Predicate Logic with Anaphora
(seven inch version)

Paul Dekker
University of Amsterdam

Abstract
In this paper I make a case for a separate treatment of (singular) anaphoric pronouns within a predicate logic with anaphora (*PLA*). Discourse representation theoretic results (from Kamp 1981) can be formulated in a compositional way, without fiddling with orthodox notions of scope and binding. In contrast with its predecessor dynamic predicate logic (Groenendijk and Stokhof 1991), the system of *PLA* is a proper extension of ordinary predicate logic and it has a genuine update semantics. Moreover, in contrast with other compositional reformulations of *DRT*, the semantics of *PLA* remains well within the bounds of ordinary, extensional type theory.

The research for this paper was supported by the ESPRIT Basic Research Project 6852, DYANA, Dynamic Interpretation of Natural Language.

Introduction

In the area of natural language semantics recent years have witnessed an increased attention for the inherent dynamics of the process of interpretation and the subsequent development of systems of dynamic semantics. According to the dynamic view on meaning, which can be traced back to the work of Stalnaker, "the meaning of a sentence does not lie in its truth conditions, but rather in the way it changes (the representation of) the information of the interpreter" (Groenendijk and Stokhof 1991). Among the variety of phenomena that have been the subject of study within dynamic semantics, the phenomenon of intersentential anaphora probably has received most attention. Also in this paper, the dynamics of interpretation is mainly restricted "to that aspect of the meaning of sentences that concerns their potential to 'pass on' possible antecedents for subsequent anaphors, within and across sentence boundaries".

Historically, one can distinguish three main types of treatments of (the semantics of) anaphoric relationships. First of all there is the so-called E-type pronoun approach (most prominently, Evans 1985; Heim 1990) which has been opposed to what has been called the bound variable approach. Among the last, generally conceived of as rooted in the work of Peter Geach, representational (Kamp 1981) and compositional (Heim 1982; Groenendijk and Stokhof 1991) approaches have been distinguished. The present paper has grown out of the last-mentioned compositional tradition, but, as I hope to show, it transcends the distinctions between the three types of approach.

In almost all *compositional* approaches to anaphora, pronouns are associated with (syntactically free, but semantically somehow bound) variables (cf., among many others, Heim 1982; Barwise 1987; Rooth 1987; Zeevat 1989; Groenendijk and Stokhof 1991; Chierchia 1992; Pagin and Westerståhl 1993). Put in a nutshell, the semantic relationship between pronouns and their antecedents is established in a compositional way by associating both with variables, and defining the interpretation algorithm as a function updating information about the possible values of these variables. Thus, information about the value of antecedent terms is available when coindexed pronouns are encountered.

The main purpose of this paper is to show that the same empirical results can be obtained without labelling the subjects introduced by candidate antecedents with variables, that is, by *not* conflating natural language pronouns with a logic's variables. In fact, I will take the lead of natural language syntax, and explicitly distinguish (anaphoric) pronouns from (bound) variables. In the system of predicate logic with anaphora (*PLA*) which is presented in this paper, anaphoric relationships are accounted for, compositionally, by keeping track of the possible values of potential antecedent terms, not of the variables they have been associated with.

Although, historically speaking, the system of *PLA* grows out of the bound variable tradition, strictly speaking it departs from that paradigm. In accordance with the E-type tradition, ordinary static (truth-conditional) aspects of interpretation are left untouched. As we will see, the *PLA* notion of interpretation can also be given a representational formulation, as a function updating type-theoretical relation terms. By the distinction between pronouns and variables, basic *DRT*- and *DPL*-results can be accounted for employing ordinary notions of representation and information from extensional, uni-sorted type theory.

The paper is organized as follows. As the point of departure I use dynamic predicate logic (*DPL*, Groenendijk and Stokhof 1991), the first most perspicuous compositional system of interpretation that models the interpretation of anaphoric relationships. I will argue that precisely the association of subjects with variables, although appropriate for the dynamic interpretation of the language of *PL* itself, needlessly complicates matters when it is used to account for anaphora in natural language.

Section 2 presents the system of *PLA*, a predicate logic which employs pronouns as an additional category of terms. The interpretation of the language of *PLA* is defined as an update function on a domain of information states, with the characteristic feature that existentially quantified formulas introduce subjects to information states. These subjects, composed of the possible values of candidate antecedent terms, are the potential 'referents' of subsequent anaphoric pronouns.

Section 3 is devoted to a comparison between *PLA*, *PL* and *DPL*, and very concisely addresses the notions of representation and information as they are employed in *DPL*, *DRT*, *PLA*, and a representational correlate *RTA* of *PLA*.

1 Modeling subjects

1.1 Variables and subjects

With their influential paper 'Dynamic predicate logic' (which has circulated through the community since 1987), Groenendijk and Stokhof present a perspicuous reformulation of the semantics of classical predicate logic, which models anaphoric relationships between pronouns and indefinite noun phrases ('donkey anaphora'). The following two examples may serve to illustrate the phenomena dealt with:

(1) A farmer owns a donkey. He beats it.
(2) If a farmer owns a donkey he beats it.

The first of these examples is taken to mean that there is a farmer who owns and beats a donkey, and the second that every farmer beats every donkey he owns. These examples pose a problem for classical theories of interpretation. In the first place, it appears that their meanings must be construed, compositionally, from those of their constituent clauses *a farmer owns a donkey*, and *he beats it*, respectively. The problem is that on no classical analysis there is a non-adhoc way of relating the interpretation of the pronouns with that of their antecedents. The possible farmers and donkeys referred back to with the second clause are no recognizable part of the truth-conditional or propositional content of the first. Consequently, their interdependence, which is made explicit in our reformulations, is left unaccounted for. I will assume that the reader is familiar with these examples and with the problems they pose for canonical (static) theories of interpretation.

A dynamic semantics appears to be well suited to deal with the donkey examples above. In a dynamic semantics, sentences or formulas are not in the first place assigned a certain information content of their own; rather, they are interpreted relative to information states, and the result of interpreting a formula in an information state is always a new, 'updated', information state. The idea is that the interpretation of a piece of discourse involves a constant update and adjustment of the information which is passed on for the processing of subsequent discourse. Clearly, this dynamic perspective upon meaning gives us a handle to deal with the donkey examples above. If only, after processing the clause *a farmer owns a donkey*, we keep track of the possible farmers and donkeys owned, then we are able to interpret subsequent pronouns as referring back to them.

