Dowty's aspect hypothesis segmented

Tim Fernando

Trinity College Dublin, Ireland
Tim.Fernando@tcd.ie

Abstract

A form of the aspect calculus hypothesized in Dowty 1979 to explain "the different aspectual properties of the various kinds of verbs" is reinterpreted by chaining segments incrementally, as in Landman 2008. Segmenting an interval brings out, it is argued, a notion of event that can be represented at bounded but variable granularities by strings of sets of temporal propositions, tracking change. Simple accounts of telicity, durativity and composition interpreting Moens and Steedman 1988 are given in terms of such strings.

1 Introduction

The hypothesis from [Dow79] that "the different aspectual properties of the various kinds of verbs can be explained by postulating a single homogeneous class of predicates – *stative predicates* – plus three or four sentential operators and connectives" (page 71) has, over the years, profoundly influenced work on lexical aspect, in spirit, if not always in letter. Propositions are interpreted relative to interval world pairs, and a proposition φ said to be *homogeneous* when it holds at an interval world pair $\langle I, w\rangle$ precisely if it holds at $\langle \{t\}, w\rangle$ for every $t \in I$. Sentential operators are then applied for "a reductionist analysis of the aspectual classes of verbs" (page 71). Under a simplified reformulation from [Rot04] (page 35) that departs from the letter, if not the spirit, of the hypothesis, the operators DO, BECOME and CAUSE yield activities (1a), achievements (1b), and accomplishments (1c), with CAUSE reworked in (1c) using a culmination function Cul and a summation operation $\sqcup_S$ producing singular entities.

(1) a. activities $\lambda e.(\mathsf{DO}(\varphi))(e)$

b. achievements $\lambda e.(\mathsf{BECOME}(\varphi))(e)$

c. accomplishments $\lambda e.\exists e'[(\mathsf{DO}(\varphi))(e') \wedge e = e' \sqcup_S \mathsf{Cul}(e)]$

As an approximation of the accounts in [Dow79, Rot04], (1) is very rough, but serves to bring out events e, conspicuously absent in [Dow79]. It is clear from, say, the *eventuality structures* in [Lan08] that interpreting propositions as sets of interval world pairs (satisfying the propositions) does not preclude events. But we can test the reductionism in [Dow79] by asking: are events not *already* implicit in the interval world interpretation of propositions? The claim of the present paper is that they are, and that they can be made explicit along the "segmental" and "incremental axes" in [Lan08].

The segmental-incremental divide is used in [LR12] to refine the notion of homogeneity above (for stative φ) to an incremental notion for eventive predicates "sensitive to the arrow of time" (page 97). Turning to telicity, note the arrow of time differentiates the second half of a run to the post-office from its preceding half. Inasmuch as a run to the post-office is telic, and its second half (but not its first) is also a run to the post-office, it is problematic to equate telic predicates with quantized predicates ([Kri98]). Let us represent the arrow of time by a linear order $\prec$ on temporal instants, lifted to intervals I and I' universally for "whole precedence"

$$I \prec I' \iff (\forall t \in I)(\forall t' \in I')\ t \prec t'.$$

A sequence $I_1 \cdots I_n$ of intervals is a *segmentation of* an interval I if $\bigcup_{i=1}^{n} I_i = I$ and $I_i \prec I_{i+1}$ for $1 \leq i < n$ ([Fer13a]). That is, a segmentation of I is a finite partition of I ordered according to $\prec$. The point in partitioning an interval is to track changes within the interval in propositions, including statives that make achievements and accomplishments telic (by culminating).

The importance of choosing a suitable segmentation is illustrated by the rule (2), where we write $I \ \mathtt{m} \ I'$ to mean the sequence II' is a segmentation of $I \cup I'$[1]

$$I \ \mathtt{m} \ I' \iff I \prec I' \text{ and } I \cup I' \text{ is an interval.}$$

(2) $\quad I \ \mathtt{m} \ I'$ and $\langle I, w \rangle \not\models \varphi$ and $\langle I', w \rangle \models \varphi \implies \varphi$ becomes w-true between I and I'

