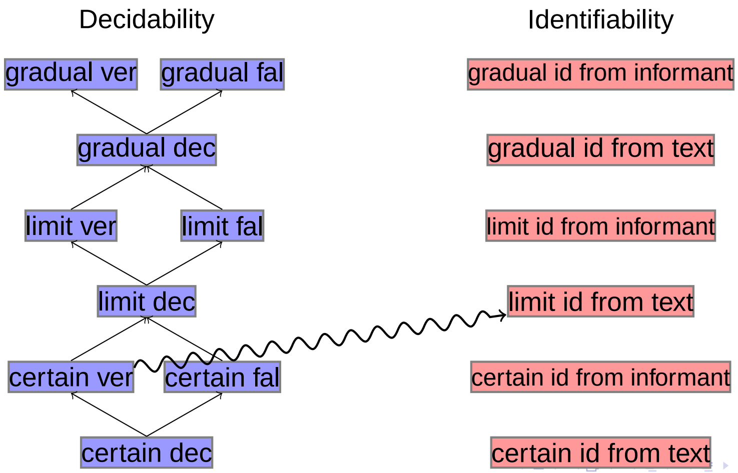

Conclusions and future work

- Epistemological role of monotonicity - additional explanation.
- Verification less difficult than falsification?
- Check connections between persistence and comprehension.
- Investigate relationship between identifiability and decidability: learning of NL semantics; new conditions of identifiability.

TH$\forall$NK YOU!