The *DPL* system models the interpretation of the above examples by defining interpretation as a function updating information about the values of variables. The result of interpreting *a farmer owns a donkey* is an information state which encodes the information that the value of a variable, say x, is a farmer who owns a donkey, which is the value of another variable, say y. By matching the pronouns *he* and *it* in the subsequent clause *he beats it* with these variables x and y, respectively, they can be co-valuated. Consequently, the interpretation of example 1 generates an information state which encodes that the value of x also beats y, or, put differently, that there is a value of a variable x which owns and beats a donkey which is the value of a variable y.

As its name already suggests, *DPL* in fact gives a dynamic semantics for the language of predicate logic. The most significant, and characteristic, fact about *DPL* interpretation is that it licenses the following equivalence, without any restriction on free occurrences of variables in ψ:

[Scope Theorem] $(\exists x\phi \wedge \psi) \Leftrightarrow \exists x(\phi \wedge \psi)$

In *DPL*, the semantic scope of an existential quantifier exceeds its syntactic scope: it can bind syntactically free variables occurring to the right of it. Thus, it mimicks the establishing of anaphoric relationships in natural language. Consider the following two intuitively equivalent examples (the first one of which is obviously similar to example 1), with the associated (simplified) translations:

(3) A man is riding through the park. He is whistling.
$(\exists x(Mx \wedge Rx) \wedge Wx)$

(4) A man who is riding through the park is whistling.
$\exists x((Mx \wedge Rx) \wedge Wx)$

In example 3, the pronoun *he* in *He is whistling* is translated with a variable (x in the subformula Wx) which occurs free from a syntactic point of view. However, the pronoun is preceded by an indefinite term which is translated with a quantifier binding the variable x. Employing the scope theorem, the free occurrence of the variable appears to be bound by this quantifier semantically, cf., the (*DPL*-equivalent) translation under 4. As a result, example 3, under this translation, is interpreted as claiming that there is a man who is riding through the park and who is whistling.

It may be beyond doubt that the semantic relationships between free variables and preceding existentially quantified structures in *DPL* resemble the ones between pronouns and their (indefinite) antecedents in natural language. Still, the *DPL* interpretation procedure can not, all by itself, be taken to model the establishing of anaphoric relationships in natural language interpretation. For, this presupposes that (occurrences of) natural language pronouns and their antecedents are uniquely associated with variables, indices, or names, the possible values of which can be kept track of in interpretation. (Similar presuppositions can be found in Heim 1982; Barwise 1987; Rooth 1987; Chierchia 1992; Dekker 1993.)

Here, I proceed upon the assumption that the natural language input to an interpretation algorithm in fact consists of unindexed syntactic structures, and, hence, a full *DPL*-style interpretation procedure capable of dealing with anaphoric relationships must achieve two things: firstly, it must decorate possible antecedents and pronouns with indices, and, secondly, it must keep track of their possible values. Now there is something odd about this way of proceeding. In view of donkey anaphora, what we have to account for is the correlation between the interpretation of pronouns and that of their antecedents. On a dynamic account of such anaphoric relationships, the possible values of (possible) antecedents must be passed on in the process of interpretation. But what is passed on in the *DPL* model of interpretation is not information about the values of possible antecedents, but information about the values of variables which are associated with potential antecedents.

The question that suggests itself here is whether it is necessary to do things in this, seemingly roundabout way. The aim of this paper is to show that it is not. As we will see, it is perfectly possible to relate the interpretation of pronouns and their antecedents in a more immediate way, without the intermediary use of variables. We will see, as well, that it is also profitable to do so.

In the system presented below, I use formulas from a language of predicate logic to represent the meanings of simple sentences or sentential clauses of natural language, like Groenendijk and Stokhof 1991 do. Furthermore, like in *DPL*, existentially quantified formulas are used to represent the context change potential of natural language indefinites. The difference with *DPL* is that the subjects introduced by these indefinites are not hooked up to the specific variables quantified over. As a consequence, (syntactically) free variables can not any longer be taken to refer back to such subjects, i.e., they no longer serve to represent the semantic contribution of anaphoric pronouns. For this reason, pronouns are introduced in this predicate logic language as a category of terms of their own.

It is worthwhile to notice that there is independent motivation for distinguishing pronouns from variables, like we do in this paper. In the first place, bound and anaphoric pronouns are also kept distinct in syntactic frameworks (cf., for instance, Reinhart 1983). In the second place, the two kinds of terms display a different semantic behaviour in the scope of modal or epistemic operators (cf., Groenendijk et al. 1994). And in the third place, precisely this distinction enables us to keep to the ordinary notions of scope and binding.

1.2 Information about subjects

In this section, I introduce the minimal notion of information which enables us to keep track of subjects which are introduced by indefinite noun phrases and which can be taken to be referred back to by pronouns. This notion has developed out of the *DPL* notion of information, and it very closely resembles the one in Dekker 1993's *EDPL*. In *DPL* and *EDPL*, the information transmitted in interpretation is information about the values of variables, cast in terms of sets of assignments of individuals to variables. Here, we will be dealing with information about these values themselves, and our information states consists of the sequences of individuals that are the values of variables in *(E)DPL*.

The subjects dealt with in this paper correspond to Lewis 1975's sets of verifying 'cases' and to what Fine 1984 calls the 'ranges' of his 'arbitrary objects'. Characteristic feature of our subjects is that they are partial, and interdependent objects. Subjects are *partial*, since their identity need not be absolutely determined. Furthermore, subjects are *interdependent*, since the value of one subject, e.g., an arbitrary number, may depend on that of another, e.g., an arbitrary *higher* number. So, although a single arbitrary object corresponds to a set of individuals, a sequence of n arbitrary objects corresponds, not to a sequence of n sets of individuals, but to a set of n-tuples of individuals, an n-place relation.

Put more concrete, an arbitrary farmer is modeled as the following set of individuals: $\{d \mid d \text{ is a farmer}\}$. And the set of pairs of individuals $\{\langle d, d'\rangle \mid d$ is a farmer and d' a donkey d beats$\}$ models an arbitrary farmer and an arbitrary donkey he owns. In terms of Lewis 1975, this last set of pairs of individuals is the set of cases verifying the clause *a farmer and a donkey he owns*.

