Crises of *Identity**

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Abstract

Identity says that any conditional with the form ⌜If p, then p⌝ is a logical truth. I show that a wide range of theories invalidate *Identity*. I argue this is due to a tension between *Identity* and *Import-Export*, and sketch a potential solution.

1 Introduction

There is much controversy about the logic of the conditional. One principle that has so far evaded controversy is *Identity*, which says that conditionals with the form ⌜If p, then p⌝ are logical truths. In the first part of this paper, I show that, despite its overwhelming plausibility, a wide variety of theories of the conditional invalidate *Identity*. I then argue that the culprit behind this failure is the *Import-Export* (*IE*) principle, which says that ⌜If p, then if q, then r⌝ and ⌜If p and q, then r⌝ are always equivalent. I show that there is a deep and surprising tension between *IE*, on the one hand, and *Identity*, on the other. In light of this tension, and the overwhelming plausibility of *Identity*, I argue we should reject *IE*. In the final part of the paper, I explore how we might reject *IE* while still accounting for the intuitive evidence that supports it.

2 Failures of *Identity*

Identity says that conditionals with the form ⌜If p, then p⌝, like those in (1), are logical truths:

(1) a. If it rained, then it rained.
  b. If it had rained, then it would have rained.

Identity is extremely natural. Nonetheless, it is invalidated by a wide range of theories of the conditional, as I will show in the rest of this section.

I work with a simple propositional language with atoms $A, B, C \ldots$, closed under '$\wedge$' ('and'), '$\neg$' ('not'), '$\vee$' ('or'), the material conditional '$\supset$' ('$p \supset q$' abbreviates '$\neg p \vee q$'), the material biconditional '$\equiv$' ('$p \equiv q$' abbreviates '$(p \supset q) \wedge (q \supset p)$'), and the natural language conditional connective '$>$' ('$p > q$' abbreviates 'If p, then q'). Lower-case italics range over sentences. Where Γ is a set of sentences, '$\Gamma \models p$' means that Γ logically entails p, in the standard classical sense, i.e. that p is true in every intended model where all the elements of Γ are true.

A key player in what follows is the *Import-Export (IE)* principle, which says that $(p > (q > r)) \equiv ((p \wedge q) > r)$ is a logical truth. In other words, *IE* says that what we do with two

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successive conditional antecedents is the same as what we do with the corresponding conjunctive antecedent. So, e.g., *IE* says the members of pairs like the following are equivalent:

(2) a. If the coin is flipped, then if it lands heads, then we'll win.
b. If the coin is flipped and it lands heads, then we'll win.

(3) a. If the coin had been flipped, then if it had landed heads, then we would have won.
b. If the coin had been flipped and it had landed heads, then we would have won.

My central claim in this section is that all extant theories, apart from the material analysis, which validate *IE* also *in*validate *Identity*. To see the point informally, consider what it takes to validate *IE*. *IE* says, in essence, that information in successive antecedents is agglomerated: a conditional with two antecedents is evaluated in the same way as a conditional with one corresponding conjunctive antecedent. That means that, to validate *IE*, we need some way of "remembering" successive conditional antecedents. Different *IE*-validating theories have different mechanisms for doing this. For instance, in McGee (1985)'s framework, conditional antecedents are added sequentially to a set of sentences; the consequent is then evaluated at the closest world where all the sentences in that set are true. In Kratzer (1981, 1991)'s framework, conditional antecedents are similarly added to the value of a "modal base" function which takes each world to a set of propositions, which in turn provides the domain of quantification for evaluating the consequent. Relevantly similar approaches are developed in von Fintel (1994); Dekker (1993); Gillies (2004, 2009). These theories differ in important ways, but they all have what I'll generically call a *domain parameter* of some kind which is in the business of somehow remembering successive conditional antecedents, so that these can be agglomerated when we arrive at the consequent. Intuitively, that is exactly what is needed in order to validate *IE*.

Structurally, this has an important consequence. What proposition a conditional expresses depends on the setting of this domain parameter. And thus, since conditional antecedents change this parameter, what proposition a conditional expresses can change depending on whether it is embedded under a conditional antecedent. In particular, consider a sentence with the form $p > p$ and suppose that p itself contains a conditional. Then the first instance of p will be interpreted relative to a domain parameter with a different value from that used to evaluate the second p: when we get to the second, the domain parameter will have been updated with the information that p is true. And that, in turn, means that the two instances of p can express different propositions, and so the conditional as a whole can be false.

