# Attitude Semantics*

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**Abstract**

Many recent theories treat indicative conditionals as restricted necessity modals. I discuss two problems for this view. First, indicative conditionals do not behave like necessity modals in embedded contexts, e.g., under 'might' and 'probably': in these contexts, conditionals do not contribute a universal quantification over epistemic possibilities. Second, when we assess the probability of a conditional, we do not assess how likely it is that the consequent is *necessary* given the antecedent, but how likely it is that it is *true* given the antecedent. I propose an account which predicts the embedding behavior of conditionals under modals and the way we assign probabilities to conditionals. The account is based on the idea that the semantics of conditionals involves only a restriction of the relevant epistemic state, and no quantification over epistemic possibilities. The relevant quantification is contributed by an attitude parameter in the semantics, which is shifted by epistemic modals. If the conditional is asserted, the designated attitude is acceptance, which contributes a universal quantifier, producing the overall effect of a restricted necessity modal.

## 1 Introduction

**The Box View.** Many recent theories of indicative conditionals analyze them as restricted epistemic necessity modals. This family includes dynamic semantics accounts (Gillies, 2004, 2009; Willer, 2014, 2018; Starr, 2014) and other semi-dynamic information-based accounts (Yalcin, 2007; Gillies, 2009, 2010; Bledin, 2014).[1] The core idea is the following:[2]

*Box View.* The semantics of an indicative conditional $p \Rightarrow q$ involves two components:

1. Restriction: restrict the set of epistemic possibilities to the $p$-worlds;
2. Quantification: check that $q$ is true at every world in the restricted set.

The conditional is true/accepted iff the check in the second step is successful.[3]

This view is motivated by an intuitively compelling analysis of how indicative conditionals are assessed, known as the *Ramsey test view* (after Ramsey (1929)). The idea is that, in order to assess $p \Rightarrow q$ in an information state $s$, we proceed in two steps: first, we add $p$ to $s$, resulting in a hypothetical state $s[p]$; second, we check whether we accept the consequent $q$ in this hypothetical state; if so (and only in this case) we accept the conditional.

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[1] The restrictor account of Kratzer (1986) also fits broadly within this line, although due to its specific assumptions about the syntax of conditionals it needs to be discussed separately. We will do so in Section 4.

[2] In this paper I focus on indicative conditionals, although the puzzles that I will raise concern subjunctive conditionals as well. Proposal extends straightforwardly to the subjunctive case. However, making subjunctive assumptions involves a different process than making indicative assumptions; spelling out the details of this process is orthogonal to our present concerns; thus, I leave subjunctive conditionals out of consideration here.

[3] In some of the theories mentioned above, the view is implemented as giving truth-conditions for conditionals relative to a world with an associated set of epistemic possibilities; in other theories, conditionals lack truth-conditions, and the semantics delivers acceptance conditions relative to an information state.

In possible world semantics, an information state $s$ is typically modeled as a set of worlds—those worlds which are possible according to the available information. Adding $p$ to $s$ corresponds to restricting to $s$ to the $p$-worlds, obtaining the state $s[p] = s \cap |p|$. Checking whether we accept $q$ in the resulting hypothetical state amounts to checking whether $q$ is true in all the worlds in $s[p]$. Thus, the two steps of the Ramsey test procedure correspond exactly to the two components of the semantics of conditionals as postulated by the Box View.

The Box View of conditionals also makes several welcome empirical predictions. For instance, it explains why the discourse in (1) sounds contradictory:

(1) If Alice left, she went to London; #but it might be that she left and went to Paris.

In a discourse of the form $p \Rightarrow q; \Diamond(p \wedge \neg q)$, the conditional requires all $p$-possibilities to be $q$-possibilities, while the might-continuation requires the existence of $p \wedge \neg q$-possibilities. In spite of these attractions, however, the Box View also faces some important problems.

**Problem 1: Embedding.** Consider a conditional in the scope of an epistemic 'might':

(2) It might be that if Alice left she went to London. $\Diamond(p \Rightarrow q)$

Intuitively, (2) is a conditional possibility claim: it says that, among the epistemic possibilities where Alice left, there are some where she went to London. This is not what the Box View would lead us to expect. According to this view, (2) should be a second-order epistemic statement: it is possible that, relative to the worlds where Alice left, it is necessary that she went to London. This does not seem right. Moreover, (2) seems to mean just the same as (3).