The information states employed in *PLA* are formally defined as follows:

Definition 1 (Information states)

- $S^n = \mathcal{P}(D^n)$ is the set of information states about n subjects
- $S = \bigcup_{n \in \mathcal{N}} S^n$ is the set of information states

A state of information about n subjects can be pictured thus:

$$\left\{ \begin{array}{c} \langle d_1, \ldots, d_n \rangle \\ \\ \vdots \\ \\ \langle d'_1, \ldots, d'_n \rangle \end{array} \right\} \quad \begin{array}{l} \nwarrow \\ \leftarrow \ \text{Possible cases} \\ \swarrow \end{array}$$

$$\nwarrow \uparrow \nearrow$$

Possible values of subjects

For a state $s \in S^n$ and $0 < j \leq n$, and for any case case $e = \langle d_1, \ldots, d_n \rangle \in s$, d_j is a possible value of the j-th subject of s, and this value will also be indicated as e_j. An information state s contains the information that it's first subject is an odd number iff all possible values of the first subject are odd numbers, i.e., iff e_1 is an odd number for every case e in s. Furthermore, such a state s contains the information that the i-th subject is a man who owns a donkey which is the j-th subject, iff, for every case e in s, e_i is a man which owns e_j, which is a donkey.

With respect to a specific number of subjects the following special types of information states are distinguished. The minimal state $\top^n$ of information about n subjects is D^n, the set of all possible n-tuples of individuals. The subjects of a minimal information state can have all values. A maximal information state about n subjects is $\{e\}$ for any $e = \langle d_1, \ldots, d_n \rangle \in D^n$. A maximal state completely determines the value of its subjects. Finally, for any number of subjects n, the absurd information state is the empty set of n-tuples, referred to as $\perp^n$. An absurd information state excludes that its subjects exist.

On the basis of the present notion of information states, we can also give a precise characterization of their subjects: the set of subjects of a state s is the set of projection functions over s. So, for $s \in S^n$, this is the set $\{f \mid \text{for some } i\text{: } 0 < i \leq n, f \text{ is the } i\text{-th projection function over } s\}$. As we will see, these subject—projection functions—serve as the interpretation of pronouns.

Let us now turn to the relation between information states that models information growth. It is useful to establish some notation conventions first:

Notation Convention 1

- If $e \in D^n$ and $e' \in D^m$, then $e \cdot e' = \langle e_1, \ldots, e_n, e'_1, \ldots, e'_m \rangle \in D^{n+m}$
- e' is an extension of e, $e \leq e'$, iff $\exists e''$: $e' = e \cdot e''$
- For $s \in S^n$ $(i \in D^n)$, N_s $(= N_i) = n$, the number of subjects of s (i)

Information growth comes about, basically, in two ways. It may consist in getting more informed about the subjects of an information state, or in getting informed about more subjects (or, of course, in a mixture of both). The first aspect of information growth boils down to reducing alternatives, i.e., to *eliminating* possible cases; the second to *extending* possible cases. Putting things together, an update of a state s is a state that consists, only, of extensions of possibilities in s:

Definition 2 (Information update) State s' is an update of state s, $s \leq s'$, iff $N_s \leq N_{s'}$, and $\forall e' \in s'\ \exists e \in s$: $e \leq e'$

If $s \in S^n$, and if $s \leq s'$, then the set of possible values of the first n subjects of s' is a subset of that of s. Hence, s' contains more information about its first n subjects than s does. Moreover s' may contain information about more subjects.

We can picture the update relation in the following way:

$$\left\{ \begin{array}{c} \langle d_1, \ldots, d_n \rangle \\ \vdots \\ \langle d'_1, \ldots, d'_n \rangle \\ \vdots \\ \langle d''_1, \ldots, d''_n \rangle \\ \vdots \\ \langle d'''_1, \ldots, d'''_n \rangle \end{array} \right\} \leq \left\{ \begin{array}{c} \langle d'_1, \ldots, d'_n, d'_{n+1}, \ldots, d'_{n+m} \rangle \\ \vdots \\ \langle d''_1, \ldots, d''_n, d''_{n+1}, \ldots, d''_{n+m} \rangle \end{array} \right\}$$

If s' is an update of s $(s \leq s')$, then all possibilities in s' are extensions of possibilities in s. Some possibilities in s, however, need not have an extension in s'. In what follows, in case a possibility $e \in s$ does have an extension in an update s' of s, I will say that e survives in s'; in case a possibility $e \in s$ does not survive in the update s', I will say that the update rejects e.

Finally, let me point at the following fact:

Observation 1 $\langle S, \leq \rangle$ is a partial order

Quite correctly, information growth is transitive, reflexive and antisymmetric.

2 Predicate Logic with Anaphora

In this section I present the system called predicate logic with anaphora. The language of this predicate logic is constructed like that of ordinary predicate logic but, apart from the categories of (individual) constants and variables, it employs an additional category of anaphoric pronouns as terms. Semantically, the difference with ordinary predicate logic is threefold. First, it is stated as an update semantics. Second, existentially quantified formulas are taken to introduce subjects to states, and, third, these subjects are the candidate referents of subsequent pronouns.

2.1 PLA, definitions

The *PLA* language is constructed from sets of relation constants R^n of arity n, from a set C of individual constants, and infinite sets V and $A = \{\mathrm{p}_i \mid i \in \mathcal{N}\}$ of variables and pronouns, respectively. The sets C, V, and A together constitute the set of terms T. The indices on pronouns determine which subjects they refer to in an information state.

The formulas of our language are defined as follows:

Definition 3 (Syntax of PLA)
The set L of *PLA* formulas is the smallest set such that:

- if $t_1, \ldots, t_n \in T$ and $R \in R^n$, then $Rt_1 \ldots t_n \in L$
- if $t_1, t_2 \in T$, then $t_1 = t_2 \in L$
- if $\phi \in L$, then $\neg\phi \in L$
- if $\phi \in L$ and $x \in V$, then $\exists x\phi \in L$
- if $\phi, \psi \in L$, then $(\phi \wedge \psi) \in L$

As usual, $\forall x\phi$ and $\phi \rightarrow \psi$ abbreviate $\neg\exists x\neg\phi$ and $\neg(\phi \wedge \neg\psi)$, respectively.

In the system of *PLA* the dynamic interpretation $s[\![\phi]\!]_{M,g}$ of a *PLA* formula ϕ in an information state s is defined with respect to a model M and an assignment g. The parameters M and g here are the ordinary ones from ordinary predicate logic, and they behave in the same, static, way. The state parameter s is, as it were, the dynamic one. The interpretation of a formula ϕ in a state s yields an update of s, and the formula that follows ϕ will be interpreted in this updated state. The interpretation $[\![\phi]\!]_{M,g}$ of a formula ϕ with respect to M and g is a (partial) update function on information states.