More concretely, think about a conditional of the form $(\neg(B > A) \wedge A) > (\neg(B > A) \wedge A)$, where A and B are arbitrary atoms. This has the form $p > p$. Now consider what happens when we arrive at the consequent of this conditional, if we validate *IE*. At that point, the antecedent will have been added to our domain parameter. So the domain parameter will now entail the antecedent, and so in particular will entail A. That means that the parameter will only make available A-worlds for the evaluation of the conditional in the consequent. The consequent, $(\neg(B > A) \wedge A)$, entails that a certain conditional, $B > A$, is false. The problem is that if our domain parameter—which gives the domain of worlds which matter for evaluating the conditional—includes only A-worlds, then $B > A$ can't be false. That means that $B > A$, as it appears in the consequent of our target conditional, must be true; so its negation must be false; so the whole consequent must be false; and so the target conditional as a whole will be false, provided its antecedent is possible.

We can illustrate this in more detail by looking at McGee's McGee (1985) theory, a variant on Stalnaker (1968)'s theory which validates *IE*. Let f be a Stalnakerian selection function from any consistent proposition and world to the "closest" world where that proposition is true. Γ is any set of sentences (a *hypothesis set*). $\mathfrak{I}$ is an atomic valuation function. McGee's theory says that any sentence is true relative to an absurd hypothesis set, i.e. $[\![p]\!]^{f,\Gamma,w}=1$ if $\bigcap_{r\in\Gamma}[\![r]\!]^{f,\varnothing}=\varnothing$; an atom is true iff it is true at the closest world where the hypothesis set is true, i.e. $[\![A]\!]^{f,\Gamma,w}=1$ iff $f(\bigcap_{r\in\Gamma}[\![r]\!]^{f,\varnothing},w)\in\mathfrak{I}(A)$; a negation is true iff the negatum is false, i.e. $[\![\neg p]\!]^{f,\Gamma,w}=1$ iff $[\![p]\!]^{f,\Gamma,w}=0$; a conjunction is true iff both conjuncts are, i.e. $[\![p\wedge q]\!]^{f,\Gamma,w}=1$ iff $[\![p]\!]^{f,\Gamma,w}=1$ and $[\![q]\!]^{f,\Gamma,w}=1$; finally, a conditional is true iff the consequent is true relative to a hypothesis set updated with the antecedent, i.e. $[\![p>q]\!]^{f,\Gamma,w}=[\![q]\!]^{f,\Gamma\cup\{p\},w}$. This matches Stalnaker's theory for simple conditionals (e.g. $A>B$ is true iff B is true at the closest A-world); but the hypothesis set lets us agglomerate successive antecedents, so we validate *IE*, unlike Stalnaker. Now consider $[\![(\neg(B>A)\wedge A)>(\neg(B>A)\wedge A)]\!]^{f,\varnothing,w}$. By the clause for conditionals, this is true iff $[\![(\neg(B>A)\wedge A)]\!]^{f,\{(\neg(B>A)\wedge A)\},w}=1$, which, by the clause for conjunction, holds only if $[\![\neg(B>A)]\!]^{f,\{(\neg(B>A)\wedge A)\},w}=1$. That in turn holds, by the clause for negation, iff $[\![B>A]\!]^{f,\{(\neg(B>A)\wedge A)\},w}=0$, which, by the clause for conditionals, holds iff $[\![A]\!]^{f,\{(\neg(B>A)\wedge A),B\},w}=0$. But $[\![A]\!]^{f,\{(\neg(B>A)\wedge A),B\},w}=1$ by the first clause, since $[\![\neg(B>A)\wedge A]\!]^{f,\varnothing}\cap[\![B]\!]^{f,\varnothing}=\varnothing$ (since $B\wedge A$ entails $B>A$ in McGee's system); so $[\![(\neg(B>A)\wedge A)>(\neg(B>A)\wedge A)]\!]^{f,\varnothing,w}=0$. Similar reasoning leads to the same conclusion for other *IE*-validating theories.

