Multialgebraic First-Order Structures for **QmbC**

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Abstract

In this paper we present an adequate semantics for the first-order paraconsistent logic **QmbC**. That semantics is based on multialgebras known as *swap structures*.

1 The logic QmbC

The class of paraconsistent logics known as *Logics of Formal Inconsistency* (**LFI**s, for short) was introduced by W. Carnielli and J. Marcos in [3] and studied in [2] and [1]. **LFI**s are characterized for having a (primitive or derived) *consistency connective* $\circ$ which allows to recover the explosion law in a controlled way. The logic **mbC** is the weakest system in the hierarchy of **LFI**s and the system **QmbC** is the extension of **mbC** to first-order languages. The goal of this paper is to introduce an algebraic-like semantics for **QmbC** based on multialgebraic structures called *swap structures* (see [1]), which naturally induce non-deterministic matrices. The semantical framework for **QmbC** we present here can be seen as a generalization of the standard semantics approach for classical first-order logic, in which a logical matrix induced by a Boolean algebra is replaced by a non-deterministic matrix induced by a Boolean algebra.

The logic **mbC** (see [2, 1]) is defined over the propositional signature $\Sigma = \{\wedge, \vee, \to, \neg, \circ\}$ by adding to **CPL**$^+$ (positive classical propositional logic) the following axiom schemas:

(Ax10) $\alpha \vee \neg\alpha$ and **(Ax11)** $\circ\alpha \to (\alpha \to (\neg\alpha \to \beta))$.

Recall that a *multialgebra* (or *hyperalgebra*) over a signature Σ' is a pair $\mathcal{A} = \langle A, \sigma_{\mathcal{A}}\rangle$ such that A is a nonempty set (the *support* of $\mathcal{A}$) and $\sigma_{\mathcal{A}}$ is a mapping assigning to each n-ary $\#$ in Σ', a function (called *multioperation* or *hyperoperation*) $\#^{\mathcal{A}} : A^n \to (\mathcal{P}(A) - \{\emptyset\})$. In particular, $\emptyset \neq \#^{\mathcal{A}} \subseteq A$ if $\#$ is a constant in Σ'. A *non-deterministic matrix (or Nmatrix)* over Σ' is a pair $\mathcal{M} = \langle \mathcal{A}, D\rangle$ such that $\mathcal{A}$ is a multialgebra over Σ' with support A, and D is a subset of A. The elements in D are called *designated* elements.

Let $\mathcal{A} = (A, \wedge, \vee, \to, 0, 1)$ be a complete Boolean algebra, and $B_{\mathcal{A}} = \{z \in A^3 \ : \ z_1 \vee z_2 = 1 \ \text{and} \ z_1 \wedge z_2 \wedge z_3 = 0\}$, where z_i denote the *ith*-projection of z. A swap structure for **mbC** over $\mathcal{A}$ is any multialgebra $\mathcal{B} = (B, \tilde{\wedge}, \tilde{\vee}, \tilde{\to}, \tilde{\neg}, \tilde{\circ})$ over Σ, such that $B \subseteq B_{\mathcal{A}}$ and, for every z and w in B:

(i) $\emptyset \neq z\tilde{\#}w \subseteq \{u \in B \ : \ u_1 = z_1 \# w_1\}$, for each $\# \in \{\wedge, \vee, \to\}$;
(ii) $\emptyset \neq \tilde{\neg}z \subseteq \{u \in B \ : \ u_1 = z_2\}$;
(iii) $\emptyset \neq \tilde{\circ}z \subseteq \{u \in B \ : \ u_1 = z_3\}$.

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The support B will be also denoted by $|\mathcal{B}|$. The *full swap structure for* ***mbC*** *over* $\mathcal{A}$, denoted by $\mathcal{B}_{\mathcal{A}}$, is the unique swap structure for **mbC** over $\mathcal{A}$ with domain $B_{\mathcal{A}}$, in which '$\subseteq$' is replaced by '$=$' in items (i)-(iii) above.