One problem with (2) is that the interval I mentioned there may have a segmentation $I_1 I_2$ such that $\langle I_1, w \rangle \not\models \varphi$, but $\langle I_2, w \rangle \models \varphi$, in which case, under (2), φ becomes w-true between I_1 and I_2 (i.e., within I), as well as between I and I'. Such circumstances can be avoided through a negation $\neg$, called *predicate* negation in [Ham71] and *strong* negation in [AF94], with $\neg\varphi$ w-satisfied by intervals that have *no* subinterval w-satisfying φ

$$\langle I, w \rangle \models \neg\varphi \iff (\forall J \sqsubseteq I) \ \langle J, w \rangle \not\models \varphi.$$

(where the *subinterval* relation $\sqsubseteq$ is $\subseteq$ restricted to intervals). We then strengthen $\langle I, w \rangle \not\models \varphi$ in (2) to $\langle I, w \rangle \models \neg\varphi$ in (3).

(3) $\quad I \ \mathtt{m} \ I'$ and $\langle I, w \rangle \models \neg\varphi$ and $\langle I', w \rangle \models \varphi \implies \varphi$ becomes w-true between I and I'

Note that $\langle I, w \rangle \models \neg\varphi$ collapses to $\langle I, w \rangle \not\models \varphi$ if the interval I is (φ, w)-*homogeneous* in that I has *no* subintervals J and J' such that $\langle J, w \rangle \models \varphi$ and $\langle J', w \rangle \not\models \varphi$ — i.e., if

$$(\exists J \sqsubseteq I) \ \langle J, w \rangle \models \varphi \iff (\forall J \sqsubseteq I) \ \langle J, w \rangle \models \varphi.$$

Under the assumptions in (3), $I \cup I'$ cannot be (φ, w)-homogeneous, though I' could (along with I), which would mean φ is w-satisfied by every subinterval of I'. Lest change get buried in a non-homogeneous interval (such as $I \cup I'$ in (3)), we will chain together homogeneous intervals (such as I and I') for a segmentation (such as II') bringing out change.

Change is detected through propositions φ, which it will be useful to assume have certain properties commonly associated with states since [BP72] — the subinterval property

$$\langle I, w \rangle \models \varphi \text{ and } J \sqsubseteq I \implies \langle J, w \rangle \models \varphi$$

and the additive property

$$\langle I, w \rangle \models \varphi \text{ and } \langle I', w \rangle \models \varphi \implies \langle I \cup I', w \rangle \models \varphi \qquad \text{whenever } I \cup I' \text{ is an interval}$$

illustrated by the entailments (4) and (5), respectively.

(4) Ed slept from 1pm to 4pm $\implies$ Ed slept from 2pm to 3pm

(5) Ed slept from 1pm to 4pm and from 2pm to 5pm $\implies$ Ed slept from 1pm to 5pm

Combining these properties, let us say φ is *w-segmented* if for all intervals I and I' whose union $I \cup I'$ is an interval,

$$\langle I, w \rangle \models \varphi \text{ and } \langle I', w \rangle \models \varphi \iff \langle I \cup I', w \rangle \models \varphi.$$

This is somewhat weaker than the notion of homogeneity for statives φ in [Dow79] (mentioned in the opening paragraph above), under which φ is *w-pointwise* in that for every interval I,

$$\langle I, w \rangle \models \varphi \iff (\forall t \in I) \ \langle \{t\}, w \rangle \models \varphi.$$

It turns out that if φ is w-segmented,

[1] In [Ham71], $\mathtt{m}$ is *abutment on the left*; in [KR93] *abutment*; and in [AF94] *meet*.

(R1) a segmentation of I consists of (φ, w)-homogeneous intervals iff it w-tracks φ in I

and if moreover, φ is w-pointwise,

(R2) an interval I has a segmentation into (φ, w)-homogeneous subintervals iff φ does not change its w-truth value infinitely often in I

where the right hand sides of (R1) and (R2) are made precise in the next section, section 2. (R1) reinforces the view that segmentations into (φ, w)-homogeneous intervals are the way to represent φ's changes in w, whereas (R2) describes the price to be paid for such representations. But why should we be interested in these representations? Because from representations of change, we can expect to extract representations of events that are parts of that change. To carry this out, we will work with not just one w-segmented proposition φ but a set X of such. We shall come under pressure to assume X is finite, and discover that X provides a useful notion of granularity that is bounded but refinable. Aspectual notions such as telicity and durativity will focus our attention in section 3 on the propositions in X, reducing, for every world w,

(i) an interval I to the set

$$X_w(I) \;=\; \{\varphi \in X \mid \langle I, w\rangle \models \varphi\}$$

of propositions $\varphi \in X$ w-satisfied at I, and

(ii) a segmentation $I_1 \cdots I_n$ to its (X, w)-*diagram*, the string

$$X_w(I_1 \cdots I_n) \;=\; X_w(I_1) \cdots X_w(I_n)$$

over the alphabet 2^X of subsets of X.