(3) If Alice left, it might be that she went to London. $p \Rightarrow \Diamond q$

This, too, is puzzling from the point of view of the Box View: if conditionals contribute a universal quantification, then $\Diamond(p \Rightarrow q)$ should correspond to a $\exists\forall$ statement, while $p \Rightarrow \Diamond q$ should correspond to a $\forall\exists$ statement, so it is hard to see how the two could be equivalent. Indeed, in all theories cited above, the equivalence $\Diamond(p \Rightarrow q) \equiv p \Rightarrow \Diamond q$ is not predicted.[4]

This phenomenon is not restricted to epistemic 'might', but concerns epistemic modals quite generally. In particular, consider the case of 'probably'.

(4) It is probable that if Alice left she went to London. $\Pr(p \Rightarrow q)$

Again, what (4) expresses is not that it is probable that Alice going to London is epistemically necessary on the supposition that she left; rather, what it expresses is that Alice going to London is probable on the supposition that she left. This reading does not involve any epistemic necessity. Moreover, in this case too, (4) sounds fully equivalent to (5):

(5) If Alice left, it is probable that she went to London. $p \Rightarrow \Pr q$

It is hard to see how this commutation can hold if $\Rightarrow$ introduces a universal quantifier. Again, to my knowledge, no implementation of the Box View predicts this commutation.

Summing up, then, when embedded under epistemic modals, conditionals do not seem to contribute a universal quantification over epistemic alternatives. Moreover, conditionals seem to commute with epistemic modals. We would like an account that predicts these observations. On the Box View, it is far from clear how this can be achieved.

[4]For recent discussion of this puzzle, see Gillies (2018). Building on data semantics (Veltman, 1981) Gillies proposes a new analysis for 'might' and claims that 'might' is ambiguous between the standard and the revised analyses. However, the fact that exactly the same problem arises with other operators, like 'probably', suggests that it is not the semantics of 'might' which needs to be amended, but rather the semantics of conditionals.

**Problem 2: Probability.** Suppose I make the following claim:

(6) If the coin was tossed, it landed heads.

If the coin is fair and no other hint is available, it is natural to assign a probability of 50% to my claim. But this is not the probability that it is *epistemically necessary* that the coin landed heads if it was tossed. That probability is zero, since we are sure that it is *not* epistemically necessary that the coin landed heads if it was tossed. This is odd: if a conditional is a restricted epistemic necessity claim, its probability should just be the probability that this claim obtains.[5]

In general, it seems that the way in which we attribute probabilities to conditionals conforms to the thesis of Adams (1975), according to which the probability of a conditional $p \Rightarrow q$ equals the conditional probability of $q$ given $p$.[6] That is, when we estimate the probability of $p \Rightarrow q$, we do not estimate, as the Box View would have it, how likely it is that $q$ is *necessary* given $p$; instead, we estimate how likely it is that $q$ is *true* given $p$.

We would like to have an account of conditionals that explains why we behave in this way. That is, we would like an account that predicts that, given what a conditional means, and given a natural way to construe the notion of probability which is the subject of our intuitive judgments, the probability of $p \Rightarrow q$ just *is* the conditional probability of $q$ given $p$.

**Aim and structure of the paper.** In this paper, I propose an account of conditionals, modals, and probabilities that achieves the following desiderata. First, it accounts for the data that motivate the Box View. In particular, it predicts that a conditional $p \Rightarrow q$ is acceptable if and only if $q$ is acceptable on the supposition of $p$, vindicating the Ramsey test idea. And it explains the infelicity of (1). Second, it accounts for the peculiar way in which conditionals embed under modal operators. This is achieved without *ad-hoc* syntactic stipulations, in particular, without denying that conditionals can take scope with respect to modals. Finally, it predicts that the probability of $p \Rightarrow q$ is the conditional probability of $q$ given $p$. This result is obtained without *ad-hoc* stipulations about probabilities of conditionals. Rather, it is derived from the semantics of conditionals and a general definition of probability.

The paper is structured as follows: in Section 2 I present my theory, called Attitude Semantics (AS, for short, in the following); in Section 3 I show that AS provides a solution to the two problems discussed above; finally, in Section 4 I discuss similarities and differences with the restrictor theory of Kratzer (1986).