Definition 1.1 ([1], Definition 7.1.5)**.** Let Θ be a first-order signature. The logic **QmbC** over Θ is obtained from the Hilbert calculus **mbC** by adding the following axioms and rules:

(Ax12) $\varphi[x/t] \to \exists x\varphi$, if t is a term free for x in φ

(Ax13) $\forall x\varphi \to \varphi[x/t]$, if t is a term free for x in φ

(Ax14) $\alpha \to \beta$, whenever α is a variant[1] of β

($\exists$-In) $\dfrac{\varphi \to \psi}{\exists x\varphi \to \psi}$, where x does not occur free in ψ

($\forall$-In) $\dfrac{\varphi \to \psi}{\varphi \to \forall x\psi}$, where x does not occur free in φ

If Θ is a first-order signature then $For(\Theta)$, $Sen(\Theta)$ and $CTer(\Theta)$ will denote the set of formulas, closed formulas and closed terms over Θ. Var is the set of individual variables. The consequence relation of **QmbC** will be denoted by $\vdash_{\mathbf{QmbC}}$. If $\Gamma \cup \{\varphi\} \subseteq For(\Theta)$, then $\Gamma \vdash_{\mathbf{QmbC}} \varphi$ will denote that there exists a derivation in **QmbC** of φ from Γ.

2 First-Order Swap Structures: Soundness

Given a swap structure $\mathcal{B}$ for **mbC**, the non-deterministic matrix induced by $\mathcal{B}$ is $\mathcal{M}(\mathcal{B}) = (\mathcal{B}, D)$ such that $D = \{z \in |\mathcal{B}| \ : \ z_1 = 1\}$. The logic **mbC** is sound and complete w.r.t. swap structures semantics, see [1], Chapter 6.

A *(first-order) structure* over $\mathcal{M}(\mathcal{B})$ and Θ is a pair $\mathfrak{A} = \langle U, I_{\mathfrak{A}}\rangle$ such that U is a nonempty set (the domain of the structure) and $I_{\mathfrak{A}}$ is an interpretation mapping which assigns to each individual constant $c \in \mathcal{C}$, an element $c^{\mathfrak{A}}$ of U; to each function symbol f of arity n, a function $f^{\mathfrak{A}} : U^n \to U$; and to each predicate symbol P of arity n, a function $P^{\mathfrak{A}} : U^n \to |\mathcal{B}|$.

Let $\mathfrak{A}$ be a structure over $\mathcal{M}(\mathcal{B})$ and Θ. A function $\mu : Var \to U$ is called an *assignment* over $\mathfrak{A}$. Let $\mathfrak{A}$ be a structure and let $\mu : Var \to U$ be an assignment. For each term t, we define $||t||^{\mathfrak{A}}_{\mu} \in U$ such that: $||c||^{\mathfrak{A}}_{\mu} = c^{\mathfrak{A}}$ if c is an individual constant; $||x||^{\mathfrak{A}}_{\mu} = \mu(x)$ if x is a variable; $||f(t_1, \ldots, t_n)||^{\mathfrak{A}}_{\mu} = f^{\mathfrak{A}}(||t_1||^{\mathfrak{A}}_{\mu}, \ldots, ||t_n||^{\mathfrak{A}}_{\mu})$ if f is a function symbol of arity n and $t_1, \ldots, t_n$ are terms. If t is closed (i.e., has no variables) we will simply write $||t||^{\mathfrak{A}}$, since μ plays no role.

Given a structure $\mathfrak{A}$ over Θ, the signature Θ_U is obtained from $\mathfrak{A}$ by adding to Θ a new individual constant $\bar{a}$ for any $a \in U$. The expansion $\widehat{\mathfrak{A}}$ of $\mathfrak{A}$ to Θ_U is defined by interpreting $\bar{a}$ as a. Any assignment $\mu : Var \to U$ induces a function $\widehat{\mu} : For(\Theta_U) \to Sen(\Theta_U)$ such that $\widehat{\mu}(\varphi)$ is the sentence obtained from φ by replacing any free variable x by the constant $\overline{\mu(x)}$.