For a fixed granularity X, we represent events and reconstruct the connectives DO and BECOME directly in terms of strings in $(2^X)^+$. Events can be conceived as truthmakers [Dav67] (page 91), relative to a notion $\models_w$ of satisfaction between a segmentation $I_1 \cdots I_n$ and a string $\alpha_1 \cdots \alpha_m \in (2^X)^+$ requiring that $m = n$ and $\langle I_i, w\rangle$ satisfy each proposition in α_i

$$I_1 \cdots I_n \models_w \alpha_1 \cdots \alpha_m \iff n = m \text{ and } (\forall i \in [1, n])(\forall \varphi \in \alpha_i)\ \langle I_i, w\rangle \models \varphi$$

(where $[i, j]$ is the set of integers $\geq i$ and $\leq j$). The world parameter w is largely inactive throughout (sitting idly along for the ride). In fact, working with strings, we can dispense with worlds at the outset, constructing as many as we wish from a notion of branching time that enriches the incremental axis of [Lan08]. But to link up with [Dow79] and the tradition from which it sprang, we have no choice but to keep the worlds around.

Section 2 provides model-theoretic justification for a string-based approach to a form of the aspect calculus of [Dow79] close to [MS88]. This approach is taken up in section 3 and can be understood, up to a point, without section 2's justification.[2] Minimizing the technical fuss, we can summarize section 2 as follows. Relative to a set Φ of propositions and its family

$$Fin(\Phi) \;=\; \{X \subseteq \Phi \mid X \text{ is finite}\}$$

of finite subsets, an interval world pair $\langle I, w\rangle$ meeting assumptions largely familiar from [Dow79] is represented by a $Fin(\Phi)$-indexed family $\{s_X\}_{X \in Fin(\Phi)}$ of strings $s_X \in (2^X)^*$ that picture $\langle I, w\rangle$ up to granularity X. Suitable substrings of the strings s_X represent (up to granularity X) events that happen within the interval world pair pictured. Section 3 bases its account of telicity and durativity on a simple choice of X. That account can be refined by expanding X.

[2] Indeed, a variant of section 3 is described in [Fer13a] without the benefit of Propositions 1 and 2 of section 2. Instead, [Fer13a] locates segmented propositions in an adjunction with (so-called) whole propositions, applying that adjunction to the imperfective/perfective divide (analyzed as viewpoint aspect).

2 From segmentations to strings

Attending promptly to (R1), let us track a w-segmented proposition φ in I through segmentations of I satisfying the following definition. A segmentation $I_1 \cdots I_n$ of I is (φ, w)-*fine* if for every subinterval I' of I, φ holds at $\langle I', w\rangle$ precisely if I' is covered by components I_i that w-satisfy φ

$$\langle I', w\rangle \models \varphi \iff I' \subseteq \bigcup\{I_i \mid i \in [1,n] \text{ and } \langle I_i, w\rangle \models \varphi\}.$$

Proposition 1. *Let φ be a w-segmented proposition, and $\mathbb{I} = I_1 \cdots I_n$ be a segmentation of I.*

(a) $\mathbb{I}$ is (φ, w)-fine iff $(\forall i \in [1,n])$ I_i is (φ, w)-homogeneous.

(b) If $\mathbb{I}$ is (φ, w)-fine, then so is any segmentation that is a finer partition of I.

(c) If $\mathbb{I}$ is (φ, w)-fine, then so is any segmentation into $n-1$ subintervals obtained from $\mathbb{I}$ by merging two components I_i and I_{i+1} such that

$$(\langle I_i, w\rangle \models \varphi \text{ and } \langle I_{i+1}, w\rangle \models \varphi) \quad \text{or} \quad (\langle I_i, w\rangle \not\models \varphi \text{ and } \langle I_{i+1}, w\rangle \not\models \varphi).$$

An easy corollary of Proposition 1(c) is that if I has a (φ, w)-fine segmentation, it has a (φ, w)-fine segmentation $J_1 \cdots J_k$ such that

$$\langle J_i, w\rangle \models \varphi \iff \langle J_{i+1}, w\rangle \not\models \varphi \qquad \text{for every } i \in [1, k-1]$$

and this is the coarsest (and shortest) of all (φ, w)-fine segmentations of I. But if we want a (φ, w)-fine segmentation of I that is also (φ', w)-fine for a different w-segmented proposition φ', the coarsest (φ, w)-fine segmentation may not do (making Proposition 1(b) relevant).