## 2 Attitude Semantics

**Language and models.** To present the theory explicitly, I will work with a formal language. The base layer of the language is a set $\mathcal{L}_0$ of *factual sentences*, whose semantics can be given in terms of truth-conditions relative to possible worlds. For our purposes, we may take $\mathcal{L}_0$ to be the language of propositional logic based on a finite set $\mathcal{P} = \{p, q, \dots\}$ of atomic sentences.

**Definition 1** (Factual language, $\mathcal{L}_0$)**.** $\alpha ::= p \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha$ where $p \in \mathcal{P}$

The full language $\mathcal{L}$ that I will work with is obtained by enriching $\mathcal{L}_0$ with operators designed to capture epistemic vocabulary: a binary operator $\Rightarrow$ for indicative conditionals; unary operators $\Box$ and $\Diamond$ for 'it must/might be that'; and a unary operator Pr for 'it is probable that'. To avoid

[5]See also Mandelkern (2018). On the more general point that our probabilistic assessment of conditionals contrasts with the hypothesis that they are universal modal claims, see DeRose (1994); Edgington (1995); Schulz (2014). For an analogous point about future discourse, see Belnap *et al.* (2001); Cariani and Santorio (2017).

[6]For an overview of the classical experimental literature, see Evans and Over (2004), §8.

further complications which are not essential to our concerns, we restrict to factual antecedents, and we do not consider Boolean compounds of epistemic sentences.[7]

**Definition 2** (Epistemic language, $\mathcal{L}$)**.** $\phi ::= \alpha \mid \alpha \Rightarrow \phi \mid \Box\phi \mid \Diamond\phi \mid \Pr\phi$ where $\alpha \in \mathcal{L}_0$

Semantically, I will work with a set $W$ of possible worlds. We can model a possible world as a valuation function $w : \mathcal{P} \to \{0, 1\}$. Notice that, since $\mathcal{P}$ is finite, the set $W$ of possible worlds is also finite.[8] The truth-value of a factual formula $\alpha$ at a world $w$, denoted $w(\alpha)$, is defined as usual. The proposition expressed by $\alpha$, denoted by $|\alpha|$, is the set of worlds where $\alpha$ is true.

**Information states and suppositions.** In order to spell out our semantics, we will need a formal notion of information states. Since we are concerned not just with qualitative notions, but also with probabilistic ones, I will take an information state to be a probability distribution on $W$. Since $W$ is finite, we can represent such a distribution simply as a map which assigns a probability to each possible world.

**Definition 3** (Information states)**.**
An information state is a function $s : W \to [0, 1]$ with the property that $\sum_{w \in W} s(w) = 1$.

A world $w$ is *ruled out* by an information state $s$ if it is assigned probability 0. Worlds which are not ruled out are referred to as the *live possibilities* in $s$.

**Definition 4** (Live possibilities)**.**
If $s$ is an information state, its set of live possibilities is $L(s) := \{w \in W \mid s(w) \neq 0\}$.

The probability of a proposition $X \subseteq W$ is just the probability that $X$ is true.

**Definition 5** (Probability of a proposition)**.** If $X \subseteq W$, $s(X) := \sum_{w \in X} s(w)$.

Next, we need a modeling of the process of making an indicative supposition. When we suppose $\alpha$ in a state $s$, we enter a new state $s[\alpha]$ in which $\alpha$ is treated as certain. Thus, all worlds in which $\alpha$ is false should be assigned probability 0. The relative probabilities of the $\alpha$-worlds are unaffected by the supposition, and should just be rescaled by a factor $1/s(|\alpha|)$ so that they sum up to 1 again. In other words, supposing can be modeled by the operation of *conditionalization.* If $\alpha$ is ruled out in $s$, i.e., if $s$ rules out all $\alpha$-worlds, then $\alpha$ is not consistently supposable in $s$.[9]

**Definition 6** (Supposing)**.** If $s$ is an information state and $\alpha$ a factual sentence with $s(|\alpha|) \neq 0$, then $s[\alpha]$ is the information state defined as follows:

$$s[\alpha](w) = \begin{cases} \frac{s(w)}{s(|\alpha|)} & \text{if } w \in |\alpha| \\ 0 & \text{if } w \notin |\alpha| \end{cases}$$

Notice that the set of live possibilities after the supposition is $L(s[\alpha]) = L(s) \cap |\alpha|$.