Definition 2.1 (**QmbC**-valuations)**.** Let $\mathcal{M}(\mathcal{B}) = (\mathcal{B}, D)$ be the non-deterministic matrix induced by a swap structure $\mathcal{B}$ for **mbC**, and let $\mathfrak{A}$ be a structure over Θ and $\mathcal{M}(\mathcal{B})$. A mapping $v : Sen(\Theta_U) \to |\mathcal{B}|$ is a **QmbC**-*valuation* over $\mathfrak{A}$ and $\mathcal{M}(\mathcal{B})$, if it satisfies the following clauses:
(i) $v(P(t_1, \ldots, t_n)) = P^{\widehat{\mathfrak{A}}}(||t_1||^{\widehat{\mathfrak{A}}}, \ldots, ||t_n||^{\widehat{\mathfrak{A}}})$, if $P(t_1, \ldots, t_n)$ is atomic;
(ii) $v(\#\varphi) \in \tilde{\#}v(\varphi)$, for every $\# \in \{\neg, \circ\}$;

[1] That is, φ can be obtained from ψ by means of addition or deletion of void quantifiers, or by renaming bound variables (keeping the same free variables in the same places).

(iii) $v(\varphi\#\psi) \in v(\varphi)\tilde{\#}v(\psi)$, for every $\# \in \{\wedge, \vee, \rightarrow\}$;
(iv) $v(\forall x\varphi) \in \{z \in |\mathcal{B}| \ : \ z_1 = \bigwedge\{v(\varphi[x/\bar{a}]) \ : \ a \in U\}\}$;
(v) $v(\exists x\varphi) \in \{z \in |\mathcal{B}| \ : \ z_1 = \bigvee\{v(\varphi[x/\bar{a}]) \ : \ a \in U\}\}$;
(vi) Let t be free for z in φ and ψ, μ an assignment and $b = ||t||^{\widehat{\mathfrak{A}}}_{\mu}$. Then:
(vi.1) if $v(\widehat{\mu}(\varphi[z/t])) = v(\widehat{\mu}(\varphi[z/\bar{b}]))$, then $v(\widehat{\mu}(\#\varphi[z/t])) = v(\widehat{\mu}(\#\varphi[z/\bar{b}]))$, for $\# \in \{\neg, \circ\}$;
(vi.2) if $v(\widehat{\mu}(\varphi[z/t])) = v(\widehat{\mu}(\varphi[z/\bar{b}]))$ and $v(\widehat{\mu}(\psi[z/t])) = v(\widehat{\mu}(\psi[z/\bar{b}]))$, then $v(\widehat{\mu}(\varphi\#\psi[z/t])) = v(\widehat{\mu}(\varphi\#\psi[z/\bar{b}]))$, for $\# \in \{\wedge, \vee, \rightarrow\}$;
(vi.3) let x be such that $x \neq z$ and x does not occur in t, and let μ^x_a such that $\mu^x_a(y) = \mu(y)$, if $y \neq x$ and $\mu^x_a(y) = a$, if $y = x$. If $v(\widehat{\mu^x_a}(\varphi[z/t])) = v(\widehat{\mu^x_a}(\varphi[z/\bar{b}]))$, for every $a \in U$, then $v(\widehat{\mu}((Qx\varphi)[z/t])) = v(\widehat{\mu}((Qx\varphi)[z/\bar{b}]))$, for $Q \in \{\forall, \exists\}$;
(vii) If α and α' are variant, then $v(\alpha) = v(\alpha')$.

Given μ and v let $v_\mu : For(\Theta_U) \rightarrow |\mathcal{B}|$ such that $v_\mu(\varphi) = v(\widehat{\mu}(\varphi))$ for every φ.