3 Locating the tension

Must we invalidate *Identity* if we validate *IE*? No: these principles are jointly consistent. A quick way to see this is that the material conditional '$\supset$' validates both. Recall that the material conditional is the truth-function such that $p\supset q$ is true whenever p is false or q true. This connective validates both *IE* and *Identity*.

But this is not much help, because the material conditional is not an adequate analysis of the natural language 'if. . . then'. This is the near consensus view, and there are myriad arguments for it. A quick way to see the implausibility of the material analysis is that, since $p\supset q$ is equivalent to $\neg p\vee q$, its negation is equivalent to the conjunction $p\wedge\neg q$. But it is clear that the negation of $p>q$ is not equivalent to $p\wedge\neg q$. For instance, 'It's not the case that, if Patch had been a rabbit, she would have been a rodent' and 'It's not the case that, if Patch is a rabbit, she is a rodent' are both clearly true, thanks just to taxonomic facts; neither entails that Patch is a rabbit, *pace* the material view.

The material conditional is the only extant theory I know of which validates both *Identity* and *IE*. It is not the only logically possible one. Nonetheless, I will argue that there is no theory which validates *Identity* and *IE* together in a *plausible* way. I will argue for this by showing that the material conditional is the only connective which validates both *Identity* and *IE* together with two very weak, and very plausible, background principles.

The first principle is a very weak monotonicity principle, which says that $p>(q\wedge r)$ entails $p>q$; call this *Left Consequent Monotonicity (LCM)*. *LCM* is as far as I know validated by every extant theory, and is of course a very limited corollary of the widely accepted principle that conditionals are monotonic in their consequents. *LCM* is what is required to predict that inferences like the following will be valid: 'If it rains, the picnic will be cancelled and Sue will be upset; therefore, if it rains, the picnic will be cancelled'. The second principle says that $\neg p$ follows from $p>q$ together with $p>\neg q$; call this principle *Ad Falsum*. *Ad Falsum*, like *LCM*, is validated by every theory I know of (with the exception of the existential theories in Bassi and Bar-Lev (2018); Herburger (2019)). The most direct evidence for *Ad Falsum* comes from logical and mathematical contexts, where it is very natural to argue that p is false by showing that, if p, then q, and if p, then not q. This reasoning, however, is only valid if *Ad Falsum* is. While this reasoning is most at home in mathematical and logical contexts, it also seems perfectly valid in non-mathematical contexts, as in Gibbard (1981)'s Sly Pete case, where we learn both 'If Pete called, he won' and 'If Pete called, he lost' and can conclude with certainty that he didn't call. More generally, it seems unimaginable that two conditionals with this form could be true, while their antecedent was also true.

But if we validate *Identity*, *IE*, *LCM*, and *Ad Falsum*, then '$>$' must be logically equivalent to '$\supset$'. We assume classical logics for '$\wedge$', '$\vee$', and '$\neg$', uniform substitutability for sentence letters, and substitutability of logical equivalents. For arbitrary p and q, given *Identity*, we have $\models (\neg(p > q) \wedge q) > (\neg(p > q) \wedge q)$; given *LCM*, we then have $\models (\neg(p > q) \wedge q) > \neg(p > q)$. *Identity* also gives us $\models ((\neg(p > q) \wedge q) \wedge p) > ((\neg(p > q) \wedge q) \wedge p)$; substitution of logical equivalents and *LCM* then give us $\models ((\neg(p > q) \wedge q) \wedge p) > q$; thus by *IE*, $\models (\neg(p > q) \wedge q) > (p > q)$. So by *Ad Falsum* we have $\models \neg(\neg(p > q) \wedge q)$. By classical logic we have $q \models p > q$. By classical logic and uniform substitution we have $((p \wedge (p > q)) \wedge \neg q) \models ((p > q) \wedge (p > \neg q))$; *Ad Falsum* thus tells us that $((p \wedge (p > q)) \wedge \neg q) \models \neg p$; by *reductio*, we have $p \wedge (p > q) \models q$, i.e. *Modus Ponens (MP)* for '$>$'. By *Identity* we have $\models ((p \supset q) \wedge p) > ((p \supset q) \wedge p)$; by substitutability of logical equivalents and *LCM* we have $\models ((p \supset q) \wedge p) > q$; by *IE* $\models (p \supset q) > (p > q)$; by *MP*, $p \supset q \models p > q$; *MP* also gives us $p > q \models p \supset q$; and so $p > q$ and $p \supset q$ are logically equivalent. In sum: the only conditional that validates *Identity, IE, LCM*, and *Ad Falsum* is the material conditional. Since we know that the conditional is not material, one of these principles must be invalid.