[7] The reason to restrict to factual antecedents is that it is just not clear how the process of supposing epistemic sentences works (though see Kolodny and MacFarlane, 2010, for a proposal that we may take on board). The reason not to look at compounds of epistemic sentences is that it is unclear, intuitively, how one should assign probabilities to sentences like $p \wedge (q \Rightarrow r)$ (Egré and Cozic, 2011). In both cases, the complications do not stem from the particular assumptions of our semantics.

[8] This assumption is not essential, although it simplifies the exposition.

[9] At least, not as an indicative assumption. One can suppose $\alpha$ as a subjunctive assumption, triggering a different kind of change. This matters for the interpretation of subjunctive conditionals, which we set aside here.

**Semantics.** Traditionally, semantics is about specifying truth-conditions for sentences in contexts. With much recent work (e.g. Dekker, 1993; Veltman, 1996; Yalcin, 2007; Willer, 2013; Hawke and Steinert-Threlkeld, 2016), we will assume that epistemic sentences, including indicative conditionals, differ essentially from factual sentences, which have truth-conditions relative to states of affairs. Rather, we will take such sentences to be devices for negotiating the epistemic attitude to be taken towards certain truth-conditional contents.[10] Accordingly, our semantics for the epistemic language $\mathcal{L}$ specifies what information it takes to bear a certain attitude to a sentence $\phi \in \mathcal{L}$. More specifically, the semantics will take the form of a ternary relation between an information state $s$, an attitude $A$, and a sentence $\phi$:

$$s \models_A \phi$$

The possible values for the attitude parameter $A$ include full acceptance (denoted $\forall$), compatibility (denoted $\exists$), and partial acceptance to degree $x \in [0, 1]$ (denoted $\pi_x$). When the word 'acceptance' is used without further qualification, what I mean is full acceptance.

**Definition 7.** The set of attitude parameters is $\mathsf{At} = \{\forall, \exists\} \cup \{\pi_x \mid x \in [0, 1]\}$

We now have all ingredients to recursively specify the semantics. For factual sentences $\alpha \in \mathcal{L}_0$ we have the following natural clauses:

- $s \models_\forall \alpha \iff s(|\alpha|) = 1$
- $s \models_\exists \alpha \iff s(|\alpha|) \neq 0$
- $s \models_{\pi_x} \alpha \iff s(|\alpha|) \geq x$

Notice that the clauses for full acceptance and compatibility can be rewritten in terms of the set of live possibilities associated with $s$:

- $s \models_\forall \alpha \iff L(s) \subseteq |\alpha|$
- $s \models_\exists \alpha \iff L(s) \cap |\alpha| \neq \emptyset$

The semantic role of an epistemic modal is that of indicating the attitude expressed towards the prejacent: acceptance for 'must', compatibility for 'might', and partial acceptance to a high degree for 'probably'. Formally, these operators work by shifting the attitude parameter:

- $s \models_A \Box\phi \iff s \models_\forall \phi$
- $s \models_A \Diamond\phi \iff s \models_\exists \phi$
- $s \models_A \mathrm{Pr}\,\phi \iff s \models_{\pi_{x_0}} \phi$ where $x_0 \in [0, 1]$ is a fixed threshold value

Finally, conditionals are interpreted by a Ramsey test clause: to bear an attitude to the conditional is to bear the attitude to the consequent, on the supposition of the antecedent. What if the antecedent is not supposable in the state? Then intuitively, the conditional cannot be assessed—we have a presupposition failure.[11] Due to space constraints, here I will not introduce presuppositions; I will just assume that, in order to bear any attitude to a conditional, the antecedent must be compatible with the evaluation state. Thus, the official clause is:

- $s \models_A \alpha \Rightarrow \phi \iff s \models_\exists \alpha$ and $s[\alpha] \models_A \phi$

Provided the antecent is supposable, this reduces to the following simpler clause:

- $s \models_A \alpha \Rightarrow \phi \iff s[\alpha] \models_A \phi$

[10] And for describing the properties of different bodies of information, as in "Alice believes it might rain." The present proposal can be extended to cover such occurrences, but I will set them aside here.

[11] It is standard in the literature to treat indicative conditionals as presupposing the epistemic possibility of their antecedent. See, a.o., Von Fintel (1998); Gillies (2009, 2010); Starr (2014); Willer (2014, 2018).