Collecting w-segmented propositions of interest into a set X, let us call a segmentation (X, w)-*fine* if it is (φ, w)-fine for every $\varphi \in X$. When does an interval have an (X, w)-fine segmentation? Clearly, a necessary condition is that *no* φ in X alternate between w-true and w-false infinitely often in I. More precisely, let us call a sequence $I_1 \cdots I_n$ of subintervals of I a (φ, w, n)-*alternation in* I if for all $i \in [1, n-1]$, $I_i \prec I_{i+1}$ and

$$\langle I_i, w\rangle \models \varphi \iff \langle I_{i+1}, w\rangle \not\models \varphi.$$

An interval I is (X, w)-*stable* if for every $\varphi \in X$, there is an integer $n > 0$ such that *no* (φ, w, n)-alternation in I exists. The necessary condition that I be (X, w)-stable is also sufficient for finite X, provided we assume that each $\varphi \in X$ is not only w-segmented but (as [Dow79] does for statives) w-pointwise, making (R2) from the introduction precise in

Proposition 2. *Fix an interval I, world w and finite set X of w-pointwise propositions.*

(a) I has an (X, w)-fine segmentation iff I is (X, w)-stable.

(b) If I is (X, w)-stable, then there is a coarsest (and shortest) (X, w)-fine segmentation of I.

To describe the coarsest (X, w)-fine segmentation of an (X, w)-stable interval, it is helpful to define a segmentation $I_1 \cdots I_n$ to be (X, w)-*compressed* if for all $i \in [1, n-1]$,

$$X_w(I_i) \neq X_w(I_{i+1})$$

where (as mentioned in the introduction) $X_w(I_i)$ is the set of propositions in X that $\langle I_i, w\rangle$ satisfy. Given an (X, w)-fine segmentation $I_1 \cdots I_n$ of I, we can merge contiguous blocks $I_i I_{i+1} \cdots I_{i+j}$ of components that w-satisfy the same propositions in X

$$X_w(I_i) = X_w(I_{i+1}) = \cdots = X_w(I_{i+j})$$

to form an (X, w)-compressed segmentation refined by $I_1 \cdots I_n$, which is, furthermore, the coarsest (and shortest) (X, w)-fine segmentation of I.

Focusing on strings, including the (X, w)-diagram $X_w(I_1) \cdots X_w(I_n)$ of a segmentation $I_1 \cdots I_n$, let us call a string $\alpha_1 \cdots \alpha_k$ *stutterless* if for all $i \in [1, k-1]$, $\alpha_i \neq \alpha_{i+1}$. Clearly, a segmentation is (X, w)-compressed iff its (X, w)-diagram is stutterless. Moreover, we can express the formation above of an (X, w)-compressed segmentation in terms of the *block compression* $\mathit{bc}(s)$ of a string s, compressing all contiguous blocks α^{j+1} of a symbol α to α

$$\mathit{bc}(s) \;=\; \begin{cases} \mathit{bc}(\alpha s') & \text{if } s = \alpha\alpha s' \\ \alpha\ \mathit{bc}(\beta s') & \text{if } s = \alpha\beta s' \text{ with } \alpha \neq \beta \\ s & \text{otherwise} \end{cases}$$

so that $\mathit{bc}(s)$ is stutterless. Next, we fix an arbitrary (possibly infinite) set Φ of w-segmented propositions, and picture a Φ-stable pair $\langle I, w\rangle$ through a function $\Delta_{I,w}$ with domain the set $\mathit{Fin}(\Phi)$ of finite subsets of Φ such that for every finite $X \subseteq \Phi$, $\Delta_{I,w}(X)$ is the (X, w)-diagram of the coarsest (X, w)-fine segmentation of I. The functions $\Delta_{I,w}$ belong to the *inverse limit* $\mathfrak{IL}(\Phi)$ *of* the $\mathit{Fin}(\Phi)$-indexed family $\{\mathit{bc}_X\}_{X \in \mathit{Fin}(\Phi)}$ of functions $\mathit{bc}_X = \rho_X; \mathit{bc} : (2^\Phi)^* \to (2^X)^*$ mapping $s \in (2^\Phi)^*$ to the block compression $\mathit{bc}(\rho_X(s))$ of the X-*restriction* $\rho_X(s)$ intersecting each component of s with X

$$\rho_X(\alpha_1 \cdots \alpha_k) \;=\; (\alpha_1 \cap X) \cdots (\alpha_k \cap X).$$

