LOGIC AND GAMES

Summary of points, start-up research seminar 16 Feb 2006

Johan van Benthem

Reasons for My Interest in the Interface Logic and Games

(a)   'Dynamic Turn' in 1980s: from logical semantics as the description of truth conditions in fixed models to richer description of logic-related actions that make information flow: evaluation procedures, information update, belief revision.

(b)   Next natural move: add many agents, and their 'social' interaction. Even basic logical activities .like dialogue and argumentation have this interactive character.

(c)   What seems needed for this purpose: marry logic with game theory. Note: not just 'application' of existing game theory in logic or of existing logic to game theory, since that creates unhealthy 'Dominator' relationships. Look for new joint topics: e.g., fine-structure of reasoning inside or about games, merge logic with game theory of various equilibria describing behaviour by (not so) rational agents.

My Own Research Topics

(d)   Abstraction levels for games as process equivalences: from extensive games to strategic forms. Analogy with spectrum of process theories in computer science. Matching logical viewpoint: design new languages (or finding old ones: not always reinvent the wheel!) matching natural description levels: modal, first-order, whatever. Best abstraction levels are not yet known.

(e)   Levels of describing relevant structure: moves, knowledge/belief, preference, probability. Each level represents a new area of investigation, and often involves some further branch of either mathematical or philosophical logic: dynamic logic, epistemic/doxastic logic, preference logic, probabilistic logics. 'Logic combinations' are popular these days, but they also pose many unsolved complexity problems!

(f)   Dynamics as a game proceeds: how does information grow in an extensive game, how do players revise beliefs as they observe moves that are played, how can their preferences change? Calls for merge between Amsterdam-style 'update logics' and game theory, which sometimes has its own take on the same phenomena.

Some key papers on these three topics (starting from 2001) can be downloaded from my website http://staff.science.uva.nl/~johan/research.html That page also reports on other interests, shared with students here, such as preference dynamics, or diversity of agents, challenging the 'homogeneity' in many current scenarios.

This way of thinking is a Grid where many current topics can be placed. Some people do pure action/outcome analysis, as in Parikh/Pauly style 'Logic of Games", others add epistemic structure, and all this either on extensive or on strategic forms.

Specific Example

I sketched a paper 'Rational Dynamics', on epistemic analysis of game-theoretic solution procedures using dynamic epistemic logic of iterated announcements. It points at analogies between epistemic puzzles like Muddy Children and successive steps in game solution procedures like Iterated Removal of Strictly Dominated Strategies, and develops a general framework in epistemic fixed-point logics for defining 'solution zones' of strategy profiles that survive the procedure.

General ambition: systematize existing 'epistemic characterizations' of game-theoretic equilibria, which now form - to my mind - a loose 'set of theorems' instead of a genuine 'theory'. I am discussing this with Krzysztof Apt, who has come to the same area with different tools and motivations, but in a highly congenial way. Once we have such a theory of solutions and equilibria, it should also be seen as part of a larger theory of rational (or irrational) action of interest beyond game theory.

Somewhat open question: Lift my iterated announcement account to extensive games. The paper has some suggestions, but no full-fledged analysis. I find this of interest also for its mixture of two processes: dynamics of update by real events, and dynamics of steps in deliberation, as we 'chew' on the information that we have.

Diversity of Topics and Perspectives

The 'logic' that you find in this encounter with game theory is pretty diverse. It ranges from mathematical logic to philosophical logic, and logics of computation. Also, there are 'logic games' played by logicians for certain purposes, and there are 'game logics' for analyzing general games. I myself think that, despite 'cultural differences', the two are very closely related, but some sane people strongly disagree.

Some Available Materials

First Wave dynamics: my 1996 book *Exploring Logical Dynamics*.

Teaching: My lecture notes *Logic in Games* (since 1999) are still available on paper, and I can get copies for those who are interested.

Specific publications: see above. I also recommend two *Open Problems* papers: one on Update Logics (http://dare.uva.nl/record/148382) and one on Logic and Games, available at our general website (http://www.illc.uva.nl/lgc/postings.html).