**Semantic equivalence.** If the semantics treats two sentences $\phi$ and $\psi$ in exactly the same way, we will say that $\phi$ and $\psi$ are semantically equivalent and write $\phi \doteq \psi$.[12]

**Definition 8** (Semantic equivalence)**.** $\phi \doteq \psi \iff \forall s \forall A : (s \models_A \phi \iff s \models_A \psi)$

**Assertion.** In the truth-conditional setting an assertion of $\phi$ is normally construed, along the lines of Stalnaker (1978), as a proposal to the conversational participants to accept the proposition expressed by $\phi$. In our setting, many sentences do not express propositions. However, the semantics gives us a notion of what it is to accept these formulas. Therefore, we do not need the detour through the proposition expressed. We can simply say that an assertion of $\phi$ is a proposal to the conversational participants to coordinate on a state which accepts $\phi$.

# 3 Predictions

**Recovering the predictions of the Box View.** Let us first show that some of the key predictions of the Box View are preserved on our account. First, consider the acceptance conditions of a conditional in a state where the antecedent is supposable. We have:

$$s \models_\forall p \Rightarrow q \iff s[p] \models_\forall q \iff L(s[p]) \subseteq |q| \iff L(s) \cap |p| \subseteq |q|$$

Thus, $\mathsf{AS}$ yields the same results as the Box View about the acceptance of factual conditionals: provided the antecedent is supposable, $p \Rightarrow q$ is accepted if and only if all live $p$-possibilities are $q$-possibilities. Since an assertion is a proposal to adopt a state in which the sentence is accepted, the effect of asserting $p \Rightarrow q$ is also in accordance with the Box View.

Notice also that the proposal fully vindicates the Ramsey test idea: accepting $p \Rightarrow q$ in a state $s$ amounts to accepting $p$ in the hypothetical state $s[p]$ resulting from the supposition of $p$.

We also predict the inconsistency of $p \Rightarrow q$ with $\Diamond(p \wedge \neg q)$, since we have $s \models_\forall \Diamond(p \wedge \neg q) \iff s \models_\exists p \wedge \neg q \iff L(s) \cap |p \wedge \neg q| \neq 0 \iff L(s) \cap |p| \not\subseteq |q|$. Thus, it is impossible for a state to accept simultaneously $p \Rightarrow q$ and $\Diamond(p \wedge \neg q)$.

**Solving the embedding problem.** Both attitude semantics and the Box View predict $p \Rightarrow q$ to be acceptable just when all the epistemically possible $p$-worlds are $q$-worlds. But there is a crucial difference: in $\mathsf{AS}$, the semantics of the conditional operator does not involve a universal quantification over epistemic possibilities. If we look back at the derivation of the acceptance conditions for $p \Rightarrow q$, we can see that the source of the universal quantification is the acceptance attitude, not the conditional operator. This makes a crucial difference: when interpreting a conditional embedded under an epistemic modal, the relevant attitude may be shifted away from acceptance; as a result, no universal quantifier will show up in the semantics. For instance, consider the acceptance conditions for a conditional embedded under 'might'. Again, let us focus on the interesting case in which the antecedent is supposable.

$$s \models_\forall \Diamond(p \Rightarrow q) \iff s \models_\exists p \Rightarrow q \iff s[p] \models_\exists q \iff \exists w \in L(s[p]) : w(q) = 1$$

Thus, we predict that $\Diamond(p \Rightarrow q)$ is not a second-order epistemic claim, but a simple claim of conditional possibility: $q$ is possible on the supposition that $p$. Moreover, $\Diamond$ commutes with $\Rightarrow$.

**Proposition 1.** $\Diamond(p \Rightarrow q) \doteq p \Rightarrow \Diamond q$

[12] Weaker notions, such as *acceptance equivalence* (having the same acceptance conditions) can also be defined. Although they are quite interesting, they are not relevant to the discussion below.