Our result strengthens a result of Gibbard (1981), which showed that only the material conditional validates all three of *IE, MP*, and *Logical Implication (LI)*, which says that, if $p \models q$, then $\models p > q$. *LI* follows from *Identity* and *LCM* given substitutability of logical equivalents, so Gibbard's result shows that only the material conditional validates all of *IE, MP, Identity*, and *LCM*. Our result replaces *MP* with *Ad Falsum*. *Ad Falsum* follows from *MP*, but is weaker than *MP*, so our result strengthens Gibbard's. This is dialectically important, because, while a strong case has been mounted against *MP* by McGee (1985), and many theories have been advanced which invalidate *MP* (including all the *IE*-validating theories cited above), I know of no case against *Ad Falsum*, and every theory I know of, *even those that invalidate* MP, still validates *Ad Falsum*.[1] Moreover, I do not see any prospects for a case against *Ad Falsum* on the basis of examples like McGee's or indeed on any other. The reaction to Gibbard's result has mainly focused on the choice between *MP* and *IE*. The present result shows that that reaction misses an important tension which already exists between *IE* and *Identity*.

4 Rejecting *Identity*?

If we accept *Ad Falsum* and *LCM*, then the present result shows we must choose between rejecting *Identity* or rejecting *IE*, given that the conditional is not material. Let us consider first whether there could be a case against *Identity*. It is hard to see *a priori* how there could be. If p holds, then it seems certain that, whatever else holds, p does. But we should not be too quick to dismiss a potential case against *Identity*; it is at least conceivable that *Identity* could fail in the case of complex conditionals, exactly where theories like McGee's predict it will.

Indeed, the foregoing discussion gives us a precise way to explore this possibility. Consider sentences with the form $(B \wedge \neg(A > B)) > \neg(A > B)$, a slightly simpler variant on the sentences considered above. Sentences with this form are predicted by *Identity* to be logical truths

[1]With the exception, again, of the existential theories mentioned above.

(assuming *LCM*). By contrast, theories that validate *IE* predict that the internal negation of such conditionals are instead logical truths. So, to assess whether a case can be made against *Identity* (and thus in favor of *IE*) on the basis of complex conditionals, we can look at sentences with forms $(B \wedge \neg(A > B)) > \neg(A > B)$ and $(B \wedge \neg(A > B)) > (A > B)$, respectively, as in:

(4) a. If the match had lit, and it's not the case that the match would have lit if it had been wet, then it's not the case that the match would have lit if it had been wet.
b. If the match had lit, and it's not the case that the match would have lit if it had been wet, then the match would have lit if it had been wet.

(5) a. If the vase had broken, and it's not the case that the vase would have broken if it had been wrapped in plastic, then it's not the case that the vase would have broken if it had been wrapped in plastic.
b. If the vase had broken, and it's not the case that the vase would have broken if it had been wrapped in plastic, then the vase would have broken if it had been wrapped in plastic.

Although these are complicated, it seems clear that the first variants in each pair are logical truths, while the second variants are logical falsehoods (assuming their antecedents are possible). Thus it seems that *Identity*, not *IE*, has the correct verdict here. This makes me pessimistic that any case can be constructed against *Identity* on the basis of complex conditionals.

5 Rejecting *Import-Export*

We should thus reject *IE*. But the motivation for this so far is relatively indirect. It would be nice to find more direct evidence against *IE*—that is, pairs with the form $p > (q > r)$ and $(p \wedge q) > r$ which are intuitively inequivalent. Strikingly, subjunctive conditionals seem to yield such pairs, while indicatives appear not to. Consider first subjunctive pairs like Etlin (2008)'s (6), and Stephen Yablo's (7) (p.c.):

(6) a. If the match had lit, then it would have lit if it had been wet.
b. If the match had lit and it had been wet, then it would have lit.