A *PLA* model $M = \langle D, F \rangle$ consists of a non-empty domain D of individuals, and an interpretation function F whichs assigns individuals in D to individual constants and sets of n-tuples of individuals to n-place relation expression. The interpretation of terms is as follows. Constants and variables are evaluated in the usual way with respect to a model and an assignment function. Pronouns are evaluated relative to an information state s and a case $e \in s$. In general, terms are evaluated relative to four parameters:

Definition 4 (Interpretation of terms)

- $[c]_{M,s,e,g} = F(c)$ for all constants c
- $[x]_{M,s,e,g} = g(x)$ for all variables x
- $[\mathrm{p}_i]_{M,s,e,g} = e_{N_s - i}$ for all pronouns p_i and e and s such that $e \in s$ and $N_s > i$

As has already been said, pronouns refer to subjects in the state of information in which they are evaluated. They receive the value assigned by a subject to the cases with respect to which the pronouns are interpreted. In the above definition a pronoun p_i is mapped onto the $i+1$-th last introduced subject of the state s with respect to which it is evaluated (if it exists, that is; otherwise, the interpretation of the pronoun is undefined). Clearly, the value which the $i+1$-th last introduced subject of a state s assigns to a case $e \in s$ is $e_{N_s - i}$. So, the pronoun with the index 0 picks out the subject introduced last, and its value is the last individual of any case with respect to which it is evaluated; the pronoun with index 1 picks out the forelast mentioned subject, etc., etc.

Before we proceed, a comment is in order on definedness and resolvedness. Since pronouns may fail a denotation in a state s, the interpretation of *PLA* formulas will be partial. For instance, if an atomic formula ϕ contains a pronoun p_j, then $s[\![\phi]\!]$ will be undefined if $N_s \leq j$. Undefinedness percolates up in the following way. If ϕ is undefined for a state s, then so are $\neg\phi$, $\exists x\phi$ and $\phi \wedge \psi$. Furthermore, if ψ is undefined for state $s[\![\phi]\!]$, then $\phi \wedge \psi$ is undefined for s. Notice, that the interpretation of a formula ψ may be partial, since it presupposes the presence of a certain number of subjects, while the interpretation of a conjunction $\phi \wedge \psi$ is total, that is, when ϕ involves the introduction of at least that number of subjects. I will say that a pronoun in a discourse is resolved if it refers to a subject introduced in that very same discourse. If all the pronouns in a discourse are resolved, and, hence, the interpretation of the discourse is total, then the discourse itself is called resolved. (Notice that definedness and resolvedness can also be characterized syntactically.)

Let us now turn to the interpretation of *PLA* formulas (in the first two clauses, if X is a set of terms, then I_X is the smallest number greater then or equal to the index of every pronoun in X):

Definition 5 (Semantics of PLA)

- $s[\![Rt_1 \dots t_n]\!]_{M,g} = \{e \in s \mid \langle [t_1]_{M,s,e,g}, \dots, [t_n]_{M,s,e,g} \rangle \in F(R)\}$ (if $N_s > I_{\{t_1,\dots,t_n\}}$)
- $s[\![t_1 = t_2]\!]_{M,g} \quad = \{e \in s \mid [t_1]_{M,s,e,g} = [t_2]_{M,s,e,g}\}$ (if $N_s > I_{\{t_1,t_2\}}$)
- $s[\![\neg\phi]\!]_{M,g} \quad = \{e \in s \mid \neg\exists e'\colon e \leq e' \;\&\; e' \in s[\![\phi]\!]_{M,g}\}$
- $s[\![\exists x\phi]\!]_{M,g} \quad = \{e' \cdot d \mid d \in D \;\&\; e' \in s[\![\phi]\!]_{M,g[x/d]}\}$
- $s[\![\phi \wedge \psi]\!]_{M,g} \quad = s[\![\phi]\!]_{M,g}[\![\psi]\!]_{M,g}$

where $s[\![Rt_1 \dots t_n]\!], s[\![t_1 = t_2]\!], s[\![\neg\phi]\!] \in S^{N_s}$, and $s[\![\exists x\phi]\!] \in S^{N_{s[\![\phi]\!]}+1}$

An atomic formula *At* is evaluated with respect to any case e in the state of information s in which interpretation takes place. If such a formula only contains variables and individual constants as terms, its evaluation is in fact independent from these cases (and from s). In such a case, $s[\![At]\!]$ either equals s, iff *At* is classically true with respect to M and g, or the absurd state, iff *At* is classically false with respect to M and g. Only when pronouns come into play the differences between the various cases in s may matter. If, for instance, the formula is $W\mathrm{p}_i$ ("the $i+1$-th last subject walks"), then its interpretation in a state s will involve the elimination of those cases e in s of which the $i+1$-th last element does not walk (in M). Put the other way around, the result of interpreting that formula in s will result in the state s', which consists of those cases $e \in s$, the $i+1$-th last element $e_{N_s - i}$ of which does walk (in M).

About the other clauses, those dealing with negation, existential quantification and conjunction, I will be short here (the clauses are illustrated in more detail in the next subsection). The interpretation of $\neg\phi$ in s preserves the cases in s that don't survive the update of s with ϕ, i.e., the cases that are rejected by that update. The interpretation of an existentially quantified formula $\exists x\phi$ is a set of cases e' extended with an individual d if e' can be found in the update with ϕ under the assignment of d to x. In keeping with the dynamic view on interpretation, sentence sequencing, or conjunction, involves the composition of the two update functions associated with the conjuncts. In order to update a state s with $\phi \wedge \psi$, first s is updated with ϕ and next the result is updated with ψ.

Before turning to some illustrations of interpretation in *PLA*, I give the *PLA* definitions of truth, or, rather, support, and entailment:

Definition 6 (Support and entailment in PLA)

- s supports ϕ wrt M and g, $s \models_{M,g} \phi$ iff
  $\forall e \in s$: $\exists e'$: $e \leq e'$ & $e' \in s[\![\phi]\!]_{M,g}$
- $\phi_1, \ldots, \phi_n$ entail ψ, $\phi_1, \ldots, \phi_n \models \psi$ iff
  $\forall M, g \forall s \in S$: $s[\![\phi_1]\!]_{M,g} \ldots [\![\phi_n]\!]_{M,g} \models_{M,g} \psi$ (if defined)

A formula ϕ is supported by s iff all cases in s survive the update with ϕ. That is, if the interpretation of ϕ in s s does not reject any case in s. So, ϕ is supported by s if s already contains the information conveyed by ϕ about s's subjects. A conclusion ψ follows from a sequence of premises $\phi_1, \ldots, \phi_n$ if the state that results from interpreting $\phi_1, \ldots, \phi_n$, in that order, always supports ψ. Like the notion of support, this notion of entailment is a dynamic one. As will be shown below, pronouns in the conclusion may refer back to subjects introduced in the premises.

