Pro-aperiodic monoids via Stone duality

Sam van Gool

University of Amsterdam

This talk is about joint work with B. Steinberg, in which we apply Stone duality and model theory to study free pro-aperiodic monoids.

The class of aperiodic monoids has long played a fundamental role in finite semigroup theory and automata theory. The famous Schützenberger theorem proved that the aperiodic monoids recognize precisely the star-free languages, which also coincides with the class of languages recognizable by counter-free automata. The connection with logic comes from a later result, which shows that the class is also exactly the class of languages definable in first order logic.

Algorithmic questions about aperiodic languages lead to challenges that the algebraic approach can often help resolve. Within this algebraic approach to aperiodic languages, free pro-aperiodic monoids are a useful tool. The structure of free pro-aperiodic monoids has been studied recently by several authors, but many difficult questions remain open. Existing results about free pro-aperiodic monoids are often about the submonoid of elements definable by ω-terms and rely on an ingenuous normal form algorithm due to McCammond, which solves the word problem for ω-terms.

Stone duality and Schützenberger's theorem together imply that elements of the free pro-aperiodic monoid may be viewed as elementary equivalence classes of pseudofinite words. Concretely, this means that one may 'compute' with elements of the free pro-aperiodic monoid as if they were finite words, in a way reminiscent of the methods of non-standard analysis. In particular, model theory provides us with saturated words in each class of pseudofinite words, i.e., words in which all possible factorizations are realized. We prove that such saturated words are stable under algebraic operations. We give several applications of this new approach, including a solution to the word problem for ω-terms that avoids using McCammonds normal forms, as well as new proofs and extensions of other structural results concerning free pro-aperiodic monoids.