An example is (6), where boxes are drawn instead of curly braces $\{,\}$ for sets as symbols.

(6) $\quad \mathit{bc}_{\{\mathrm{E}\}}(\boxed{\mathrm{E}}\boxed{\mathrm{E,R}}\boxed{\mathrm{E}}) \;=\; \mathit{bc}(\rho_{\{\mathrm{E}\}}(\boxed{\mathrm{E}}\boxed{\mathrm{E,R}}\boxed{\mathrm{E}})) \;=\; \mathit{bc}(\boxed{\mathrm{E}}\boxed{\mathrm{E}}\boxed{\mathrm{E}}) \;=\; \boxed{\mathrm{E}}$

For the record, $\mathfrak{IL}(\Phi)$ is the set of functions f with domain $\mathit{Fin}(\Phi)$ such that

$$f(X) = \mathit{bc}_X(f(Y)) \quad \text{whenever } X \subseteq Y \in \mathit{Fin}(\Phi).$$

Φ-stable pairs $\langle I, w\rangle$, all of which are represented in $\mathfrak{IL}(\Phi)$, vary wildly, giving $\mathfrak{IL}(\Phi)$ an intensional dimension. *Incremental inclusion* $\subseteq_i$ from [Lan08] on intervals I and I'

$$I \;\subseteq_i\; I' \quad\Longleftrightarrow\quad I \sqsubseteq I' \text{ and not } (\exists t' \in I')\ \{t'\} \prec I$$

can be rendered into strings s and s' as the prefix relation

$$s \ \mathsf{prefix}\ s' \quad\Longleftrightarrow\quad (\exists s'')\ ss'' = s'$$

construing s within a string $s\alpha$ as the past of α. We lift prefix to $\mathfrak{IL}(\Phi)$, building into an irreflexive relation $\prec_\Phi$ on $\mathfrak{IL}(\Phi)$ the prefix requirement at every granularity X

$$f \prec_\Phi f' \quad\Longleftrightarrow\quad f \neq f' \text{ and } (\forall X \in \mathit{Fin}(\Phi))\ f(X)\ \mathsf{prefix}\ f'(X).$$

As noted in [Fer13a], $\prec_\Phi$ is tree-like ([Dow79]); i.e. transitive and left linear: for all $f \in \mathfrak{IL}(\Phi)$,

$$f_1 \prec_\Phi f_2 \quad\text{or}\quad f_2 \prec_\Phi f_1 \quad\text{or}\quad f_1 = f_2 \qquad \text{whenever } f_1, f_2 \prec_\Phi f.$$

No element of $\mathfrak{IL}(\Phi)$ is $\prec_\Phi$-maximal, which is to say all intervals I that form Φ-stable pairs (I, w) can be extended further, whether or not they are conceived within w to stretch through an entire timeline. That said, some small part of $\mathfrak{IL}(\Phi)$ may suffice for a specific purpose. In particular, a fixed $f \in \mathfrak{IL}(\Phi)$ may do, inducing at every granularity $X \in Fin(\Phi)$ an interpretation of $\varphi \in X$ from $f(X) = \alpha_1 \cdots \alpha_n$, with temporal instants $t \in [1, n]$ and interval satisfaction

$$[i, j] \models \varphi \iff (\forall t \in [i, j])\ \varphi \in \alpha_t$$

whenever $1 \leq i \leq j \leq n$. Worlds are absent in this interpretation, and, borrowing a phrase from [Kah11], what you see is all there is (WYSIATI).[3]

3 Composing transitions, telicity and durativity

We now step from propositions (interpreted over intervals) that represent states to strings $\alpha_1 \cdots \alpha_n$ of length $n > 1$ (interpreted over segmentations) that represent events with precondition α_1 and postcondition α_n. We build on [MS88] for (7) and (8) below.[4]