Theorem 2.2 (Substitution Lemma)**.** *Given $\mathcal{M}(\mathcal{B})$, $\mathfrak{A}$, a **QmbC**-valuation v and an assignment μ, if t is free for z in φ and $b = ||t||^{\widehat{\mathfrak{A}}}_{\mu}$, then: $v_\mu(\varphi[z/t]) = v_\mu(\varphi[z/\bar{b}])$.*

Definition 2.3. Given $\mathcal{M}(\mathcal{B})$ and $\mathfrak{A}$, let $\Gamma \cup \{\varphi\} \subseteq For(\Theta_U)$. Then φ is a *semantical consequence of* Γ *over* $(\mathfrak{A}, \mathcal{M}(\mathcal{B}))$, denoted by $\Gamma \models_{(\mathfrak{A},\mathcal{M}(\mathcal{B}))} \varphi$, if the following holds: for every **QmbC**-valuation v over $(\mathfrak{A}, \mathcal{M}(\mathcal{B}))$, if $v_\mu(\gamma) \in D$, for every $\gamma \in \Gamma$ and every μ, then $v_\mu(\varphi) \in D$, for every μ. And φ is said to be a *semantical consequence of* Γ *in **QmbC** w.r.t. first-order swap structures*, denoted by $\Gamma \models_{\mathbf{QmbC}} \varphi$, if $\Gamma \models_{(\mathfrak{A},\mathcal{M}(\mathcal{B}))} \varphi$ for every $(\mathfrak{A}, \mathcal{M}(\mathcal{B}))$.

Theorem 2.4 (Soundness of **QmbC** w.r.t. first-order swap structures)**.** *For every set $\Gamma \cup \{\varphi\} \subseteq For(\Theta)$: if $\Gamma \vdash_{\mathbf{QmbC}} \varphi$, then $\Gamma \models_{\mathbf{QmbC}} \varphi$.*

3 First-Order Swap Structures: Completeness

Let $\Delta \subseteq Sen(\Theta)$ and let C be a nonempty set of constants of the signature Θ. Then, Δ is called a *C-Henkin theory* in **QmbC** if it satisfies the following: for every sentence of the form $\exists x\varphi$ in $Sen(\Theta)$, there exists a constant c in C such that if $\Delta \vdash_{\mathbf{QmbC}} \exists x\varphi$ then $\Delta \vdash_{\mathbf{QmbC}} \varphi[x/c]$.

Let Θ_C be the signature obtained from Θ by adding a set C of new individual constants. The consequence relation $\vdash^C_{\mathbf{QmbC}}$ is the consequence relation of **QmbC** over the signature Θ_C.

Recall that, given a Tarskian and finitary logic $\mathbf{L} = \langle For, \vdash\rangle$ (where For is the set of formulas of $\mathbf{L}$), and given a set $\Gamma \cup \{\varphi\} \subseteq For$, the set Γ is said to be *maximally non-trivial with respect to* φ *in* $\mathbf{L}$ if the following holds: (i) $\Gamma \nvdash \varphi$, and (ii) $\Gamma, \psi \vdash \varphi$ for every $\psi \notin \Gamma$.

Proposition 3.1 ([1], Corollary 7.5.4)**.** *Let $\Gamma \cup \{\varphi\} \subseteq Sen(\Theta)$ such that $\Gamma \nvdash_{\mathbf{QmbC}} \varphi$. Then, there exists a set of sentences $\Delta \subseteq Sen(\Theta)$ which is maximally non-trivial with respect to φ in **QmbC** (by restricting $\vdash_{\mathbf{QmbC}}$ to sentences) and such that $\Gamma \subseteq \Delta$.*

Definition 3.2. Let $\Delta \subseteq Sen(\Theta)$ be a C-Henkin and non-trivial theory in **QmbC**. Let $\equiv_\Delta \subseteq Sen(\Theta)^2$ be the relation in $Sen(\Theta)$ defined as follows: $\alpha \equiv_\Delta \beta$ iff $\Delta \vdash_{\mathbf{QmbC}} \alpha \rightarrow \beta$ and $\Delta \vdash_{\mathbf{QmbC}} \beta \rightarrow \alpha$.