(7) a. If I had been exactly $6'$ tall, then if I had been a bit taller than $6'$, I would have been $6'1''$.
b. If I had been exactly $6'$ tall and a bit taller than $6'$, I would have been $6'1''$.

These pairs instantiate *IE* but are intuitively inequivalent. (6-a) can be false in some circumstances, whereas (6-b) cannot. Conversely, (7-a) is plausibly true in some circumstances; whereas (7-b) feels like nonsense (if it is ever true, then it is surely only true trivially, unlike (7-a)). These felt inequivalences target the two directions of *IE*. Pairs like this suggest that neither direction is valid for subjunctives.

When we turn to indicatives, however, things look different. Consider the indicative versions of the pairs we have just looked at:

(8) a. If the match lit, then it lit if it was wet.
b. If the match lit and it was wet, then it lit.

(9) a. If I am exactly $6'$ tall, then if I am a bit taller than $6'$, then I am $6'1''$.
b. If I am exactly $6'$ tall and a bit taller than $6'$, then I am $6'1''$.

Unlike the corresponding subjunctive pairs, these appear to be pairwise equivalent: both of the sentences in (8) feel equally like logical truths; and both of the sentences in (9) feel equally incoherent. More generally, I have not been able to find concrete counterexamples to *IE* for indicatives: the very same pairs which seem to witness the *invalidity* of *IE* for subjunctives seem to provide further support for its *validity* when it comes to indicatives.

6 Strawson validity

We thus need a theory which (i) accounts for the contrast just observed between indicatives and subjunctives; and (ii) accounts for the *felt* validity of *IE* in the case of indicatives without also invalidating *Identity*. Let us focus for the moment on indicatives. Summing up our evidence, we have strong indirect evidence that *IE* is invalid, but we do not seem to find concrete counterinstances to it. One response to this kind of situation is to say that the inference pattern in question is not logically valid, but still preserves truth in an important subset of cases. This, in turn, makes it hard to find concrete counterinstances to it. Following Strawson (1952); von Fintel (1999), we can distinguish *logical entailment* (preservation of truth in all intended models) from *Strawson entailment*: preservation of truth in all intended models *where we might actually use the sentences in question.* More precisely, we can associate sentences with presuppositions—conditions on their felicitous use—and then define Strawson entailment as follows:

> **Strawson entailment**: Γ Strawson entails p iff for any intended model $\mathcal{M}$, context c and world $w \in c$, if the presuppositions of all the elements of Γ and of p are satisfied in $\langle c, w\rangle$ in $\mathcal{M}$ and all the elements of Γ are true at $\langle c, w\rangle$ in $\mathcal{M}$, so is p.

If an inference is Strawson valid, it doesn't necessarily preserve truth in all worlds in all intended models. But it does preserve truth in any context-world pair where all the premises and the conclusion have their presuppositions satisfied—which includes all contexts where the sentences in question can be naturally used. So, if an inference is Strawson valid, there won't be *natural* concrete counterexamples to it, even if the inference is not *logically* valid.

There are various ways we can model presuppositions, and thus Strawson validity. Here I follow multi-dimensional approaches (Herzberger (1973); Karttunen and Peters (1979)) which distinguish truth and presupposition as two dimensions of content.[2] The first dimension, which I underline for mnemonic reasons, records presupposition satisfaction; the second records truth. So, where $*$ is shorthand for either 1 or 0, the inference from p to q is *Strawson* valid iff, for any intended model and any c and $w \in c$, $[\![q]\!]^{c,w}$ is $\langle \underline{1}, 0\rangle$ if $[\![p]\!]^{c,w}$ is $\langle \underline{1}, 1\rangle$ in that model; whereas the inference is *logically* valid iff, for any intended model and any c and $w \in c$, $[\![q]\!]^{c,w}$ is $\langle \underline{*}, 1\rangle$ if $[\![p]\!]^{c,w}$ is in that model.

7 The local indicative constraint

With this discussion in hand, I can state my proposal: to find an account of the presuppositions of indicative conditionals which predicts that *IE* is Strawson valid for indicatives, but not subjunctives. There is, in fact, an existing proposal which associates indicatives with a presupposition which subjunctives lack. The idea, proposed in Stalnaker (1975), slightly strengthened in von Fintel (1998), and widely accepted, is that indicatives presuppose that their antecedent is evaluated at a contextually possible world. One motivation for this is contrasts like (10):

[2] The kind of presupposition I focus on here is *expressive*, not semantic, insofar as it projects universally, and cannot be filtered; I leave the universal projection rules implicit.