To see that this is the case, take any state $s$ and attitude $A$. We have:

$$\begin{aligned} s \models_A \Diamond(p \Rightarrow q) &\iff s \models_\exists p \Rightarrow q \\ &\iff s \models_\exists p \text{ and } s[p] \models_\exists q \\ &\iff s \models_\exists p \text{ and } s[p] \models_A \Diamond q \\ &\iff s \models_A p \Rightarrow \Diamond q \end{aligned}$$

The predictions about conditionals embedded under 'probably' are analogous: Pr shifts the attitude parameter to $\pi_{x_0}$, and thus the acceptance conditions for $\mathrm{Pr}(p \Rightarrow q)$ involve no universal quantification over epistemic possibilities. Instead, we get:

$$s \models_\forall \mathrm{Pr}(p \Rightarrow q) \iff s \models_{\pi_{x_0}} p \Rightarrow q \iff s[p] \models_{\pi_{x_0}} q \iff s[p](|q|) \geq x_0$$

Thus, $\mathrm{Pr}(p \Rightarrow q)$ is acceptable in case, in the state resulting from the supposition of $p$, the probability of $q$ is high. As we will see, this means that the conditional probability $s(q|p)$ is high. This is the intuitively correct prediction. Moreover, with a proof analogous to the one we saw for $\Diamond$, we can show that Pr and $\Rightarrow$ commute.

**Proposition 2.** $\mathrm{Pr}(p \Rightarrow q) \doteq p \Rightarrow \mathrm{Pr}\, q$

Thus, we have a solution to the embedding problem: we can explain why conditionals embedded under $\Diamond$ and Pr do not contribute a universal quantification over epistemic possibilities, although the acceptance conditions for unembedded conditionals involve such a quantification; and we can predict the commutation of epistemic modals with conditionals.

**Solving the probability problem.** How to characterize the probability $\mathbb{P}_s(\phi)$ of a sentence in our language relative to a state $s$? For factual sentences, this is clear: the probability of $\alpha \in \mathcal{L}_0$ is the probability that $\alpha$ is true:

$$\mathbb{P}_s(\alpha) = s(|\alpha|)$$

Conditionals—we are assuming—don't express propositions, so they cannot be assigned probabilities in this way. Still, as we discussed, it seems that we can meaningfully attribute probabilities to conditionals. Of course, we could simply follow Adams (1975) and stipulate that, for a factual conditional $\alpha \Rightarrow \beta$, its probability is just the conditional probability of the consequent given the antecedent. But this would not explain *why* we assign probabilities to conditionals in this way. Is there a sense in which, when we are estimating the probability of $p \Rightarrow q$, we are estimating *the same thing* as when we estimate the probability of $q$?

Attitude semantics allows us to define such a general notion: take the probability of a sentence $\phi$ in a state $s$ to be the highest degree to which $\phi$ is accepted in $s$ (if $\phi$ is not accepted in $s$ to any degree, let the probability be zero). The idea is that the probability of a sentence in a state is a measure of *how acceptable* the sentence is based on the available information.

**Definition 9** (Probability of a sentence)**.** $\mathbb{P}_s(\phi) := \sup_{[0,1]}\{x \mid s \models_{\pi_x} \phi\}$

Let us look at the predictions that this proposal makes. First, we can prove that the probability of a factual sentence is just the probability that the corresponding proposition is true.

**Proposition 3** (Probabilities of factual sentences)**.** If $\alpha \in \mathcal{L}_0$, $\mathbb{P}_s(\alpha) = s(|\alpha|)$

The proof is simple, since we have $\mathbb{P}_s(\alpha) = \sup\{x \mid s \models_{\pi_x} \alpha\} = \sup\{x \mid s(|\alpha|) \geq x\} = s(|\alpha|)$.

Next, let us look at a conditional $p \Rightarrow q$. Based on the semantics of conditionals and the definition of probability, we can now *prove* Adams thesis: in a state in which the antecedent is supposable, the probability of $p \Rightarrow q$ is the conditional probability of $q$ given $p$.