2.2 PLA, some illustrations

The definitions from the preceding section will now be illustrated with some examples. Here, reference to a model and to an assignment function is suppressed whenever convenient.

Existential quantification As is fairly usual, existentially quantified formulas are used to express the semantic contribution of indefinite noun phrases in natural language. Following Karttunen we have taken indefinites to 'set up' discourse referents, which may remain available for future anaphoric (co-)reference. Employing the terminology developed in section 1, we can say that the quantified formulas associated with indefinites introduce subjects to information states. For instance, the sentence *A man walks*, which can be translated as $\exists x(Mx \wedge Wx)$, involves the addition of men who walk to the cases in the state with respect to which interpretation takes place.

The interpretation of an existentially quantified formula $\exists x\phi$ with respect to some assignment g is stated in terms of the interpretation of ϕ with respect to any assignment $g[x/d]$ which at most differs from g in that it assigns an individual d to x. This is as usual. What is new, is that for any such individual d, d gets added to the cases considered possible after interpreting ϕ with respect to $g[x/d]$. In order to see what this amounts to, consider the following example:

(5) There is a man.

$$\begin{aligned} s[\![\exists x Mx]\!]_g &= \{e \cdot d \mid e \in s[\![Mx]\!]_{g[x/d]}\} \\ &= \{e \cdot d \mid e \in s \;\&\; d \text{ is a man}\} \; (= s') \end{aligned}$$

The interpretation of this example yields a state consisting of all the cases $e \in s$ extended with an individual d which is a man. (The witnessess of x involved in supporting the formula quantified over.) The last subject in the resulting state (which I will refer to as s' in the following examples) simply is an arbitrary man. According to the definition of support, the sentence *There is a man* is supported by a, non-absurd, state s iff no cases in s get eliminated when interpreting the sentence, that is, iff there in fact is a man (in the model).

Notice that, since we distinguish (information about the values of) variables from (information about the possible values of) pronouns, it is no more than natural that we keep to the standard scheme for defining existential quantification. In standard predicate logic $[\![\exists x\phi]\!]_g$ can be defined as $\bigcup_{d\in D}[\![\exists x\phi]\!]_{g[x/d]}$, and our definition can be also be stated as $s[\![\exists x\phi]\!]_g = \bigcup_{d\in D}(s[\![\phi]\!]_{g[x/d]} \times \{d\})$. There are only two differences: interpretation is defined relative to information states, and an existential quantifier is taken to introduce a subject.

Anaphoric pronouns Pronouns refer to subjects in the state of information in which they are interpreted. These subjects can be equated with projection functions over (the cases in) s, and for each case in s, the value of a pronoun is the individual the projection function assigns to that case. Above, I have stipulated that the i-th last subject of a state is referred to by the pronoun with index $i-1$. So, pronoun p_0 refers to the subject introduced last, as in the following example:

(6) (There is a man.) He walks.

$$\begin{aligned} s'[\![W\mathrm{p}_0]\!] &= \{e' \in s' \mid \text{the last element of } e' \text{ walks}\} \\ &= \{e \cdot d \mid e \in s \;\&\; d \text{ is a man} \;\&\; d \text{ walks}\} \end{aligned}$$

The interpretation of this example involves the elimination of all those cases $e' \in s'$ the last element of which does not walk. The last subject in the resulting state is an arbitrary walking man.

The preceding examples may serve to show how the anaphoric connection between a pronoun and its antecedent gets established. In fact, the state that results from interpreting *There is a man. He walks* in s is identical to the one that results from interpreting *There is a man who walks* in s:

(7) There is a man who walks.

$$\begin{aligned} s[\![\exists x(Mx \wedge Wx)]\!] &= \{e \cdot d \mid e \in s[\![Mx \wedge Wx]\!]_{g[x/d]}\} \\ &= \{e \cdot d \mid e \in s[\![Mx]\!]_{g[x/d]}[\![Wx]\!]_{g[x/d]}\} \\ &= \{e \cdot d \mid e \in s \;\&\; d \text{ is a man} \;\&\; d \text{ walks}\} \end{aligned}$$

The examples 5+6 and 7 are supported a by a non-absurd state s iff in fact there is man who walks.

Negation The interpretation of $\neg\phi$ in a state s is defined in terms of s and the interpretation of ϕ in s. All cases in s that survive the update with ϕ are cases that support ϕ, and, hence, must be taken to be rejected by $\neg\phi$. For example, consider the interpretation of the sentence *Nobody knows him* in state s', under its translation $\neg\exists x K x \mathrm{p}_0$:

(8) (There is a man.) Nobody knows him.

$$
\begin{aligned}
s'[\![\neg\exists x K x \mathrm{p}_0]\!] &= \{e' \in s' \mid \neg\exists e'' \colon e' \leq e'' \;\&\; e'' \in s[\![\exists x K x \mathrm{p}_0]\!]_{M,g}\} \\
&= \{e' \in s' \mid \neg\exists d' \colon e' \cdot d' \in s[\![K x \mathrm{p}_0]\!]_{M,g[x/d']}\} \\
&= \{e' \in s' \mid \neg\exists d' \colon d' \text{ knows the last element of } e'\} \\
&= \{e \cdot d \mid e \in s \;\&\; d \text{ is a man } \&\; \neg\exists d' \colon d' \text{ knows } d\}
\end{aligned}
$$

The last subject in the resulting state is an arbitrary, unknown man.

Here, I will not discuss the *PLA* notions of universal quantification and implication, as given by their definition in terms of negation, existential quantification and conjunction. For their characteristic properties I have to refer the reader to Dekker 1993. It may suffice to observe here that the donkey implication and the universally quantified donkey sentence receive their so-called strong readings.

Support and entailment As has already been said, the support and entailment relation are dynamic. A pronoun in a supported formula may refer to a subject in the supporting state, and pronouns in entailed formulas may refer back to subjects introduced by the premises of the entailment. Consider the state that results from interpreting example 8 above. This state supports that, say, John doesn't know *him*:

(9) There is a man. Nobody knows him. So, John doesn't know him.

$$s[\![\exists x M x]\!][\![\neg\exists y K y \mathrm{p}_0]\!] \models \neg K \mathrm{j} \mathrm{p}_0$$

Clearly, and correctly, an arbitrary man nobody knows is not known by anybody. The dynamics of the support relation carries over to the entailment relation. Consider one more example:

(10) If a man comes from Rhodes, he likes pineapple-juice. A man I met yesterday comes from Rhodes. So, he likes pineapple-juice.

$$\exists x(Mx \wedge Rx) \rightarrow L\mathrm{p}_0, \exists x(Mx \wedge Rx) \models L\mathrm{p}_0$$

This concludes our exposition of the system of *PLA*. The logical properties of the entailment relation will be studied in more detail in section 3.