(10) Beau Balou won the race.

a. #If he didn't win, we lost a lot of money.
b. If he hadn't won, we would have lost a lot of money.

In general, using '$>_i$' now for the indicative conditional and '$>_s$' for the subjunctive, $p >_i q$ is infelicitous when p has been ruled out in a context, while $p >_s q$ remains acceptable. Another motivation is the felt validity of the 'or'-to-'if' inference for indicatives, but not subjunctives:

(11) a. It was the gardener or the butler, and it might have been either.
b. $\rightsquigarrow$ If it wasn't the gardener, it was the butler.
c. $\not\rightsquigarrow$ If it hadn't been the gardener, it would have been the butler.

In general, the inference from $p \vee q$ to $\neg p >_i q$ feels legitimate (provided p is contextually possible), while the inference to $\neg p >_s q$ does not.

To capture these two patterns, start with the semantics for the conditional from Stalnaker (1968), on which a conditional says that the closest antecedent-world (according to the contextual selection function) is a consequent-world. Then we say that $p >_i q$ presupposes that, for any context world w, the closest p-world to w is in the context; while $p >_s q$ does not have a parallel presupposition. This accounts for our two generalizations.

This presupposition, which I will call the *indicative constraint*, does not on its own help with *IE*. But a close extension does. The motivation for the extension comes from the observation that the compatibility requirement which motivates the indicative constraint resurfaces at a *local* level. Consider (12):

(12) I don't know whether Bob came to the party.

a. #But suppose that Bob came, and that if he didn't, he went to work.
b. But suppose that Bob came, and that if he hadn't, he would have gone to work.

The embedded indicative in (12-a) is infelicitous, in contrast to the subjunctive variant in (12-b). This is surprising because, relative to the global context in (12), it is possible that Bob didn't go to the party, and so the indicative antecedent is compatible with the global context. To account for the contrast in (12), it looks like we need to compute our compatibility requirement for indicatives relative to a *local* context which takes into account the information in the left conjunct in (12-a)—that Bob came to the party. Similar contrasts in other environments support this point. For instance, consider nested conditionals. Suppose we have a die which is either weighted towards evens or odds; we don't know which, and we don't know whether the die was thrown. Compare (13-a) and (13-b):

(13) a. #If the die was thrown and landed four, then if it didn't land four, it landed two or six.
b. If the die had been thrown and landed four, then if it hadn't landed four it would have landed two or six.

The antecedent of the embedded conditional in (13-a) and (13-b)—that the die didn't land four—is compatible with the global context. But, embedded under a conditional antecedent that entails that the die landed four, only the subjunctive variant in (13-b) seems acceptable, while the indicative variant is not. Once more, it looks like the indicative conditional's compatibility constraint in the consequent of a conditional is calculated relative to a local context: in this case, one which entails the information in the conditional's antecedent.[3]

[3] Boylan and Schultheis (2019) provide yet another motivation for a local version of the indicative constraint,

That the indicative constraint is calculated locally in fact looks unsurprising from the point of view of the recent literature on epistemic modality. That literature has suggested that epistemic accessibility is calculated in a local manner in general, and so it is not surprising that indicative's epistemic compatibility constraint is also local.[4] There are different ways we could capture the locality of the indicative constraint. For each "local" theory of epistemic modality, we could build a corresponding local indicative constraint in roughly similar fashion. Here I will build loosely on the bounded theory of Mandelkern (2019). That theory borrows Schlenker (2009)'s account of local contexts from his theory of presupposition. A local context, on Schlenker's account, is a set of worlds which represents the information locally available relative to a given syntactic environment and global context: in other words, the information that could be added to that environment without changing the contextual meaning of the sentence as a whole. The local context for a conditional's consequent, for instance, entails its antecedent; the local context for a right conjunct entails the left conjunct. The bounded theory posits that epistemic modals presuppose that their accessibility relation is *local* in the sense that only local context worlds can be accessed from local context worlds.