(7)

	non-durative	durative
telic	achievement $\boxed{\neg\varphi}\boxed{\varphi}$	accomplishment $\boxed{\neg\varphi}\boxed{\neg\varphi,\psi}\boxed{\neg\varphi,\psi}^{+}\boxed{\varphi}$
atelic	semelfactive $\boxed{}\boxed{\psi}$	activity $\boxed{}\boxed{\psi}\boxed{\psi}^{+}$

(8) a. $\alpha_1 \cdots \alpha_n$ is *durative* if its length n is ≥ 3

b. $\alpha_1 \cdots \alpha_n$ is *telic* if there is some φ in α_n such that for all $i \in [1, n-1]$, the negation $\neg\varphi$ of φ appears in α_i

c. $\mathsf{iterate}_;(\boxed{}\boxed{\psi}) \ = \ \boxed{}\boxed{\psi}\boxed{\psi}^{+}$

d. $\boxed{}\boxed{\psi}\boxed{\psi}^{+} \ ; \ \boxed{\neg\varphi}\boxed{\varphi} \ \approx \ \boxed{\neg\varphi}\boxed{\neg\varphi,\psi}\boxed{\neg\varphi,\psi}^{+}\boxed{\varphi}$

(8a) and (8b) give simple definitions of durative and telic strings, examplified in (7). (8c) reduces an activity in $\boxed{}\boxed{\psi}\boxed{\psi}^{+}$ to the iteration of the semelfactive $\boxed{}\boxed{\psi}$, while (8d) decomposes an accomplishment into an activity and achievement, not unlike (1c) in the opening paragraph above (with $\sqcup_S$ as ;). (8c) and (8d) depend on operations, ; and $\mathsf{iterate}_;$, defined in (9). In (9a), β and α range over subsets of some fixed set Φ of propositions, while s and s' range over strings in $(2^\Phi)^*$, including the empty string ϵ mentioned in (9b). L and L' in (9b) and (9c) are languages, understood as subsets of $(2^\Phi)^*$.

(9) a. $s\beta \ ; \ \alpha s' \ = \ s(\beta \cup \alpha)s'$

b. $L; L' \ = \ \{s; s' \mid s \in L - \{\epsilon\} \text{ and } s' \in L' - \{\epsilon\}\}$ (regular if L and L' are)

c. $\mathsf{iterate}_;(L) \ = \ $ the $\subseteq$-least set Z such that $L; L \subseteq Z$ and $Z; L \subseteq Z$

Observe that ; is essentially string concatenation, except that the last symbol β of the first string is merged with the first symbol α of the second string. The rationale behind this overlap is a notion of inertia that is expressed, for instance, in page 49 of [Com76]

[3] More about (and around) Propositions 1 and 2 in [Fer13b].

[4] (7) here is comparable to Figure 1, page 17 of [MS88], and (8) to Figure 2, page 18, with non-durative $\rightsquigarrow$ atomic, durative $\rightsquigarrow$ extended, telic $\rightsquigarrow$ +conseq, atelic $\rightsquigarrow$ $-$conseq, $\varphi \rightsquigarrow$ consequent state, $\psi \rightsquigarrow$ progressive state, and achievement/accomplishment/semelfactive/activity $\rightsquigarrow$ culmination/culminated process/point/process.

unless something happens to change [a] state, then the state will continue

together with the assumption that in $s; s'$, nothing happens between the end of s and the start of s'. That same notion of inertia accounts for the discrepancy behind the wavy $\approx$ in (8d)

$$\boxed{\ }\boxed{\psi}\boxed{\psi}^{+} \;;\; \boxed{\neg\varphi}\boxed{\varphi} \;=\; \boxed{\ }\boxed{\psi}^{+}\boxed{\neg\varphi,\psi}\boxed{\varphi} \;\neq\; \boxed{\neg\varphi}\boxed{\neg\varphi,\psi}\boxed{\neg\varphi,\psi}^{+}\boxed{\varphi}$$

with $\neg\varphi$ spreading (within the accomplishment) back from the achievement $\boxed{\neg\varphi}\boxed{\varphi}$. But then why doesn't ψ in the activity $\boxed{\ }\boxed{\psi}\boxed{\psi}^{n}$ also spread? Quoting more fully from [Com76], page 49

> With a state, unless something happens to change that state, then the state will continue ... With a dynamic situation, on the other hand, the situation will only continue if it is continually subject to a new input of energy.