**Proposition 4** (Probabilities of conditionals)**.** Let $s$ be a state with $s(|p|) \neq 0$. Then:

$$\mathbb{P}_s(p \Rightarrow q) = s[p](|q|) = \frac{s(|p \wedge q|)}{s(|p|)}$$

*Proof.* $\mathbb{P}_s(p \Rightarrow q)$ is defined as the maximum $x$ for which $s \models_{\pi_x} p \Rightarrow q$. Since $s \models_{\pi_x} p \Rightarrow q \iff s[p] \models_{\pi_x} q \iff s[p](|q|) \geq x$, the maximum value of $x$ for which this holds is $s[p](|q|)$. This gives the first identity. As for the second identity, we have:

$$s[p](|q|) = \sum_{w \in |q|} P_{s[p]}(w) = \sum_{w \in |p| \cap |q|} \frac{s(w)}{s(|p|)} = \sum_{w \in |p \wedge q|} \frac{s(w)}{s(|p|)} = \frac{s(|p \wedge q|)}{s(|p|)}$$

Thus, now we can explain *why* the probability of a conditional is the conditional probability of the antecedent given the consequent in terms of our semantics of conditionals and our construal of probability. The reason is that all a conditional does is to restrict the evaluation state—and *not* introduce any quantification of its own: to accept $p \Rightarrow q$ to degree $x$ in state $s$ is just to accept $q$ to degree $x$ in the restricted state $s[p]$. Therefore, the probability of $p \Rightarrow q$ in $s$ is equal to the probability of $q$ in $s[p]$, and this is just the conditional probability of $q$ given $p$ in $s$.[13]

## 4 Comparison with the restrictor theory

According to the restrictor theory of conditionals (Kratzer, 1986) the embedding problem discussed above stems from a fundamental mistake about the syntax of conditionals: there is no operator $\Rightarrow$ corresponding to the 'if … then' construction in natural language. Rather, *if*-clauses spell out the restrictor of a modal operator. On this view, sentences (2) and (3) have exactly the same logical form, namely, $\Diamond_p q$. The restricted modal $\Diamond_p$ works like the original operator $\Diamond$, except that its domain is restricted to the $p$-worlds. In this way, the right interpretation for (2) is derived (unlike in AS, this is achieved by denying that (2) involves a modal embedding a conditional). The problem of accounting for the commutation of modals and conditionals also vanishes, since there is no conditional operator with respect to which the modal can take scope. Thus, the embedding problem is dissolved, or rather, turned into a problem of syntax-semantics interface. However, the probability problem still remains. Consider a plain conditional like (7):

(7) If the coin was tossed, it landed heads. $\Box_p q$

In this sentence, no modal occurs. In order to analyze it, the restrictor theory postulates that it contains a silent epistemic 'must', so its logical form is $\Box_p q$. Thus, (7) makes a certain epistemic claim: that the consequent is epistemically necessary given the antecedent. Then, its probability should be the probability that this claim is true. In the case of (7), in a prototypical

[13] On the idea that Adams' thesis can be explained in terms of the restricting role of if-clauses, see also Egré and Cozic (2011). The present work improves on that proposal in two ways. First, we give a general definition of probability of a sentence from which both the case of factual sentences and the case of conditionals follow as particular cases. Second, Egré and Cozic's approach deals with conditionals embedded under probabilistic modals (e.g., "There's a 50% chance that") but not with the probabilities of simple, unmodalized conditionals. Since Egré and Cozic build on Kratzer's restrictor view, they face the problem described in the next section.

scenario this will be zero: it is certainly not the case that the outcome heads is epistemically necessary given the toss. That is not the right prediction.[14] Another way to put the problem is to observe that, on the restrictor theory, (7) has exactly the same logical form as (8).

(8) If the coin was tossed, it must have landed heads. $\Box_p q$

Thus, (7) and (8) should be judged identically in terms of probabilities. This seems wrong: intuitively, (8) can be rejected with certainty; if we have to assign a probability to it, it would be 0.

Comparing AS to the restrictor theory, we find one key similarity and one key difference. The similarity is that, in both theories, conditional constructions do not contribute a quantifier. The source of quantification lies elsewhere. This is what allows both theories to predict, e.g., that (2) involves only an existential quantifier over possible worlds, and no universal quantifier.

The key difference lies in where the two theories locate the source of quantification: in the restrictor theory, the source of quantification is a modal operator; therefore, we need to assume that a modal operator is always present in a sentence involving conditionals, whence the need to postulate silent necessity modals in the logical form of bare conditionals. Besides lacking independent motivation, this stipulation is empirically problematic, since probability judgments show that bare conditionals are not assessed in the same way as conditionals containing an overt 'must'. In AS, by contrast, the source of quantification is the attitude parameter; while modal operators can shift this parameter, we need not assume that every sentence contains a modal operator. Thus, AS obviates the need for covert modals. If a sentence does *not* contain an epistemic modal that fixes the attitude parameter, the relevant quantification is not determined once and for all from within the sentence, but it is determined from the outside, by the attitude under consideration. If the sentence is asserted, the relevant attitude is full acceptance, which is responsible for introducing a universal quantification, producing the same effect as if the sentence had contained a 'must'. But if the sentence is assessed for probability, the process will not involve any universal quantification, and the result will differ from that of the corresponding 'must' sentence. This allows AS to provide a solution to the probability problem.