Clearly $\equiv_\Delta$ is an equivalence relation. In the quotient set $A_\Delta \overset{\text{def}}{=} Sen(\Theta)/_{\equiv_\Delta}$ define $\wedge, \vee, \rightarrow$ as follows: $[\alpha]_\Delta\#[\beta]_\Delta \overset{\text{def}}{=} [\alpha\#\beta]_\Delta$ for any $\# \in \{\wedge, \vee, \rightarrow\}$, where $[\alpha]_\Delta$ is the equivalence class of α w.r.t. Δ. Then, $\mathcal{A}_\Delta \overset{\text{def}}{=} \langle A_\Delta, \wedge, \vee, \rightarrow, 0_\Delta, 1_\Delta\rangle$ is a Boolean algebra with $0_\Delta \overset{\text{def}}{=} [\varphi \wedge \neg\varphi \wedge \circ\varphi]_\Delta$ and $1_\Delta \overset{\text{def}}{=} [\varphi \vee \neg\varphi]_\Delta$, for any φ. Moreover, for every formula $\psi(x)$ with at most x occurring free, $[\forall x\psi]_\Delta = \bigwedge_{\mathcal{A}_\Delta}\{[\psi[x/t]_\Delta \ : \ t \in CTer(\Theta)\}$, and $[\exists x\psi]_\Delta = \bigvee_{\mathcal{A}_\Delta}\{[\psi[x/t]_\Delta \ : \ t \in CTer(\Theta)\}$.

Let $\mathcal{CA}_\Delta$ be the MacNeille-Tarski completion of $\mathcal{A}_\Delta$ and let $* : \mathcal{A}_\Delta \to \mathcal{CA}_\Delta$ be the associated monomorphism. Let $\mathcal{B}_\Delta$ be the full swap structure over $\mathcal{CA}_\Delta$ with associated Nmatrix $\mathcal{M}(\mathcal{B}_\Delta) \stackrel{\text{def}}{=} (\mathcal{B}_\Delta, D_\Delta)$. Note that $(([\alpha]_\Delta)^*, ([\beta]_\Delta)^*, ([\gamma]_\Delta)^*) \in D_\Delta$ iff $\Delta \vdash_{\mathbf{QmbC}} \alpha$.

Definition 3.3. Let $\Delta \subseteq Sen(\Theta)$ be C-Henkin and non-trivial in **QmbC**, let $\mathcal{M}(\mathcal{B}_\Delta)$ be as above, and let $U = CTer(\Theta)$. The *canonical structure induced by* Δ is the structure $\mathfrak{A}_\Delta = \langle U, I_{\mathfrak{A}_\Delta}\rangle$ over $\mathcal{M}(\mathcal{B}_\Delta)$ and Θ such that: $c^{\mathfrak{A}_\Delta} = c$ for each constant c; $f^{\mathfrak{A}_\Delta} : U^n \to U$ is such that $f^{\mathfrak{A}_\Delta}(t_1, \ldots, t_n) = f(t_1, \ldots, t_n)$ for each n-ary function symbol f; and $P^{\mathfrak{A}_\Delta}(t_1, \ldots, t_n) = (([\varphi]_\Delta)^*, ([\neg\varphi]_\Delta)^*, ([\circ\varphi]_\Delta)^*)$ where $\varphi = P(t_1, \ldots, t_n)$, for each n-ary predicate symbol P.

Let $(\cdot)^\triangleright : Ter(\Theta_U) \to Ter(\Theta)$ be the mapping such that $(t)^\triangleright$ is the term obtained from t by substituting every occurrence of a constant $\bar{s}$ by the term s itself. Observe that $(t)^\triangleright = \widehat{||t||^{\mathfrak{A}_\Delta}}$ for every $t \in CTer(\Theta_U)$. This mapping can be naturally extended to a mapping $(\cdot)^\triangleright : For(\Theta_U) \to For(\Theta)$ such that $(\varphi)^\triangleright$ is the formula in $For(\Theta)$ obtained from $\varphi \in For(\Theta_U)$ by substituting every occurrence of a constant $\bar{t}$ by the term t itself.