I propose to localize the indicative constraint in parallel to this. Recall that the indicative constraint says that the indicative selection function must take any context world and indicative antecedent to a context world. We need only change 'context' for 'local context' to get an appropriately local version. In other words, where κ represents the local context for the conditional, our *local indicative constraint* says that $p >_i q$ presupposes that, for any world w in κ, the closest p-world to w is also in κ. By contrast, the subjunctive has no similar constraint.

For unembedded conditionals, the local indicative constraint is equivalent to the global indicative constraint. But things are different for embedded conditionals. The local context for a right conjunct will be the intersection of the global context and the left conjunct.[5] So, in a global context c, the local context for the conditional in $\neg p \wedge (p >_i q)$ will be $c \cap \neg p$. That means that, whenever the local indicative constraint is satisfied, for any world w' in $c \cap \neg p$, $f_i(p, w') \in (c \cap \neg p)$. But this is incoherent, since $f_i(p, w')$ must be a p-world (if p is consistent), and thus cannot be in $c \cap \neg p$.[6] So there will be no way to satisfy the local indicative constraint for an indicative conditional in a conjunction like this. Crucially, this reasoning goes through whether $\neg p \wedge (p >_i q)$ is embedded or unembedded, accounting for the infelicity of sentences like (12-a). Likewise, the local context for the consequent of an indicative conditional is the global context intersected with the antecedent. So, in a conditional with the form $p >_i (\neg p >_i q)$ in global context c, the local context for the consequent is $c \cap p$; so the local indicative constraint for the embedded conditional will require that the closest $\neg p$-world to the closest p-world be in $c \cap p$, and once more will be unsatisfiable. Since subjunctives do not have a local indicative constraint, none of this reasoning will go through for them, accounting for the contrasts above.

Summing up, with f_i the indicative selection function and f_s the subjunctive selection function (for readability I sometimes write 'p' for '$[\![p]\!]^{\kappa, f_i, f_s}$' when there is no danger of ambiguity):

- $[\![p >_s q]\!]^{\kappa, f_i, f_s, w} = \langle \underline{1}, 1 \rangle$ iff $[\![q]\!]^{\{f_s(p, w') : w' \in \kappa\}, f_i, f_s, f_s(p,w)} = \langle \underline{1}, 1 \rangle$

from embedded cases of 'or'-to-'if'.

[4]E.g. Groenendijk et al. (1996); Aloni (2001); Yalcin (2007); Dorr and Hawthorne (2013); Mandelkern (2019).

[5]In general, incorporating local contexts for the Boolean connectives yields: $[\![p \wedge q]\!]^{\kappa, f_i, f_s} = [\![p]\!]^{\kappa \cap [\![q]\!]^{\kappa, f_i, f_s}, f_i, f_s} \cap [\![q]\!]^{\kappa \cap [\![p]\!]^{\kappa, f_i, f_s}, f_i, f_s}$; $[\![p \vee q]\!]^{\kappa, f_i, f_s} = [\![p]\!]^{\kappa \cap [\![\neg q]\!]^{\kappa, f_i, f_s}, f_i, f_s} \cup [\![q]\!]^{\kappa \cap [\![\neg p]\!]^{\kappa, f_i, f_s}, f_i, f_s}$; and $[\![\neg p]\!]^{\kappa, f_i, f_s} = \mathcal{W} \setminus [\![p]\!]^{\kappa, f_i, f_s}$. I omit the world parameter to denote the set of worlds where the sentence in question is true, so $[\![p]\!]^{\kappa, f_i, f_s} = \{w : [\![p]\!]^{\kappa, f_i, f_s, w} = \langle \underline{*}, 1 \rangle\}$. I leave implicit universal projection rules for presupposition satisfaction.

[6]Selection functions must meet four constraints. Where φ and ψ are propositions: *Strong Centering*: $f(\varphi, w) = w$ iff $w \in \varphi$; *Success*: $f(\varphi, w) \in \varphi$ provided $\varphi \neq \varnothing$; *CSO*: if $f(\varphi, w) \in \psi$ and $f(\psi, w) \in \varphi$, then $f(\varphi, w) = f(\psi, w)$; and *Absurdity*: $f(\varnothing, w) = \lambda$, where λ is an absurd world that makes all sentences true.