# References

Adams, E. (1975). *The logic of conditionals: An application of probability to deductive logic*, volume 86. Springer Science & Business Media.

Belnap, N., Perloff, M., and Xu, M. (2001). *Facing the future: agents and choices in our indeterminist world*. Oxford University Press.

Bledin, J. (2014). Logic informed. *Mind*, **123**(490), 277–316.

Cariani, F. and Santorio, P. (2017). Will done Better: Selection Semantics, Future Credence, and Indeterminacy. *Mind*, **127**(505), 129–165.

Dekker, P. (1993). *Transsentential Meditations. Ups and Downs in Dynamic Semantics*. Ph.D. thesis, ILLC, University of Amsterdam.

DeRose, K. (1994). Lewis on 'might' and 'would' counterfactual conditionals. *Canadian Journal of Philosophy*, **24**(3), 413–418.

[14] One might respond that, when asked to judge how probable (7) is, what we do is to make claims of the form $(\mathsf{P}^\alpha)_p q$, where $\mathsf{P}^\alpha$ is the modal "it is $\alpha$-probable that". But this essentially a restatement of the problem. Why would we do that, if such claims bear no relation to the probability of the statement we are asked to judge? For discussion of this point, see also Mandelkern (2018).

Edgington, D. (1995). On conditionals. *Mind*, **104**, 235–329.

Egré, P. and Cozic, M. (2011). If-clauses and probability operators. *Topoi*, **30**.

Evans, J. and Over, D. (2004). *If*. Oxford University Press.

Gillies, A. (2004). Epistemic conditionals and conditional epistemics. *Noûs*, **38**(4), 585–616.

Gillies, A. (2009). On truth-conditions for if (but not quite only if). *Philosophical Review*, **118**(3), 325–349.

Gillies, A. S. (2010). Iffiness. *Semantics and Pragmatics*, **3**, 4–1.

Gillies, A. S. (2018). Updating Data Semantics. *Mind*.

Hawke, P. and Steinert-Threlkeld, S. (2016). Informational dynamics of epistemic possibility modals. *Synthese*, pages 1–34.

Kolodny, N. and MacFarlane, J. (2010). Ifs and oughts. *The Journal of philosophy*, **107**(3), 115–143.

Kratzer, A. (1986). Conditionals. *Chicago Linguistics Society*, **22**(2), 1–15.

Mandelkern, M. (2018). Talking about worlds. *Philosophical Perspectives*, **32**(1), 298–325.

Ramsey, F. P. (1929). General propositions and causality. In H. Mellor, editor, *F. Ramsey, Philosophical Papers*. Cambridge University Press.

Schulz, M. (2014). Counterfactuals and Arbitrariness. *Mind*, **123**(492), 1021–1055.

Stalnaker, R. (1978). Assertion. *Syntax and Semantics*, **9**, 315–332.

Starr, W. (2014). A uniform theory of conditionals. *Journal of Philosophical Logic*, **43**(6), 1019–1064.

Veltman, F. (1981). Data semantics. In J. Groenendijk, T. Janssen, and M. Stokhof, editors, *Formal Methods in the Study of Language*. Mathematical Centre.

Veltman, F. (1996). Defaults in update semantics. *Journal of Philosophical Logic*, **25**(3), 221–261.

Von Fintel, K. (1998). The presupposition of subjunctive conditionals. *The interpretive tract*, **25**, 29–44.

Willer, M. (2013). Dynamics of epistemic modality. *Philosophical Review*, **122**(1), 45–92.

Willer, M. (2014). Dynamic thoughts on ifs and oughts. *Philosophers' Imprint*, **14**(28), 1–30.

Willer, M. (2018). Simplifying with free choice. *Topoi*, **37**(3), 379–392.

Yalcin, S. (2007). Epistemic modals. *Mind*, **116**(464), 983–1026.