Anaphoric linking It may be clear from the discussion sofar that *DPL*'s characteristic scope theorem is not valid in *PLA*, but this does not go to show that *PLA* fails a proper normalization procedure. In *PLA*, the semantic relationships between pronouns and their antecedents can be displayed by bringing them in the scope of their antecedents and *replacing* them by variables bound by these antecedents. Such a substitution of anaphoric pronouns with (bound) variables must be somewhat sophisticated, however, since other anaphoric relationships should not get distorted.

To conclude this section, consider the following two reductions by means of which the semantic connection between two pronouns and their antecedents is brought to (*PL*) light:

(11) $\exists x(Mx \wedge \exists y(Wy \wedge Cxy)) \wedge I\mathrm{p}_0\mathrm{p}_1$

A man courts a widow. He impresses her.

(12) $\exists u(Mu \wedge \exists y(Wy \wedge Cuy) \wedge Iu\mathrm{p}_0)$

A man courts a widow and impresses her.

(13) $\exists u(Mu \wedge \exists v(Wv \wedge Cuv \wedge Iuv))$
A man courts and impresses a widow.

The reductions preserve truth-conditional content. The reader is referred to the full paper for a fully general statement of the reduction of resolved *PLA* formulas into truth-conditionally equivalent *PL* formulas.

3 Properties and prospects

With the system of *PLA* I have given an account of intersentential anaphoric relationships in which pronouns and (free) variables are explicitly distinguished. As I want to argue in this section, this way of dealing with things has a number of appealing features: the account is stated as a proper extension of ordinary predicate logic, it fully lacks the unintuitive property of *DPL*-style systems to license arbitrary dumping of subjects, and its entailment relation appears to be better characterizable. Moreover, as is argued in a little more detail in section 3.2, the account remains properly extensional.

3.1 Characteristic properties

This first subsection states some observations about the relations between ordinary predicate logic (*PL*), *PLA*, and (versions of) *DPL*.

PLA and PL Quite unlike *DPL*, *PLA* obeys the following ordinary substitution law:

Observation 2 (α-conversion) $\exists x\phi \Leftrightarrow \exists y[y/x]\phi$ if y is free for x in ϕ and y does not occur free in ϕ

Such a substitution is not allowed in *DPL*, since, there, it changes the binding potential of the quantified formula. As we see here, in *PLA* the ordinary notions of scope and binding apply.

It is easily observed that *PLA* behaves in a more classical way than *DPL* not only on this score. The subsystem of *PLA* without pronouns is fully equivalent with classical predicate logic:

Observation 3 (PL and PLA) For any *PL* formula ϕ:
$PL \models_{M,g} \phi$ iff $s \models_{M,g} \phi$ (for any state s)

Something similar does *not* hold for *DPL*.

The last observation may go to show that *PLA* preserves all the theorems from ordinary *PL*. Of course, *PLA* generates new theorems, viz., ones in which pronouns occur. Furthermore, given the dynamic nature of the *PLA* entailment relation, entailments involving pronouns will not automatically pattern with ordinary *PL* inference *schemes*. However, as will be illustrated in a little more detail at the end of this subsection, also such inference schemes can be preserved by appropriate pronoun substitutions.

The two observations above show that the *PLA*-system is a proper extension, not modification, of ordinary predicate logic. In this respect, *PLA* stands on a par with so-called E-type pronoun approaches, claimed advantage of which has always been that they keep as much as possible to classical semantics. In the full paper, this issue is discussed more extensively. There one may also find a sketch of how one can combine the present treatment of donkey anaphora with a (semantic version of) an account of E-type anaphora along the lines of Does 1994.

Update semantics It is readily established that interpretation in *PLA* always produces information update:

Observation 4 (Update) $\forall s$: $s \leq s[\![\phi]\!]$ (if defined)

The system of *PLA* simply models the introduction of subjects (by existentially quantified formulas) and the update of information about these subjects (by means of pronouns). It is fairly obvious that this reflects the natural language practice of indefinitely setting up and anaphorically referring back to subjects, for as far as that can be modeled in an extensional first order theory at all.

Still, this result is not at all that trivial. For instance, bound variable approaches to anaphora do not have the update property. In such approaches, only one of a number of existential quantifiers binding a variable x can be taken to (semantically) bind a (syntactically) free occurrence of x. As a consequence, in such approaches, the introduction of a subject as the value of a variable x involves the elimination of a subject introduced earlier as the value of x. Put crudely, in a bound variable approach unfortunately indexing or translating natural language leads to 'dumping of subjects'. This whole possibility simply does not arise in a *PLA*-style system of interpretation.

The above observation can be strenghthened in the following way:

Observation 5 (Registration) For all s, $e \in D^{N_s}$:
$e \in s$ & $\{e\} \models \phi$ iff $\exists e'$: $e \leq e'$ & $e' \in s[\![\phi]\!]$

The update of a state s with ϕ contains (only) cases that register, i.e., extend, the cases in s that all by themselves support ϕ. For this reason, it is appropriate to define support and entailment in the way we did, i.e., in terms of a state s and the update of s with ϕ. In *DPL* such a definition would have given improper results, precisely because its lack of update and registration.

As a further pay off, update and registration imply that *PLA* can be straightforwardly extended with the account of epistemic modalities in Groenendijk et al. 1994. A treatment of epistemic modalities along these lines presupposes an update notion of support. For this reason, Groenendijk et al. 1994 have to complicate their *DPL*-style information states, not only with information about the world, but also with an additional layer of variables, which enables the formulation of the required type of update semantics. No such complications are involved in extending *PLA* with information about the world and with epistemic modalities. Simply by not using variables to label subjects, that is, by a mere simplification of the mediating notion of information, such complications are not in order.

PLA entailment Since *PLA* is a proper extension of ordinary predicate logic with (indexed) anaphoric pronouns, deviations from predicate logic inference schemes can be characterized in terms of (the indices on) pronoun occurrences. Structural inference schemes are preserved modulo appropriate pronoun substitutions which preserve induced anaphoric relationships. To conclude this section I give three examples. (Again, I have to refer to the full paper for more discussion.) First, however, notice that *PLA*, like *DPL*, licenses the deduction theorem (here, and in what follows, Γ represents an arbitrary sequence of premises):

Observation 6 (Deduction theorem) $\Gamma, \phi_n \models \psi$ iff $\Gamma \models \phi_n \rightarrow \psi$

Clearly, no anaphoric relationships get distorted when moving from the left-hand side of this equivalence to the right-hand side.