[Stalnaker truth conditions]

- $[\![p >_i q]\!]^{\kappa, f_i, f_s, w}$

$$= \langle \underline{1}, * \rangle \text{ iff } \forall w' \in \kappa : f_i(p, w') \in \kappa \qquad \text{[local indicative constraint]}$$
$$= \langle \underline{*}, 1 \rangle \text{ iff } [\![q]\!]^{\kappa \cap p, f_i, f_s, f_i(p,w)} = \langle \underline{1}, 1 \rangle \qquad \text{[Stalnaker truth conditions]}$$

Now back to *IE*: strikingly, the local indicative constraint guarantees that *IE* is Strawson valid for indicatives, i.e. that the conjunction $((p >_i (q >_i r)) \supset ((p \wedge q) >_i r)) \wedge (((p \wedge q) >_i r) \supset (p >_i (q >_i r)))$ is never $\langle \underline{1}, 0 \rangle$ for any c and $w \in c$ in any intended model. Take each conjunct in turn, considering first $(p >_i (q >_i r)) \supset ((p \wedge q) >_i r)$. Suppose there is an index $\mathfrak{i} = \langle c, f_i, f_s, w \rangle$ with $w \in c$ such that the presuppositions of all the indicative conditionals are satisfied at $\mathfrak{i}$ but this material conditional has a true antecedent and false consequent at $\mathfrak{i}$. The local context for the consequent of a material conditional is the global context together with its antecedent. Thus $(p \wedge q) >_i r$ is false at $\langle c \cap (p >_i (q >_i r)), f_i, f_s, w \rangle$. By the local indicative constraint, since $w \in c \cap (p >_i (q >_i r))$, $f_i(p \wedge q, w) \in c \cap (p >_i (q >_i r))$. But any index that makes both $p \wedge q$ true and makes $p >_i (q >_i r)$ true makes r true; so $(p \wedge q) >_i r$ is true at $\langle c \cap (p >_i (q >_i r)), f_i, f_s, w \rangle$, contrary to assumption. Next consider $((p \wedge q) >_i r) \supset (p >_i (q >_i r))$. Suppose the presuppositions of all the indicative conditionals are satisfied at some index $\mathfrak{i} = \langle c, f_i, f_s, w \rangle$ with $w \in c$ but the material conditional is false at $\mathfrak{i}$. Then $p >_i (q >_i r)$ is false at $\langle c \cap ((p \wedge q) >_i r), f_i, f_s, w \rangle$, and $w \in c \cap ((p \wedge q) >_i r)$. By the local indicative constraint, $f_i(p, w) \in c \cap ((p \wedge q) >_i r)$, and by *Success* $f_i(p, w) \in c \cap ((p \wedge q) >_i r) \cap p$. So again by the local indicative constraint, $f_i(q, f_i(p, w)) \in c \cap ((p \wedge q) >_i r) \cap p$, and so will be a $p \wedge q$-world and a $(p \wedge q) >_i r$-world and hence an r-world, so $p >_i (q >_i r)$ is true at $\langle c \cap ((p \wedge q) >_i r), f_i, f_s, w \rangle$, contrary to assumption. This reasoning turns crucially on the local indicative constraint, so nothing similar follows for subjunctives.

8 Conclusion

The only way to validate *Identity*, *IE*, *LCM* and *Ad Falsum* together is with the material conditional. This helps explain why every extant theory of the conditional which validates *IE*, other than the material analysis, invalidates *Identity*. *Identity*, however, appears to be valid. And I cannot see any case against *LCM* or *Ad Falsum*. So we must reject *IE*. This fits well with the empirical evidence in the case of subjunctives, where the existing literature contains concrete counterinstances to *IE*. But the same pairs of conditionals that constitute counterexamples in the subjunctive mood feel pairwise equivalent in the indicative mood. This makes it difficult to reject *IE* for indicatives. I have explored one way we might account for this situation: ascribe to indicative conditionals a presupposition which predicts that *IE* is Strawson valid, but not logically valid. That presupposition, the *local indicative constraint*, is independently motivated on the basis of unrelated contrasts between indicatives and subjunctives, and naturally accounts for the apparent differences in their logics which I have emphasized here.

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