The reason $\neg\varphi$ flows in (8d) while ψ does not is that $\neg\varphi$ represents a state in $\boxed{\neg\varphi}\boxed{\varphi}$, whereas in an activity $\boxed{\ }\boxed{\psi}\boxed{\psi}^{n}$, the proposition ψ represents a dynamic situation that persists only if forced (moving loosely from a scalar, energy, to a vector, force). A precise mechanism for regulating inertial flow in the absence and presence of force is described in [Fer08], under which $\neg\varphi$ is inertial, whereas ψ is not.

But just what is ψ? For an answer, it is instructive to cast $\boxed{\neg\varphi}\boxed{\varphi}$ as $\boxed{\ }\boxed{\hat{\varphi}}$ with $\hat{\varphi}$ set to $\varphi \wedge \langle \mathtt{mi} \rangle \neg\varphi$, where $\langle \mathtt{mi} \rangle$ is the diamond modal operator given by the inverse $\mathtt{mi}$ of meet $\mathtt{m}$

$$\langle I, w \rangle \models \langle \mathtt{mi} \rangle \chi \iff (\exists I' \;\mathtt{m}\; I)\; \langle I', w \rangle \models \chi$$

so that $\hat{\varphi}$ describes a dynamic situation

$$\langle I, w \rangle \models \varphi \wedge \langle \mathtt{mi} \rangle \neg\varphi \iff (\exists I' \;\mathtt{m}\; I)\; I'I \models_w \boxed{\neg\varphi}\boxed{\varphi}.$$

A segmentation cannot w-satisfy $\boxed{\ }\boxed{\hat{\varphi}}^{n}$ for $n \geq 2$ any more than it can w-satisfy a string in $\boxed{\neg\varphi}\boxed{\neg\varphi,\varphi}^{+}\boxed{\varphi}$, which under (9c), is $\mathsf{iterate}_{;}(\boxed{\neg\varphi}\boxed{\varphi})$. But we can modify $\hat{\varphi}$ so that strings in the set $\boxed{\ }\boxed{\psi}\boxed{\psi}^{+} = \mathsf{iterate}_{;}(\boxed{\ }\boxed{\psi})$ are w-satisfiable. One way is through a linearly ordered set D of degrees to which a function $\deg^{D}_{\varphi,w}$ maps temporal instants. Propositions $d < \varphi$-deg and φ-deg $\leq d$ are then interpreted over intervals by universal quantification (as with whole precedence $\prec$)

$$\langle I, w \rangle \models d < \varphi\text{-deg} \iff (\forall t \in I)\; d < \deg^{D}_{\varphi,w}(t)$$

$$\langle I, w \rangle \models \varphi\text{-deg} \leq d \iff (\forall t \in I)\; \deg^{D}_{\varphi,w}(t) \leq d$$

(conflating, for simplicity, semantic entities d, $<$ and $\leq$ to the left of $\models$ with their syntactic representations to the right of $\models$). It is natural to assume a contextually given threshold $\hat{d}$ reducing φ to $\hat{d} < \varphi$-deg. Existentially quantifying degrees, we put

$$\varphi^{D}_{\uparrow} \;=\; (\exists d \in D)\; (d < \varphi\text{-deg} \wedge \langle \mathtt{mi} \rangle \varphi\text{-deg} \leq d)$$

and stretch instantaneous change $\boxed{\neg\varphi}\boxed{\varphi}$ to a more graduated sequence $\boxed{\ }\boxed{\varphi^{D}_{\uparrow}}^{n}$ associating an increasing sequence $d_1 < d_2 < \cdots < d_n$ of degrees with a segmentation $I_0 I_1 \cdots I_n$

$$I_0 I_1 \cdots I_n \models_w \boxed{\ }\boxed{\varphi^{D}_{\uparrow}}^{n} \iff (\exists d_1 \in D) \cdots (\exists d_n \in D)(\forall i \in [1, n])$$
$$(\forall t \in I_{i-1})(\forall t' \in I_i)\; \deg^{D}_{\varphi,w}(t) \leq d_i < \deg^{D}_{\varphi,w}(t').$$