Although it *is* generally the case in *PLA*, as it is in *DPL*, that if $\Gamma \models \psi$ then $\phi, \Gamma \models \psi$, still we cannot generally add premises without further ado. For instance, $\exists x(Mx \wedge Rx)$ (*There is a man from Rhodes*) entails $R\mathrm{p}_0$ (*He is from Rhodes*), but $\exists x(Mx \wedge Rx) \wedge \exists x(Mx \wedge Ax)$ (*There is a man from Rhodes, and there is a man from Athens*) does not. Pronouns might have to be substituted to give the right results:

Observation 7 (Monotonicity) If $\Gamma \models \psi$ then $\Gamma, \phi \models \psi'$ if ψ' is obtained by replacing every unresolved pronoun p_n in ψ by $\mathrm{p}_{n+IS(\phi)}$

Here, the substitutions in ψ are required to preserve the anaphoric relationships.

The *PLA* entailment relation is not unconditionally reflexive either. For instance, $\exists xGx\mathrm{p}_0 \not\models \exists xGx\mathrm{p}_0$ since the pronoun in the conclusion does not refer to the same subject as the one in the premise. Reflexivity is saved if we make sure that (unresolved) anaphors in the conclusion are matched with those in the premise:

Observation 8 (Reflexivity) $\Gamma, \phi \models \phi'$ if ϕ' is obtained from ϕ by replacing every unresolved pronoun p_n in ϕ by $\mathrm{p}_{n+IS(\phi)}$

So, we do find that $\exists xGx\mathrm{p}_0 \models \exists xGx\mathrm{p}_1$.

As a final example I turn to the (non-)transitivity of the *PLA* notion of entailment. The dynamics of the entailment relation only allows for the following adjusted form of transitivity:

Observation 9 (Transitivity) if χ' can be obtained from χ by replacing every unresolved pronoun p_n in χ by an unresolved pronoun $\mathrm{p}_{n-IS(\psi)}$, and if $\phi \models \psi$ and $\psi \models \chi$, then $\phi \models \chi'$

In the first condition I have deliberately used *can*: if we cannot obtain a formula χ' in the way indicated, it is because an unresolved pronoun in χ refers back to a subject introduced by ψ when concluding χ from ψ. In that case we cannot neglect the subjects introduced by ψ, and entail (a substitute of) χ from ϕ. In all other cases, there is a substitute χ' of χ such that $\phi \models \psi$ and $\psi \models \chi$ imply that $\phi \models \chi'$.

The above inference schemes already display the two characteristic features of *PLA* deduction. From an existentially quantified premise ($\exists xFx$) one may derive a conclusion with a pronoun ($F\mathrm{p}_0$), and in a derivation one has to preserve anaphoric links by appropriate pronoun substitutions. Especially this last feature does induce some additional bookkeeping. However, deduction in *PLA* is not troubled by the use and possible reuse of variables to label subjects, the aspect of interpretation that really complicates deduction in *DPL*.

3.2 Representation and information

Both *DRT* and *DPL* are examples of a dynamic semantics, and each of the two exemplifies one, relatively natural, way of turning a static semantics into a dynamic one. In *DRT*, the interpretation of a sentence is, in the first place, defined in terms of updates of *representations*. In *DPL* it is defined in terms of updates of *information*. The *DPL* reformulation of *DRT* is, by and large, motivated by considerations concerning compositionality, which, I think, most would agree should be secured if it is not really too expensive. However, this compositionality issue is intertwined with the issue of representationalism.

In this paper I will not try to make a point for a representational or non-representational position. Instead, I will sketch a representation theory for anaphora (*RTA*) which is the representational correlate of *PLA*. There are two reasons for doing so. In the first place it shows that a *PLA* analysis of anaphoric pronouns is not at odds with a representational position. More importantly, it shows that an account of anaphoric relationships can be stated within an ordinary extensional type-theoretical framework.

The last result is not trivial. For, *DRT*'s *DRS* language has an idiosyncratic interpretation, and compositional elaborations of *DPL*-style systems have had to be stated in terms non-standard models.

The purposes of this section are similar to those of Muskens 1994, but somewhat more ambitious. We will see that precisely the distinction between pronouns and variables enables us to obtain *DRT* and *DPL* results in a composition way in an unconstrained extensional type theory with only one basic type, that of individuals.

PLA interpretation can be defined representationally as a function 'updating' type-theoretical relation expressions. In order to keep the correspondence with the *PLA* semantics as close as possible, I will assume Orey's relational models, together with Muskens 1989's analysis of abstraction and application (notice that these are harmless assumptions). Moreover, I will assume that *PLA*'s variables are variables (of type e) of the type-theoretical language $\mathcal{L}$ and that the constants of $\mathcal{L}$ are those of *PLA*. Thus, we can assume models $M = \langle D, F\rangle$ for $\mathcal{L}$ which are also *PLA* models.

In what follows, I use σ^n to indicate the type of n-ary relation expressions. Furthermore, the following notation conventions will be employed. If $\vec{x}^n$ is a sequence of variables $x_1, \ldots, x_n$ (all of type e), then:

- $A(\vec{x}^n) = A(x_1)\ldots(x_n)$ (of type σ^m, for A of type σ^{n+m})
- $\lambda\vec{x}^n B = \lambda x_1 \ldots \lambda x_n B$ (of type σ^{n+m}, for B of type σ^m)
- $\exists\vec{x}^n B = \exists x_1 \ldots \exists x_n B$ (of type σ^0, for B of type σ^0)

Finally, for s of type σ^n, I will write $\downarrow s$ for the closure $\exists\vec{x}^n s(\vec{x}^n)$ of s of type σ^0.