Definition 3.4. Let $\Delta \subseteq Sen(\Theta)$ be a C-Henkin theory in **QmbC** for a nonempty set C of individual constants of Θ, such that Δ is maximally non-trivial with respect to φ in **QmbC**, for some sentence φ. The *canonical **QmbC**-valuation induced by* Δ *over* $\mathfrak{A}_\Delta$ *and* $\mathcal{M}(\mathcal{B}_\Delta)$ is the mapping $v_\Delta : Sen(\Theta_U) \to |\mathcal{B}_\Delta|$ such that $v_\Delta(\psi) = (([(\psi)^\triangleright]_\Delta)^*, ([\neg(\psi)^\triangleright]_\Delta)^*, ([\circ(\psi)^\triangleright]_\Delta)^*)$.

Remark 3.5. Note that $v_\Delta(\psi) \in D_\Delta$ iff $\Delta \vdash_{\mathbf{QmbC}} \psi$, for every sentence $\psi \in Sen(\Theta)$.

Theorem 3.6. *The canonical **QmbC**-valuation v_Δ is a **QmbC**-valuation over $\mathfrak{A}_\Delta$ and $\mathcal{M}(\mathcal{B}_\Delta)$.*

Theorem 3.7 (Completeness for sentences of **QmbC** w.r.t. first-order swap structures)**.** *Let $\Gamma \cup \{\varphi\} \subseteq Sen(\Theta)$. If $\Gamma \vDash_{\mathbf{QmbC}} \varphi$ then $\Gamma \vdash_{\mathbf{QmbC}} \varphi$.*

Proof. Suppose that $\Gamma \nvdash_{\mathbf{QmbC}} \varphi$. Then, there exists a C-Henkin theory Δ^H over Θ_C in **QmbC** for a nonempty set C of new individual constants such that $\Gamma \subseteq \Delta^H$ and, for every $\alpha \in Sen(\Theta)$: $\Gamma \vdash_{\mathbf{QmbC}} \alpha$ iff $\Delta^H \vdash^C_{\mathbf{QmbC}} \alpha$. Hence, $\Delta^H \nvdash^C_{\mathbf{QmbC}} \varphi$ and so, by Proposition 3.1, there exists a set of sentences $\overline{\Delta^H}$ in Θ_C extending Δ^H which is maximally non-trivial with respect to φ in **QmbC**, such that $\overline{\Delta^H}$ is a C-Henkin theory over Θ_C in **QmbC**. Now, let $\mathcal{M}(\mathcal{B}_{\overline{\Delta^H}})$, $\mathfrak{A}_{\overline{\Delta^H}}$ and $v_{\overline{\Delta^H}}$ as above. Then, $v_{\overline{\Delta^H}}(\alpha) \in D_{\overline{\Delta^H}}$ iff $\overline{\Delta^H} \vdash^C_{\mathbf{QmbC}} \alpha$, for every α in $Sent(\Theta_C)$. From this, $v_{\overline{\Delta^H}}[\Gamma] \subseteq D_{\overline{\Delta^H}}$ and $v_{\overline{\Delta^H}}(\varphi) \notin D_{\overline{\Delta^H}}$. Finally, let $\mathfrak{A}$ and v be the restriction to Θ of $\mathfrak{A}_{\overline{\Delta^H}}$ and $v_{\overline{\Delta^H}}$, respectively. Then, $\mathfrak{A}$ is a structure over $\mathcal{M}(\mathcal{B}_{\overline{\Delta^H}})$, and v is a valuation for **QmbC** over $\mathfrak{A}$ and $\mathcal{M}(\mathcal{B}_{\overline{\Delta^H}})$ such that $v[\Gamma] \subseteq D_{\overline{\Delta^H}}$ but $v(\varphi) \notin D_{\overline{\Delta^H}}$. This shows that $\Gamma \nvDash_{\mathbf{QmbC}} \varphi$. □

It is easy to extend **QmbC** by adding a standard equality predicate $\approx$ such that $(a \approx^{\mathfrak{A}} b) \in D$ iff $a = b$. On the other hand, the extension of **QmbC** to other first-order **LFI**s based on well-known axiomatic extensions of **mbC** is straightforward, both syntactically and semantically.

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