But is $\varphi_{\uparrow}^{D}$ w-segmented? Not necessarily. Let us say that $\deg_{\varphi,w}^{D}$ *is increasing within* an interval I if for all $t, t' \in I$, if $t \prec t'$ then $\deg_{\varphi,w}^{D}(t) < \deg_{\varphi,w}^{D}(t')$. Given $\langle I, w\rangle \models \varphi_{\uparrow}^{D}$, one can show

$$\begin{aligned}\deg_{\varphi,w}^{D} \text{ is increasing within } I &\iff (\forall J \sqsubseteq I)\ \langle J, w\rangle \models \varphi_{\uparrow}^{D} \\ &\iff \langle I, w\rangle \models [\sqsupseteq]\varphi_{\uparrow}^{D}\end{aligned}$$

where $[\sqsupseteq]$ is the necessity operator induced by the inverse of the subinterval relation. The proposition $[\sqsupseteq]\varphi_{\uparrow}^{D}$ can be assumed to be w-segmented, whereas $\varphi_{\uparrow}^{D}$ cannot (unless interpreted against say, a string supporting WYSIATI, as described in the previous section). It is not entirely obvious that as a representation of a dynamic situation, the proposition ψ in an activity $\boxed{}\boxed{\psi}\boxed{\psi}^{n}$ should be required to be w-segmented. What is clear from the lively literature on aspectual composition is that there is much more to say about what ψ to box (not to mention how to do so compositionally from syntactic input) under the scheme (7) – (9) or some revision thereof. The strings in (7) are simple examples of how to flesh out (8) and (9) based on a minimal set X of propositions that we can, under the inverse limit $\mathfrak{IL}(\Phi)$ in section 2, expand indefinitely, refining granularity (to express, for instance, forces and grammatical aspect).[5]

References

[AF94] James F. Allen and George Ferguson. Actions and events in interval temporal logic. *J. Logic and Computation*, 4(5):531–579, 1994.

[BP72] Michael Bennett and Barbara Partee. Toward the logic of tense and aspect in English. Technical report, System Development Corporation, Santa Monica, California, 1972. Reprinted in Partee 2008.

[Com76] Bernard Comrie. *Aspect*. Cambridge University Press, 1976.

[Dav67] Donald Davidson. The logical form of action sentences. In Nicholas Rescher, editor, *The Logic of Decision and Action*, pages 81–95. University of Pittsburgh Press, 1967.

[Dow79] David R. Dowty. *Word Meaning and Montague Grammar*. Reidel, Dordrecht, 1979.

[Fer08] Tim Fernando. Branching from inertia worlds. *J. Semantics*, 25(3):321–344, 2008.

[Fer13a] Tim Fernando. Segmenting temporal intervals for tense and aspect. In *13th Meeting on the Mathematics of Language (MoL 13)*, pages 30–40. ACL Archive, 2013.

[Fer13b] Tim Fernando. Partitions representing change homogeneously. Manuscript (5 pages), 2013.

[Ham71] Charles Leonard Hamblin. Instants and intervals. *Studium Generale*, 24:127–134, 1971.

[Kah11] Daniel Kahneman. *Thinking, Fast and Slow*. Farrar, Straus and Giroux, 2011.

[KR93] Hans Kamp and Uwe Reyle. *From Discourse to Logic*. Kluwer, 1993.

[Kri98] Manfred Krifka. The origins of telicity. In Susan Rothstein, editor, *Events and Grammar*, pages 197–235. Kluwer, 1998.

[Lan08] Fred Landman. 1066: On the differences between the tense-perspective-aspect systems of English and Dutch. In Susan Rothstein, editor, *Theoretical and Crosslinguistic Approaches to the Semantics of Aspect*, pages 107–166. John Benjamins, 2008.

[LR12] Fred Landman and Susan Rothstein. The felicity of aspectual for-phrases, part 2: Incremental homogeneity. *Language and Linguistics Compass*, 6(2):97–112, 2012.

[MS88] Marc Moens and Mark Steedman. Temporal ontology and temporal reference. *Computational Linguistics*, 14(2):15–28, 1988.

[Rot04] Susan Rothstein. *Structuring Events*. Blackwell, Oxford, 2004.

[5]Not a single durative string in (7) is stutterless. Such a string cannot make it into $\mathfrak{IL}(\Phi)$ on its own, but can do so as part of a stutterless string, for example, $\boxed{}\boxed{\psi, R}\boxed{\psi}$ with a Reichenbachian reference time R.