We may now turn to the definition of $s^n((\phi))$, the representational update of a relation term s^n of type σ^n by ϕ. The result of this, if defined, will always be a relation term of some type σ^{n+m}:

Definition 7 (RTA)

- $[c]_{\vec{x}^n} = c$ • $[x]_{\vec{x}^n} = x$ • $[\mathrm{p}_i]_{\vec{x}^n} = x_{n-1}$ (if it exists)
- $s^n((Rt_1 \ldots t_m)) = \lambda\vec{x}^n\ s(\vec{x}^n) \wedge R([t_1]_{\vec{x}^n}, \ldots, [t_m]_{\vec{x}^n})$
- $s^n((t_1 = t_2)) \quad = \lambda\vec{x}^n\ s(\vec{x}^n) \wedge [t_1]_{\vec{x}^n} = [t_2]_{\vec{x}^n}$
- $s^n((\neg\phi)) \quad = \lambda\vec{x}^n\ s(\vec{x}^n) \wedge \neg\downarrow s((\phi))(\vec{x}^n)$
- $s^n((\exists y\phi)) \quad = \lambda\vec{x}^{n+m}\lambda y\ s((\phi))(\vec{x}^{n+m})$ (for $s((\phi))$ of type σ^{n+m})
- $s^n((\phi \wedge \psi)) \quad = s((\phi))((\psi))$

observing appropriate variable conventions

It is relatively easily established that the *RTA* update of a representation s denotes the *PLA* update of the denotation of s (here, $[\alpha]$ indicates the type-theoretical interpretation of α):

Observation 10 (Soundness of RTA) $[s((\phi))]_{M,g[s/\mathbf{s}]} = \mathbf{s}[\![\phi]\!]_{M,g}$

In short, *RTA* is the representational correlate of *PLA*. Here, we come across an interesting difference between *DRT* and *DPL* on the one hand, and *RTA* and *PLA* on the other. For, notice that *DRT*- and *DPL*-style systems of interpretation somehow must assume or guarantee that the domains of interpretation contain some kind of correlates of variables, or of variable assignments. Now we can see that, basically by not conflating pronouns with variables, we can as well account for the semantics of natural language anaphoric relationships using ordinary, type-theoretical notions of representation and information.

Let us conclude this section with one potential objection to the enterprise we have engaged in here. One might be tempted to think that with the development of an *RTA* or *PLA*-style semantics we are only pushing our theory to one of the two extremes of representationalism and non-representationalism, while a more viable or promising alternative might have to be sought somewhere in the middle of the two positions. Now I would agree that a comprehensive theory of natural language interpretation in the end probably is in need of informational structures in which all relevant aspects of representation and information are integrated. However, as, I hope, the preceding has shown, the phenomena we have been concerned with here appear to have no immediate bearing on such an issue. The syntactic aspects of the *DPL* notion of information, or the semantic idiosyncracies of *DRT*'s *DRSs*, simply are no inalienable ingredient of an account of the semantics of anaphoric links.

Conclusion

In this paper I have presented a predicate logic with anaphora. Taking the lead from natural language syntax, I have developed a system of predicate logic which does not conflate anaphoric pronouns with variables. Like its predecessor *DPL*, *PLA* gives a compositional account of intersentential anaphoric relationships dealt with in basic *DRT*.

The semantic connections between pronouns and their antecedents are accounted for in terms of update of information, not about the possible values of the variables associated with these antecedent terms, but about the possible values of these antecedent terms themselves. This simplified way of doing things has a number of advantages. In the first place, it enables us to account for the dynamics of establishing anaphoric relationships by means of a proper extension, not modification, of ordinary logical systems. In the second place, it avoids certain complications which pertain to a *DPL*-style approach to natural language anaphora. In particular, interpretation in *PLA* does not generate arbitrary downdate of information.

Last but not least, the system of *PLA* shows that a compositional treatment of the semantics of anaphoric relationships does not enforce upon us an idiosyncratic representation language, or some specialized intensional or many-sorted logic. The system of *PLA* and its representational correlate *RTA* remain well within the bounds of ordinary extensional type theory.

References

Barwise, J.: 1987, Noun phrases, generalized quantifiers and anaphora, in P.

Gärdenfors (ed.), *Generalized Quantifiers: Linguistic and Logical Approaches*, pp 1–29, Reidel, Dordrecht

Chierchia, G.: 1992, Anaphora and dynamic binding, *Linguistics and Philosophy* 15(2), 111–83

Dekker, P.: 1993, *Transsentential Meditations*, Ph.D. thesis, ILLC/Department of Philosophy, University of Amsterdam, Amsterdam

Does, J. v. d.: 1994, The dynamics of sophisticated laziness, in P. Dekker and M. Stokhof (eds.), *Proceedings of the ninth Amsterdam colloquium*, pp 229–248, Amsterdam

Evans, G.: 1985, *Collected papers*, Foris, Dordrecht

Fine, K.: 1984, A defense of arbitrary objects, in F. Landman and F. Veltman (eds.), *Varieties of Formal Semantics*, pp 123–42, Kluwer, Dordrecht

Groenendijk, J. and Stokhof, M.: 1991, Dynamic predicate logic, *Linguistics and Philosophy* 14, 39–100

Groenendijk, J., Stokhof, M., and Veltman, F.: 1994, Coreference and modality, in R. Cooper and J. Groenendijk (eds.), *Integrating Semantic Theories II*, University of Amsterdam, Amsterdam, DYANA-2 deliverable R2.1.B

Heim, I.: 1982, *The Semantics of Definite and Indefinite Noun Phrases*, Ph.D. thesis, University of Massachusetts, Amherst, Published in 1989 by Garland, New York.

Heim, I.: 1990, E-type pronouns and donkey anaphora, *Linguistics and Philosophy* 13, 137–78

Kamp, H.: 1981, A theory of truth and semantic representation, in J. Groenendijk, T. Janssen, and M. Stokhof (eds.), *Formal Methods in the Study of Language*, pp 277–322, Mathematical Centre, Amsterdam

Lewis, D.: 1975, Adverbs of quantification, in E. L. Keenan (ed.), *Formal Semantics of Natural Language*, pp 3–15, Cambridge University Press, Cambridge, U.K.

Muskens, R.: 1989, A relational formulation of the theory of types, *Linguistics and Philosophy* 12, 325–46

Muskens, R.: 1994, Compositional discourse representation theory, in P. Dekker and M. Stokhof (eds.), *Proceedings of the ninth Amsterdam colloquium*, pp 467–86, ILLC, Amsterdam

Pagin, P. and Westerståhl, D.: 1993, Predicate logic with flexibly binding operators and natural language semantics, *Journal of Logic, Language and Information* 2(2), 89–128

Reinhart, T.: 1983, Coreference and bound anaphora: A restatement of the anaphora question, *Linguistics and Philosophy* 6, 47–88

Rooth, M.: 1987, NP interpretation in Montague grammar, file change semantics and situation semantics, in P. Gärdenfors (ed.), *Generalized Quantifiers*, pp 237–69, Kluwer, Dordrecht

Veltman, F.: 1990, Defaults in update semantics, in H. Kamp (ed.), *Conditionals, Defaults, and Belief Revision*, University of Edinburgh, Edinburgh, DYANA deliverable R2.5.A

Zeevat, H.: 1989, A compositional approach to discourse representation theory, *Linguistics and Philosophy* 12, 95–131